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TEACHING APPLIED MATHEMATICS FOR ENGINEERS – A NEW TEACHING PARADIGM BASED ON INDUSTRIAL MATHEMATICS

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INTRODUCTION

My interest in pedagogical aspects of engineering mathematics arose when I was working as a scientist in a Finnish chemical company called Kemira. My role was to help the engineers to solve problems of mathematical nature in chemical and biotechnical engineering. In the early years of my career I was astonished to notice the lack of understanding, even in some very elementary mathematical concepts, and the inability to describe problems in mathematical terms. Astonishingly enough, some engineers seemed to be almost proud of not having needed mathematics in their works. Yet, all the engineers I worked with had studied a huge amount of mathematics, at least in terms of time spent for it, typically 12 years in school and 2 to 3 years at university. Why were the results so poor, is mathematics really so difficult, or are we not teaching it in the right way? Could we do something differently? One of the objectives of this lecture is to discuss these questions.

Another objective of this lecture is to discuss whether the present engineering mathematics courses meet with the challenges emerging from applied industrial mathematics, and if not, what kind of changes are needed. Especially, we shall discuss the role of numerical mathematics, multivariate methods and applied statistics.

The ideas in this lecture are based on 10 years experience in teaching engineering mathematics at EVTEK University of Applied Sciences, mostly to students whose major subject is chemical or biotechnical engineering. However, my firm belief, partly based on discussions with teachers and students of other fields of engineering, is that many of these ideas and conclusions can be generalized into these other fields as well. The generality of the conclusions is naturally somewhat limited by the fact that my experience is mostly based on the European educational systems.

OBJECTIVES OF ENGINEERING MATHEMATICS

Before discussing pedagogical aspects or contents of engineering mathematics we have to be aware of the objectives of studying mathematics. We also have to know what a typical engineer expects to learn, what he is really interested in. For a mathematically oriented person, or a becoming mathematician, mathematics is interesting in itself. He or she is fascinated by the beauty, generality and logic of mathematics. An engineer thinks differently: he or she wants to solve engineering problems and mathematics is just a tool among others. In order to gain permanent results, one has to be able to motivate the engineering students that mathematics is an essential tool in solving real problems.

It is certainly true that many real engineering problems of industrial origin lead to mathematics that is beyond the mathematical abilities and skills of an average engineer. In such cases, a project team of engineers needs to consult a professional applied mathematician for a successful completion of their project. However, such cooperation is bound to fail if the engineers do not know in what kind of sub-problems mathematics can help, or if they are not able to communicate with the mathematician due to the ignorance of mathematical terms or methods. Of course, there is a similar demand for the mathematician of having some understanding of the physical, chemical or biological and engineering aspects of the problem in question.

The Problems in The Traditional Way of Teaching Mathematics for Engineers

Still, in most universities, mathematics and some other subjects, e.g. physics, are taught almost solely during the first two years of engineering studies. After that, it is assumed that the student has achieved such a mastery of mathematical skills that he or she is able to apply them in the engineering subjects of the following years. This sounds good, and it would be ideal if it worked - but it doesn't. And the less there are practical examples within the math courses, linking the theory to real problems – or, to be more accurate, simplified real life problems, the less this kind of pedagogy works. Why is that so? The main reason is very simple: only a very small minority of people is able to model problems in mathematical terms without practicing. Just as a simple example, consider that of a student, who just has learned the definition of the derivative. It is very unlikely that he is able to apply it e.g. in physics without practicing it by examples of increasing complexity. But there are other reasons as well. Probably the one of the greatest importance is the question of motivation. A typical engineering student is not well motivated to study abstract mathematics, rather he or she would like to be convinced that what he or she is doing is useful in engineering problems – and very few people are convinced only for the reason that the mathematics teacher says that this will be useful in the future. A good reason is also the fact that many practically oriented students, who may become very good engineers, understand mathematical concepts best by practical examples.

Are there alternative ways of teaching mathematics for engineers? This will be discussed within the question of a need to change also the contents of mathematics courses.

The Nature of Industrial Engineering Mathematics

In the introduction we discussed how the recent development in computer technology has influence on the relevance of different branches of industrial and engineering mathematics. Let us look closer at some important aspects that also have influence on how to teach math.

The principle of parsimony

The main goal in solving industrial engineering problems is to get a solution, as quickly as possible and at as small expenses as possible. Of course, before any decisions, also an estimate of reliability of the solution is needed. This leads to the conclusion that the problem should be solved by the simplest possible method. The problem is that the academic world does not emphasize this kind of thinking. Academic research is looking forward to new methods, which usually are more complicated than the old ones. Naturally, the professors want to make publications in which the novelty of methods is a typical requirement. As a consequence, most graduate and especially PhD theses involve the newest most, and often complicated methodology. Exaggerating slightly it can be said that, if a simple method solves a problem quickly and cost effectively, it does not have academic value. Of course, academic research has to produce new methodology, but it is important make it clear to the engineering student what is important in industry. However, one must differentiate between the simplicity of methods and simplicity in thinking: a simple method combined with simple thinking leads to erroneous conclusions, but insightful thinking, i.e. mathematical understanding, helps to find the simplest method.

To be able to show this to the students, exercises allowing solutions of different complexity levels are very important in teaching. The following examples illustrate the importance of simple mathematical methods combined with mathematical understanding.

Example 1: Quite often an engineer is looking for correlations in experimental data. Figure 1 depicts a typical case where the dependent variable is y and the explanatory variables are x_1 and x_2. The question is: can we conclude that y tends to increase along with both x_1 and x_2?

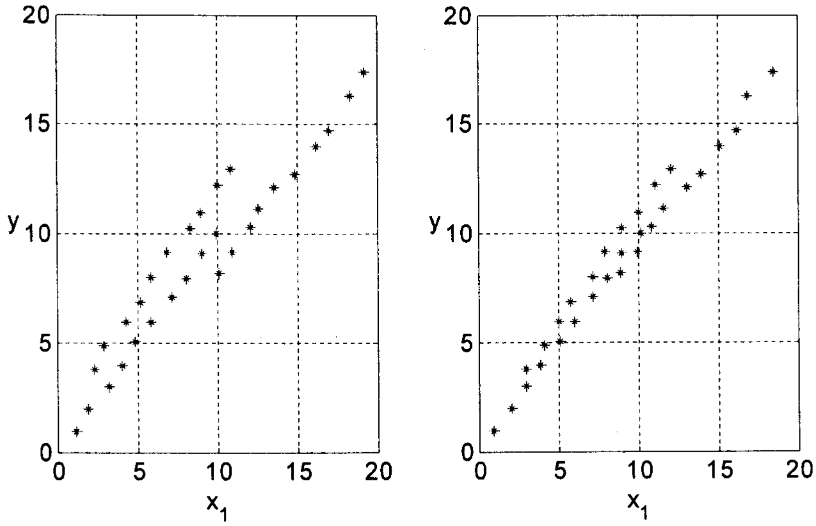

Figure 1

If the engineer hasn't studied any statistics or statistical design of experiments the probable answer is affirmative, which in this case is wrong. This would probably be the case even if he or she would look at the original data table. Actually, these data have been calculated using a model $y = 0.25 - 0.75x_1 + 1.75x_2 + E$, where E is a normally distributed random number, and thus y decreases along with x_1. An engineering with some knowledge of basic statistics would immediately realize that the reason is the correlation between x_1 and x_2, commonly exhibited in real process data. The true dependencies could of course be easily detected by simple multiple linear regression, or by some more sophisticated graphical tools.

Example 2: Consider a case where 5 absorbances at different wavelengths have been measured from 21 food oil samples. The task is to find out whether any of them is a fake that would cause exceptional ratios in absorbances. The table of arbitrarily scaled absorbances is given below.

Oil nr	1. absorbance	2. absorbance	3. absorbance	4. . absorbance	5. . absorbance
1	6.99	7.07	2.92	10.82	11.48
2	7.47	6.68	3.38	10.53	11.75
3	9.30	4.63	5.39	8.95	12.67
4	7.52	6.59	3.45	10.45	11.77
5	10.38	5.30	5.72	9.49	13.57
6	5.79	7.71	1.91	11.31	10.74
7	6.21	6.70	2.63	10.52	10.85
8	4.92	7.26	1.61	10.95	10.03
9	9.88	4.66	4.85	8.96	12.03
10	7.07	6.40	3.27	10.30	11.41
11	8.58	5.12	4.75	9.32	12.25
12	8.17	5.20	4.47	9.38	11.97
13	11.02	4.44	6.49	8.82	13.87
14	8.98	5.67	4.73	9.75	12.64
15	9.13	5.37	4.95	9.52	12.69
16	7.12	7.01	3.02	10.78	11.56
17	5.12	6.97	1.86	10.72	10.12
18	10.86	4.40	6.41	8.79	13.74
19	9.06	5.07	5.05	9.29	12.59
20	7.77	5.71	4.00	9.77	11.78
21	4.87	7.64	1.41	11.24	10.07

Now, it is quite obvious that the task is not easy just by looking at the numbers, neither by elementary graphical tools, e.g. simple scatter plots. Actually, the problem is really difficult unless one is familiar with elementary multivariate methods. On the other hand, it is very easy if one just knows the simplest multivariate method, the principal component analysis (PCA). Figure 2 shows the so-called score plot of the second and third principal components.

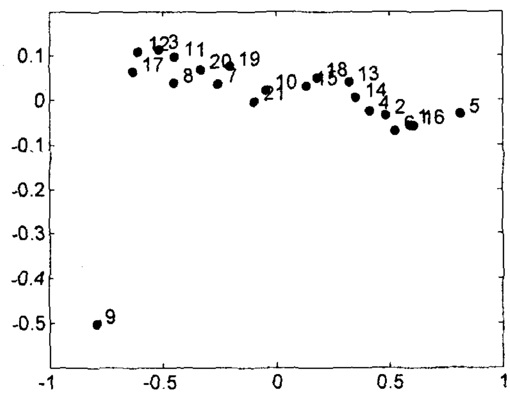

Figure 2

From this figure it is obvious that the oil number 9 is different from all the other and consequently possibly a fake oil. Such tasks of finding outliers, or groups in general, in multivariate data, also in process data, are quite common in industrial problems. In many cases they can be solved with such simple methods as the one above.

The importance of modeling

Modeling or mathematical formulation is the first step in all mathematical problems of industrial origin. In spite of this obvious fact, modeling has a minor role in the elementary courses of engineering mathematics. As a consequence, the students typically have difficulties in separating the tasks of formulating a problem and solving the problem. Yet, to solve any complicated enough problem, it is essential first to formulate the problem, i.e. to get a model and after that, to analyze and classify the model. After classifying the model, the choice of efficient tools for solving the problem is usually quite straightforward. The following simple optimization example illustrates these aspects.

Example 3: This is an exercise that is given to the first chemical and biotechnology engineering students. To formulate the problem, only elementary mathematics is needed, but to solve the problem accurately, some basic knowledge of optimization is required. However, an approximate solution can be easily found e.g. by a grid search. The problem is given in the following form.

The product (B) of a chemical factory is produced in a batch reactor at constant temperature. The raw material (A) is fed into the reactor and it is let to react a given time (t). After this, the reactor is emptied and the product is purified. The reaction is assumed to be a first order chemical reaction, i.e. $A(t) = A_0 e^{-kt}$ and $B(t) = A_0 - A(t)$, where $A(t)$ and $B(t)$ are the concentrations at time t and A_0 is the initial concentration of the raw material. The reaction rate constant t depends on the temperature T according to the Arrhenius equation $k = A_f e^{\frac{}{R(273.15+T)}}$, where A_f is the frequency factor E_a is the activation energy and R is the gas constant.

The income from the product is T_p € per produced amount of B (m_B). The costs are K_0 € per batch and the purification costs are K_p € per the amount of A (m_A) left after reaction time t. The costs due to the heating of the reactor are $K_T T^2$ € and the time dependent costs during the reaction are $K_t\, t$ €. The time between two consecutive batches depends linearly on m_A, i.e. on $t_0 + k_t\, m_A$. The volume of the reactor is V.

The task is to optimize the yearly profit (= income - costs) with respect to the temperature (T) and the reaction time (t). The batches can be produced without breaks assuming 330 operating days per year.

The values for the parameters are (the units of A_f, E_a and R are compatible, when time is given in hours):

$A_f = 10^{12}$	$E_a = 90000$	$R = 8.3$	$A_0 = 0.1$ kg/l
$V = 1000$ l	$T_p = 100$ €/kg	$K_p = 10$ €/kg	$K_0 = 2000$ €
$K_T = 0.2$ € / °C^2	$K_t = 100$ €/h	$t_0 = 2$ h	$k_t = 0.5$ h/kg

For a mathematician, approaching the problem in the right way is not difficult, i.e. by first forming a composed function of T and t for both income and costs, and then classifying and solving the optimization task. However, for a beginning engineering student it is not easy. It is very typical that he or she is mixed up with solution and formulation. The idea of first building a model of the problem is something that has to be learned by practicing and purely mathematical problems, unrelated to real problems, or practising only mechanical calculation, simply is not enough.

Such examples should also be an essential ingredient of elementary engineering mathematics courses. As a consequence, the elementary engineering mathematics courses cannot be faculty independent. Mass lectures of general mathematics will not serve purpose. It is also very challenging for the mathematics teacher: in order to make up interesting and motivating examples, he or she should be familiar with the kind of engineering the students are educated for.

Numerical vs. symbolic math

Due to the fast development of both computer hardware and mathematics software, the role of applied mathematics in solving industrial problems has increased. Industrial problems typically lead to complicated, large-scale problems, which can be solved only numerically by computers, emphasizing the role of numerical methods and matrix algebra. It is quite typical in engineering curricula that numerical mathematics is taught in separate courses. A more efficient way is to include numerical methods as an essential part of the elementary engineering mathematics courses. For example, numerical differentiation should be taught together with teaching the concept of a derivative and differentiation rules.

The importance of multivariate methods

Contemporary automatic process monitoring systems and modern analytical instruments produce inherently multivariate data. In interpreting such data, both conventional statistical and multivariate statistical methods have a key role. In biotechnical engineering, the emergence of bioinformatics has increased the need of understanding multivariate methods. Also process data analysis, one of the most common problems in industrial mathematics, is essentially an application of multivariate methods. Many of the speech and image processing techniques are based on multivariate methods as well. Here again, the principle of parsimony is good to remember. The emphasis should be in the simplest, yet powerful methods, such as the principal component analysis (PCA). The following industrial example shows the power of PCA in solving a quality problem.

Example 4: A factory produces a granulated product. The problem was that the particle size distribution of the product varied too much. However, in spite of a lot of effort, the reason of the variation remained unclear. Finally, it was decided to look at the size distributions using PCA score plots. Each size distribution of the daily average of the product was treated as an 8-dimensional vector, and each such vector was projected onto a plane of the first two principal components. This plot is shown in figure 3.

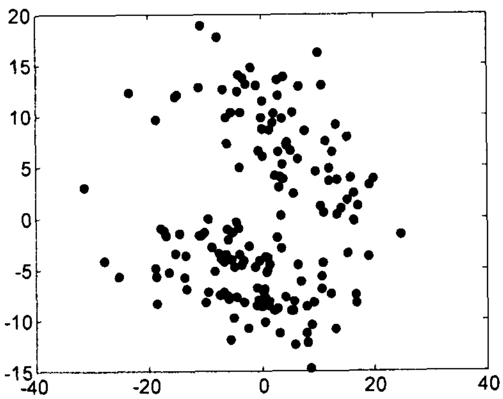

Figure 3

The score plot showed clear grouping into two almost distinct groups. A detailed analysis showed that every 4 to 5 consecutive points, i.e. projections of distributions, belonged to either of the groups. At first, nobody in the factory could explain the peculiar grouping, but finally a production engineer remembered that there were actually two identical granulation units in the process line and that the unit had to be cleaned after 4 to 5 days of operation. Now the reason for the unwanted variation was clear: although the units should have been identical and they were controlled identically, they produced a product of different size distributions. After this, the solution was obvious; the other granulation unit just had to be controlled differently.

Naturally, one might claim that it should have been obvious to suspect the cause of the problem to be in the granulation units. However, the periodicity in the size distributions did not show up in the mean particle size, nor in the standard deviation or in any other univariate measure. In addition, the similarity of the units was taken for granted. Thus, the problem might not have been solved without using multivariate methods.

For an engineer to be able to understand and apply multivariate methods, it is important to introduce the basics of multivariate mathematics at as early stage of studies as possible. This is possible in a natural way in connection with vector algebra and systems of linear equations. The student should find a 10 or 1000 dimensional vector as natural as an ordinary 2 or 3 dimensional vector. Again, it is a challenge for the mathematics teacher to provide motivating examples of high dimensional vectors related with real engineering problems.

The importance of statistical methods

Most industrial engineering problems involve use and analysis of measurement data. Without proper understanding of measurement uncertainty and statistical nature of such data, truly meaningful conclusions can hardly be drawn. Statistical methods are essential also in designing new processes and products of high quality. Nowadays, most big companies use statistically based quality policies, such as Six Sigma. Consequently, a course in statistical design of experiments (DOE) should be included in all fields of engineering. The same holds also for statistical process control (SPC), an area where new interesting multivariate techniques have been developed, e.g. multivariate SPC. However, all that is impossible without a good knowledge of the basics: probability distributions, confidence intervals, estimation, the logic of statistical testing, regression analysis etc. But again, efficient and motivating teaching of the basics is impossible without simple examples that are related to real engineering problems.

ENHANCING MOTIVATION

In the previous chapters, we have mainly discussed the kind of mathematics that is needed in order to be able to solve typical engineering problems of modern industrial processes. It has also been stated that a typical engineer, or engineering student, is not much interested in mathematics in itself, rather he or she sees it as a tool to serve quite practical purposes. However, to be able to apply mathematics a good understanding of the basic mathematical concepts and the 'language' of mathematics are necessary. To obtain such understanding and knowledge requires hard work and a lot of effort. This, in turn, requires motivation and a firm belief that the effort is worthwhile. The three basic ingredients of enhancing motivation are: 1) good examples in the field of the main subjects of the students, 2) integration of mathematics studies with physics, chemistry, biology, computer science and engineering subjects and 3) use of mathematical software already in the early stages of studies. Let us have a closer look at these topics.

Making up good examples is far from being easy. The minimum requirements of a good example are: *it illustrates the mathematical concept that is being taught*, it is interesting, it is simple, but still it is related to a real problem. At least part of the examples should lead to numerical methods, to statistical aspects and to questions of the correct degree of simplification.

Integration of different studies is even more difficult. However, it is possible in small steps, if just the professors and lecturers are open-minded and willing to do it. Key ingredients of any success are interdisciplinary conversations and good timing. Especially important is that not all mathematics is taught in the first two years. An ideal way would be that if an engineering subject requires some special mathematics, a suitable mathematics course is given concurrently. In many cases this may be impossible in practice, but it is a goal worth aiming at. Some mathematical subjects are especially suitable for the 1 or 2 last years of the studies. Such is, for example, statistical design of experiments, whose proper understanding requires a lot of experience in experimentation and measurements. The famous applied statistician George Box has once stated that nobody should study statistics before having worked for at least 5 years after studies. This might be exaggerating the matter, but it contains the essential idea.

The use of mathematical software, properly done, allows the student to focus on understanding instead of training skills that are not any more important. However, the right balance is difficult to achieve. It is not always obvious what kind of mechanical training actually is helpful for the understanding. Let us take, for example matrix multiplication. If the student has understood the dot product of vectors and its geometric interpretation, it is possible to apply matrix multiplication in several fields ant there is not much sense to practice matrix multiplication by hand. Similarly, inverting a matrix by hand does not help in understanding what the matrix inverse is. For that purpose the analogy between ordinary linear equations and systems of linear equations is much better. Using computers allows taking up more realistic examples already during the first mathematics courses. The use of computers also makes it easier to give good examples of how the problems are divided into to the steps of modeling, solving and assessment of the results.

ENHANCING UNDERSTANDING

Finally, we shall discuss what kind of mathematical understanding and knowledge form the basics on which applications can be built on. In a way, learning to apply mathematics is to learn a new language and, as in learning any new language, one has to learn both the vocabulary and the syntax of forming meaningful sentences. The very basics of the language of mathematics are algebra and geometry. Learning any more advanced mathematical concepts or techniques is impossible without fluent mastery of algebra. This fact is often neglected, and the engineering students entering a university are typically assumed to know more than they actually do. Learning algebra resembles learning a language in the sense that it requires a lot of repetition. The required fluency is a product of a massive amount of practicing which does not end in the school.

An equally important matter is to find a balance between learning mechanical mathematical skills and understanding the key concepts. The former is especially important in learning solution techniques and the latter in problem formulation (modeling). Let us take an example, the derivative. If a beginning engineering student is asked to explain what a derivative of a function is, typical answers are usually related to differentiation rules, i.e. to the skills and not to the concept. Yet, to be able to apply the derivative, understanding the concept is what counts, not being able to differentiate elementary functions. Of course the latter is an important skill in solving problems but useless in applications without understanding what the derivative is. It is important to realize that numerical methods can be useful, not only in solving problems where analytical methods fail, but also in gaining understanding. For example, simple numerical differentiation based on the difference quotient is good practice for understanding what the derivative really is (of course emphasizing that it is not applicable for differentiation using measured values of a function). Naturally, geometrical interpretations are good for this purpose as well. Another good example is the concept of a function. The students typically have great difficulties in learning to form (program) their own functions in any mathematical software. The problem is in the abstractness and the generality of the concepts. Therefore, teaching such concepts should not be separated from programming or numerical methods and these should be taught simultaneously.

Finally, one should not forget that an important part of the mathematical understanding is to realize that intuition can be very misleading, especially in probability and in multivariate dependencies. Examples serving also this purpose, such as example 1 here, should be included whenever applicable.

One of the consequences of teaching mathematics that is not related to real problems containing measured quantities is that students start to think of mathematical models as absolute truths. Though it is important to understand the rigorous nature of mathematical reasoning, it is equally important to understand that mathematical models are only approximate descriptions of real phenomena. Again, the best way to achieve this understanding is to show good examples, preferably such that the students can conduct the experiments themselves for comparing the model with the experimental data.

Example 5: The students are asked to model the cooling of a beer can in cold water using the Newton's law for cooling. After finding the correct ordinary differential equation, they have to solve it. After this, they have to conduct an experiment and to estimate the thermal resistance using the measured temperature profile. Finally, they are asked to discuss the goodness of the model, and to give explanations for the differences between the modeled and observed temperature in the can.

This is a typical example that combines modeling, experimentation, statistical analysis and consequences of the simplifications made in the model.

SUMMARY

What is the "new paradigm"? It is impossible express it in one or two words, but if one had to; the closest might be the "holistic approach". The expression can be justified by the fact that the conclusions above lead to a greater intermixing of mathematics with engineering and natural sciences subjects, typically expressed in the form of examples of simplified real problems. They also lead to a greater intermixing of subjects within mathematics so that the courses should have less separation e.g. between symbolic and numerical mathematics.

The conclusions also lead to the spreading the mathematics courses throughout all study years, not just the first two years. Of course, this should be done with great care in order to guarantee studies that are logically linked together.

The new paradigm also means that the needs arising from industrial mathematics must be taken into account in the contents of engineering mathematics courses. Such topics are e.g. multivariate methods, statistics and use of mathematical software.

What are we expected to gain from the paradigm shift? The primary benefit should be in obtaining more productive engineers equipped with a better degree of mathematical preparedness for engineering problems. But in addition, it should also promote more intensive use of applied mathematics and easier communication with professional mathematicians, often needed in complicated industrial problems.

Finally, it can be noted that the new paradigm is in harmony with the basic ideas of the CDIO (Conceive – Design – Implement – Operate) initiative for producing the next generation of engineers [1]. New ideas for engineering education can be found also in the homepage of SEFI (European Society for Engineering Education) [2].

1. http://www.cdio.org/

2. http://www.sefi.be/