

Clarifying Warhead Separation from the Reentry Vehicle Using a Novel Tracking Algorithm

Cheng-Yu Liu and Yu-Ming Sung

Abstract: Separating a reentry vehicle into warhead and body is a conventional and efficient means of producing a huge decoy and increasing the kinetic energy of the warhead. This procedure causes the radar to track the body, whose radar cross section is larger, and ignore the warhead, which is the most important part of the reentry vehicle. However, the procedure is difficult to perform using standard tracking criteria. This study presents a novel tracking algorithm by integrating input estimation and modified probabilistic data association filter to solve this difficulty in a clear environment. The proposed algorithm with a new defined association probability in this filter provides a good tracking capability for the warhead ignoring the radar cross section. The simulation results indicate that the errors between the estimated and the warhead trajectories are reduced to a small interval in a short time. Therefore, the radar can produce a beam to illuminate to the right area and keep tracking the warhead all the way. In conclusion, this algorithm is worthy of further study and application.

Keywords: Input estimation, probabilistic data association filter, reentry vehicle, tracking algorithm, trajectory estimation.

1. INTRODUCTION

Separation of the reentry vehicle (RV) into the warhead and the body is a simple and common way to confuse radar and increase a warhead's damage capacity by producing two sets of measurements from a radar beam. The radar estimates and predicts the target's trajectory at the next sampling period from the measurement with the higher signal to noise ratio and forms a beam to illuminate it. The body is then tracked and intercepted first, since its radar cross section is inevitably larger than that of the warhead. The warhead, which is the most significant part of the reentry vehicle, becomes a new track or is completely ignored all the way. This problem is difficult to solve unless the standard tracking criteria is modified.

If a warhead does not alter its trajectory during separation, then it typically follows closely the original trajectory with slightly higher speed and lower drag force. However, for the body, the cylinder

configuration causes its speed to drop rapidly to zero. Then, the innovation with respect to the warhead, given by the difference between the estimated and warhead trajectories, is smaller than that with respect to the body. This value is an important hint for designing an useful algorithm. Hence, the tracking problem concerns quickly generating a reliable innovation to determine which set of measurements originates from the RV in the track. A reliable innovation is calculated by an on-line precise trajectory estimation approach. The origination of the set of measurement is based on data association technique.

The on-line estimation of the trajectory of an RV is very important for radar tracking. The main problems related to trajectory estimation relate to model validation, due to model error between the mathematical model and physical system. The model error is normally the result of the simplifying assumptions, maneuvering and unpredictable external forces during flight, parameter uncertainty, and other sources. The extended Kalman filter (EKF) is a well-known and helpful state estimation scheme for a nonlinear dynamic system, but fails to reach the required accuracy in a short time. Input estimation (IE) provides a good solution to this problem. IE has been successfully employed to estimate inputs for solving tracking [1-4] and inverse heat conduction problems [5-7]. Lee and Liu presented a filter associating EKF with IE to handle model validation problems and provided an accurate trajectory estimation approach

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for the RV [8]. Their proposed filter has lower innovation and estimation errors than the original EKF, and hence can be applied to this problem.

Bar-Shalom and Tse designed a suboptimal Bayesian algorithm, probabilistic data association filter (PDAF) [9], for tracking a single target in a cluttered environment, which addresses all radar returns including clutters. The predicted and updated states for all returns weighted by a *posteriori* probability, known as association probability, formed the combined states of the target. The association probability indicates the probability of each return originating from the object in track. The PDAF was successfully utilized in sonar and radar systems to increase their tracking capacity [10,11] and in other fields [12-15]. Recalling the problem faced herein, IE and PDAF can provide an accurate innovation and idea for obtaining the combined state from these two sets of measurement. However, the clutter seldom appears in high elevation angle region where is the main search area for detecting the RV. For simplicity, it is properly to develop a method, namely modified probabilistic data association filter (MPDAF), to detect the RV ignoring the clutter.

This study defines the MPDAF and links with the EKF and IE to form an algorithm to estimate and predict the warhead trajectory from two sets of measurements. Section 2 presents a problem statement given in to elucidate the problem and to formulate dynamic equations for the RV, warhead, and body. Section 3 outlines the IE algorithm. Section 4 presents the MPDAF in a clear environment with the redefined association probability for each measurement set, and the flow chart of the proposed filter. Section 5 presents the simulation analysis of the proposed algorithm in different cases, which demonstrate that the proposed approach perform well, and presents its future applications.

2. DYNAMIC EQUATIONS

Consider a vehicle in the reentry phase over a flat and nonrotating earth as illustrated in Fig. 1. Assume the RV to be a point mass with constant weight following a ballistic trajectory in which two significant forces, drag and gravity, act on the RV. Extra forces are induced by model error when assumptions are violated or the RV undertakes a maneuver. The RV trajectory model in radar coordinate (O_R, X_R, Y_R, Z_R) centered at the radar site can be written as

$$\dot{v}_x = -\frac{\rho v^2}{2C} g \cos \gamma_1 \sin \gamma_2 + u_4, \quad (1)$$

$$\dot{v}_y = -\frac{\rho v^2}{2C} g \cos \gamma_1 \cos \gamma_2 + u_5, \quad (2)$$

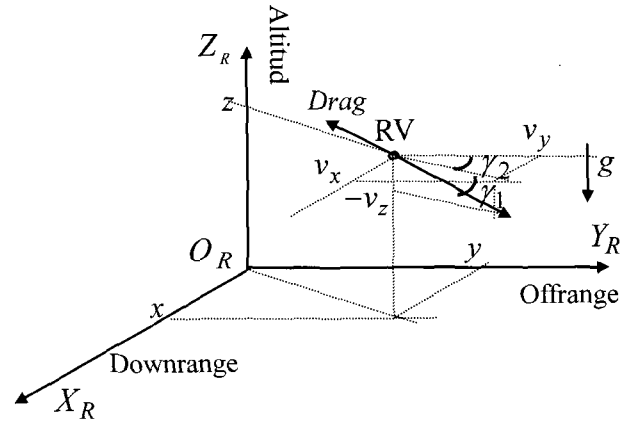


Fig. 1. Reentry vehicle flight geometry.

$$\dot{v}_z = \frac{\rho v^2}{2C} g \sin \gamma_1 - g + u_6, \quad (3)$$

with position initial conditions $x(0)$, $y(0)$, $z(0)$ and velocity initial conditions $v_x(0)$, $v_y(0)$, and $v_z(0)$ in X_R , Y_R and Z_R , respectively. In this model, C denotes the ballistic coefficient,

$$C = \frac{W}{SC_{D0}},$$

$$\gamma_1 = \tan^{-1}\left(-\frac{v_z}{\sqrt{v_x^2 + v_y^2}}\right), \quad \gamma_2 = \tan^{-1}\left(\frac{v_x}{v_y}\right),$$

v means the total velocity of the RV, v_x , v_y , and v_z express velocity components along X_R , Y_R , and Z_R , respectively; u_4 , u_5 , and u_6 are unmodeled accelerations generated by the model errors along each axis; C_{D0} , S and W denote zero-lift drag coefficient, reference area and, weight respectively. ρ stands for air density and is a function of altitude [16]. The well known normal gravity g model is extensively used because the RV normally flies over heights of several hundred kilometers [17].

The RV separates into two objects, the warhead and the body, at a given altitude or a certain time. The warhead moves toward to a spot near the RV's destination, along a slight different trajectory from that of the original RV. The body then falls on its own rapidly after several seconds. The difference among the equations of motion for the RV, warhead, and body is only in the ballistic coefficient. Equations of motion for the warhead with the ballistic coefficient C_w after separation can be expressed as

$$\dot{v}_{wx} = -\frac{\rho v_w^2}{2C_w} g \cos \gamma_{w1} \sin \gamma_{w2} + u_{w4}, \quad t > t_s, \quad (4)$$

$$\dot{v}_{wy} = -\frac{\rho v_w^2}{2C_w} g \cos \gamma_{w1} \cos \gamma_{w2} + u_{w5}, \quad t > t_s, \quad (5)$$

$$\dot{v}_{wz} = \frac{\rho v_w^2}{2C_w} g \sin \gamma_{w1} - g + u_{w6}, \quad t > t_s \quad (6)$$

with position initial conditions $x_w(t_s)$, $y_w(t_s)$, $z_w(t_s)$ and velocity initial conditions $v_{wx}(t_s)$, $v_{wy}(t_s)$, and $v_{wz}(t_s)$ in X_R , Y_R , and Z_R , respectively, where t_s means the time to separate, v_w denotes the total velocity of the warhead, γ_{w1} and γ_{w2} express the elevation and flight path angles, respectively; u_{w4} , u_{w5} , and u_{w6} are unpredictable input accelerations acting on the warhead. For the body with the ballistic coefficient C_b ,

$$\dot{v}_{bx} = -\frac{\rho v_b^2}{2C_b} g \cos \gamma_{b1} \sin \gamma_{b2} + u_{b4}, \quad t > t_s, \quad (7)$$

$$\dot{v}_{by} = -\frac{\rho v_b^2}{2C_b} g \cos \gamma_{b1} \cos \gamma_{b2} + u_{b5}, \quad t > t_s, \quad (8)$$

$$\dot{v}_{bz} = \frac{\rho v_b^2}{2C_b} g \sin \gamma_{b1} - g + u_{b6}, \quad t > t_s, \quad (9)$$

with position initial conditions $x_b(t_s)$, $y_b(t_s)$, $z_b(t_s)$, and velocity initial conditions $v_{bx}(t_s)$, $v_{by}(t_s)$, and $v_{bz}(t_s)$ in X_R , Y_R , and Z_R , respectively. The parameters and variables are defined as in the warhead. Let the state vectors be

$$\begin{aligned} \mathbf{X}(t) &= [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6]^T \\ &= [x \quad y \quad z \quad v_x \quad v_y \quad v_z]^T, \end{aligned} \quad (10)$$

$$\begin{aligned} \mathbf{X}_w(t) &= [x_{w1} \quad x_{w2} \quad x_{w3} \quad x_{w4} \quad x_{w5} \quad x_{w6}]^T \\ &= [x_w \quad y_w \quad z_w \quad v_{wx} \quad v_{wy} \quad v_{wz}]^T, \end{aligned} \quad (11)$$

$$\begin{aligned} \mathbf{X}_b(t) &= [x_{b1} \quad x_{b2} \quad x_{b3} \quad x_{b4} \quad x_{b5} \quad x_{b6}]^T \\ &= [x_b \quad y_b \quad z_b \quad v_{bx} \quad v_{by} \quad v_{bz}]^T. \end{aligned} \quad (12)$$

The nonlinear state equations can be written as

$$\dot{\mathbf{X}}(t) = F(\mathbf{X}) + \phi \mathbf{u} + \mathbf{I}_{6 \times 6} \zeta, \quad t \leq t_s, \quad (13)$$

$$\dot{\mathbf{X}}_w(t) = F(\mathbf{X}_w) + \phi \mathbf{u}_w + \mathbf{I}_{6 \times 6} \zeta_w, \quad t > t_s, \quad (14)$$

$$\dot{\mathbf{X}}_b(t) = F(\mathbf{X}_b) + \phi \mathbf{u}_b + \mathbf{I}_{6 \times 6} \zeta_b, \quad t > t_s, \quad (15)$$

where ζ , ζ_w , and ζ_b stand for the process noise vectors with variance \mathbf{Q} , \mathbf{Q}_w , and \mathbf{Q}_b , respectively, \mathbf{I} denotes the identity matrix,

$$\begin{aligned} F(\mathbf{X}) &= \begin{bmatrix} x_4 \\ x_5 \\ x_6 \\ -\frac{\rho}{2C_b}(x_4^2 + x_5^2 + x_6^2)g \cos \gamma_1 \sin \gamma_2 \\ -\frac{\rho}{2C_b}(x_4^2 + x_5^2 + x_6^2)g \cos \gamma_1 \cos \gamma_2 \\ \frac{\rho}{2C_b}(x_4^2 + x_5^2 + x_6^2)g \sin \gamma_1 - g \end{bmatrix}, \\ \phi &= \begin{bmatrix} \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{I}_{3 \times 3} \end{bmatrix}, \\ \mathbf{u} &= [0 \quad 0 \quad 0 \quad u_4 \quad u_5 \quad u_6]^T, \\ \mathbf{u}_w &= [0 \quad 0 \quad 0 \quad u_{w4} \quad u_{w5} \quad u_{w6}]^T, \\ \mathbf{u}_b &= [0 \quad 0 \quad 0 \quad u_{b4} \quad u_{b5} \quad u_{b6}]^T. \end{aligned}$$

The precision phased array radar, which is a digital radar, is used in tracking and is the only instrument in the system for detecting the RV. It predicts target's position at the next sampling period according to a set of measurement, and generates a radar beam to illuminate the predicted area to track the target. The measurement equation for the RV is then given by

$$\mathbf{Z} = \mathbf{H}\mathbf{X} + \varepsilon, \quad t \leq t_s, \quad (16)$$

where ε denotes the measurement noise vector, which is assumed to be normally distributed with mean zero and variance \mathbf{R} , and \mathbf{H} is the 6×6 identity matrix. (13) and (16) are the dynamic equations for the RV during reentry. When $t > t_s$, two sets of measurement are detected at the sampling time, one for the warhead and one for the body, that is

$$\mathbf{Z}_w = \mathbf{H}\mathbf{X}_w + \varepsilon, \quad t > t_s, \quad (17)$$

$$\mathbf{Z}_b = \mathbf{H}\mathbf{X}_b + \varepsilon, \quad t > t_s. \quad (18)$$

(14), (17), (15), and (18) are the dynamic equations for the warhead and body, respectively, after separation. The radar is trying to estimate and predict state vectors from these two sets of measurement.

3. EXTENDED KALMAN FILTER WITH INPUT ESTIMATION

Input estimation means the method that estimating unknown inputs in state equations from pseudo innovations, which are the difference between the estimated states by the EKF with no input and measurements. At $t \leq t_s$, the predicted and updated states vectors of the RV by the EKF from $t=n\Delta t$ to

$t=(n+1)\Delta t$, $n=0,1,2,\dots$, under input vector $\mathbf{u}(n)$ at $t=n\Delta t$ are given by, respectively, [18]

$$\hat{\mathbf{X}}(n+1|n) = \phi(n)\hat{\mathbf{X}}(n|n) + \varphi_{\Delta}\mathbf{u}(n), \quad (19)$$

$$\hat{\mathbf{X}}(n+1|n+1) = \hat{\mathbf{X}}(n+1|n) + \mathbf{K}(n+1)[\mathbf{Z}(n+1) - \mathbf{H}\hat{\mathbf{X}}(n+1|n)], \quad (20)$$

where $\varphi_{\Delta} = \phi\Delta t$, Δt is the sampling period, $\mathbf{Z}(n+1)$ denotes radar measurements at $t=(n+1)\Delta t$, and the transition matrix

$$\phi(n) = \mathbf{I}_{6 \times 6} + \left. \frac{\partial \mathbf{F}(\mathbf{X})}{\partial \mathbf{X}} \right|_{\mathbf{X}=\hat{\mathbf{X}}(n|n)} \Delta t,$$

$\mathbf{K}(n+1)$ is the Kalman gain.

Let $\bar{\mathbf{X}}(n+1|n+1)$ denote the updated states for the EKF with no input at $t=(n+1)\Delta t$. For simplicity, let

$$\hat{\mathbf{X}}(n+1) = \hat{\mathbf{X}}(n+1|n+1),$$

$$\bar{\mathbf{X}}(n+1) = \bar{\mathbf{X}}(n+1|n+1),$$

and define

$$\mathbf{M}(n+1) = [\mathbf{I} - \mathbf{K}(n+1)\mathbf{H}]\phi(n),$$

$$\mathbf{N}(n+1) = [\mathbf{I} - \mathbf{K}(n+1)\mathbf{H}]\varphi_{\Delta}.$$

Assume that the abrupt deterministic inputs are applied during $k\Delta t \leq t \leq (k+s)\Delta t$,

$$\mathbf{u} = \begin{cases} 0 & t < k\Delta t, t > (k+s)\Delta t & k, s > 0 \\ \mathbf{u}(k+l) & k\Delta t \leq t \leq (k+s)\Delta t & l = 0, 1, 2, \dots, s, \end{cases} \quad (21)$$

where $\mathbf{u}(k+l)$ is a constant vector over the sampling interval. Then, $\hat{\mathbf{X}}(k) = \bar{\mathbf{X}}(k)$ during $t \leq k\Delta t$. The difference induced by the abrupt inputs between these two formations during $k\Delta t \leq t \leq (k+s)\Delta t$ can then be written as

$$\begin{aligned} \Delta \mathbf{X}(k+l) &= \hat{\mathbf{X}}(k+l) - \bar{\mathbf{X}}(k+l) \\ &= \mathbf{M}(k+l)\Delta \mathbf{X}(k+l-1) + \mathbf{N}(k+l)\mathbf{u}(k+l-1). \end{aligned}$$

Define the measurement residual for the EKF formation without and with inputs to be, respectively,

$$\bar{\mathbf{Z}}(k+l) = \mathbf{Z}(k+l) - \mathbf{H}\bar{\mathbf{X}}(k+l),$$

$$\hat{\mathbf{Z}}(k+l) = \mathbf{Z}(k+l) - \mathbf{H}\hat{\mathbf{X}}(k+l).$$

The recursive least-squares input estimator can be derived as [8]

$$\begin{aligned} \hat{\mathbf{u}}(k+l-1) &= \hat{\mathbf{u}}(k+l-2) \\ &+ \mathbf{G}(k+l)[\hat{\mathbf{Y}}(k+l) - \Phi(k+l)\hat{\mathbf{u}}(k+l-2)], \end{aligned} \quad (22)$$

where

$$\hat{\mathbf{Y}}(k+l) = \bar{\mathbf{Z}}(k+l) - \mathbf{H}\mathbf{M}(k+l)\Delta \hat{\mathbf{X}}(k+l-1),$$

$$\Phi(k+l) = \mathbf{H}\mathbf{N}(k+l),$$

$$\begin{aligned} \Delta \hat{\mathbf{X}}(k+l-1) &= \mathbf{M}(k+l)\Delta \hat{\mathbf{X}}(k+l-2) \\ &+ \mathbf{N}(k+l)\hat{\mathbf{u}}(k+l-2). \end{aligned}$$

The gain $\mathbf{G}(i)$ and variance of $\hat{\mathbf{u}}(i)$, $\mathbf{V}(i)$, are

$$\mathbf{G}(k+l) = \mathbf{V}(k+l-1)\Phi(k+l)\xi^{-1},$$

$$\begin{aligned} \mathbf{V}(k+l-1) &= \mathbf{V}(k+l-2) - \mathbf{V}(k+l-2)\Phi(k+l)^T \\ &\quad \times [\Phi(k+l)\mathbf{V}(k+l-2)\Phi(k+l)^T + \xi]^{-1} \\ &\quad \times \Phi(k+l)\mathbf{V}(k+l-2), \end{aligned}$$

where ξ means the variance matrix of $\hat{\mathbf{Z}}(k+l)$ and can be easily derived as

$$\xi = \mathbf{R} + \mathbf{H}\mathbf{P}(k+l+1|k+l)\mathbf{H}^T,$$

and $\mathbf{P}(k+l+1|k+l)$ expresses the covariance matrix of the predicted states for the EKF with no input.

In (21), k and s respectively denote the starting and stopping indices of the system input, which can be determined by testing. The test for detection of input is expressed as

$$\left| \frac{\hat{\mathbf{u}}(i)}{\sqrt{V(i,i)}} \right| > t_{st}, \quad \text{existence of } u(i) \text{ for } i=4,5,6, \quad (23)$$

otherwise $u(i)$ is absent where $V(i,i)$ is the ii -th element of \mathbf{V} and $[-t_{st}, t_{st}]$ represents the confidence interval which can be obtained from the cumulative normal distribution table for a certain preset confidence coefficient $1-\alpha$.

Once the input is estimated, the EKF is corrected with the estimated input at the same time. By incorporating the on-line input estimator into the EKF, the predicted and updated states at time interval $k\Delta t \leq t \leq (k+s)\Delta t$ are given by

$$\begin{aligned} \hat{\mathbf{X}}^v(k+l|k+l-1) &= \phi(k+l-1)\hat{\mathbf{X}}^v(k+l-1|k+l-1) \\ &\quad + \varphi_{\Delta}\hat{\mathbf{u}}(k+l-1), \end{aligned} \quad (24)$$

$$\begin{aligned} \hat{\mathbf{X}}^v(k+l|k+l) &= \hat{\mathbf{X}}^v(k+l|k+l-1) + \mathbf{K}^v(k+l)[\mathbf{Z}(k+l) \\ &\quad - \mathbf{H}\hat{\mathbf{X}}^v(k+l|k+l-1)]. \end{aligned} \quad (25)$$

The Kalman gain becomes

$$\begin{aligned} \mathbf{K}^v(k+l) &= \mathbf{P}^v(k+l|k+l-1)\mathbf{H}^T \\ &\quad [\mathbf{H}\mathbf{P}^v(k+l|k+l-1)\mathbf{H}^T + \mathbf{R}]^{-1}, \end{aligned} \quad (26)$$

with the covariance matrices at $k\Delta t \leq t \leq (k+s)\Delta t$

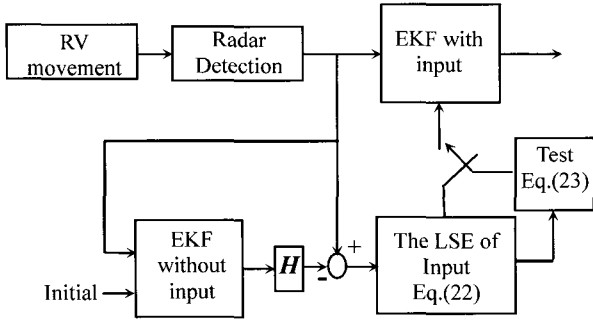


Fig. 2. The mechanism of the Kalman Filter with input estimation.

being

$$\begin{aligned} \mathbf{P}^v(k+l|k+l-1) &= \mathbf{P}(k+l|k+l-1) \\ &+ \phi(k+l-1)\mathbf{L}(k+l)\phi^T(k+l-1) \quad (27) \\ &+ \varphi_\Delta \mathbf{V}(k+l-1)\varphi_\Delta^T, \end{aligned}$$

$$\mathbf{P}^v(k+l|k+l) = [\mathbf{I} - \mathbf{K}^v(k+l)\mathbf{H}]\mathbf{P}^v(k+l|k+l-1), \quad (28)$$

where

$$\begin{aligned} \mathbf{L}(k+l) &= 0, \quad \mathbf{L}(k+2) = \mathbf{N}(k+2)\mathbf{V}(k)\mathbf{N}^T(k+2), \\ \mathbf{L}(k+l) &= \mathbf{M}(k+l-1)\mathbf{L}(k+l-1)\mathbf{M}^T(k+l-1). \end{aligned}$$

For time beyond the interval $t < k\Delta t$ and $t < (k+s)\Delta t$, state estimation is based upon the original EKF. Note that the initial states and covariance matrices at $t < (k+s)\Delta t$ are reinitiated by $\hat{\mathbf{X}}^v(k+s|k+s)$ and $\mathbf{P}^v(k+s|k+s)$. (22)-(28) form the algorithm and Fig. 2 schematically depicts the proposed filter.

At $t > t_s$, measurements \mathbf{Z}_w and \mathbf{Z}_b are sensed. The estimated inputs $\hat{\mathbf{u}}_w$ and $\hat{\mathbf{u}}_b$ are then calculated using (22) if \mathbf{Z} is replaced by \mathbf{Z}_w and \mathbf{Z}_b , respectively. Substituting $\hat{\mathbf{u}}_w$ and $\hat{\mathbf{u}}_b$ into (24) and (25) yields the predicted and updated states $\hat{\mathbf{X}}_w^v(k+l|k+l-1)$, $\hat{\mathbf{X}}_w^v(k+l|k+l)$, and $\hat{\mathbf{X}}_b^v(k+l|k+l-1)$, $\hat{\mathbf{X}}_b^v(k+l|k+l)$ for the warhead and body.

4. THE MODIFIED PROBABILISTIC DATA ASSOCIATION FILTER

PDAF is designed to track a single target in a cluttered environment. The filter provides the combined predicted and updated state vectors by weighting the probability of originating from the target. The RV reenters with high elevation angle to the radar. The clutter seldom appears in this region. For simplicity, the clutter effects may be ignored in the PDAF. This section presents the modified method, the MPDAF, by defining proper associate

probabilities without considering the clutter.

During $t > t_s$, two sets of measurement are detected and the MPDAF is utilized. Assume each target to be detected in a clear environment with probability 1. Let \mathbf{Z}^{n+1} be the vector collecting all measurements from $t=0$ to $t=(n+1)\Delta t$, where $n \geq t_s/\Delta t$. Define the events at time $t=(n+1)\Delta t$ to be

$$\theta_w(n+1) = \{ \mathbf{Z}_w(n+1) \text{ is the target-originated measurement} \}, \quad (29)$$

$$\theta_b(n+1) = \{ \mathbf{Z}_b(n+1) \text{ is the target-originated measurement} \}, \quad (30)$$

with association probabilities conditioned on \mathbf{Z}^{n+1} ,

$$\beta_w(n+1) = \Pr\{\theta_w(n+1) | \mathbf{Z}^{n+1}\}, \quad (31)$$

$$\beta_b(n+1) = \Pr\{\theta_b(n+1) | \mathbf{Z}^{n+1}\}. \quad (32)$$

Then, the association probabilities should satisfy

$$\beta_w(n+1) + \beta_b(n+1) = 1.$$

The updated state for the EKF at time $t=(n+1)\Delta t$ can be regarded as

$$\hat{\mathbf{X}}^v(n+1|n+1) = E[\mathbf{X}(n+1) | \mathbf{Z}^{n+1}].$$

Applying the Bayes' rule yields

$$\begin{aligned} E[\mathbf{X}(n+1) | \mathbf{Z}^{n+1}] &= E[\mathbf{X}(n+1) | \theta_w(n+1), \mathbf{Z}^{n+1}] \\ &\times \Pr\{\theta_w(n+1) | \mathbf{Z}^{n+1}\} \\ &+ E[\mathbf{X}(n+1) | \theta_b(n+1), \mathbf{Z}^{n+1}] \\ &\times \Pr\{\theta_b(n+1) | \mathbf{Z}^{n+1}\}. \end{aligned}$$

Then, $\hat{\mathbf{X}}^v(n+1|n+1)$, known as the combined updated state vector, becomes

$$\begin{aligned} \hat{\mathbf{X}}^v(n+1|n+1) &= \beta_w(n+1)\hat{\mathbf{X}}_w^v(n+1|n+1) \\ &+ \beta_b(n+1)\hat{\mathbf{X}}_b^v(n+1|n+1), \quad (33) \end{aligned}$$

where $\hat{\mathbf{X}}_w^v(n+1|n+1)$ and $\hat{\mathbf{X}}_b^v(n+1|n+1)$ denote the updated state vectors of the warhead and body, respectively, and are given by

$$\hat{\mathbf{X}}_w^v(n+1|n+1) = \hat{\mathbf{X}}_w^v(n+1|n) + \mathbf{K}_w^v(n+1)v_b(n+1), \quad (34)$$

$$\hat{\mathbf{X}}_b^v(n+1|n+1) = \hat{\mathbf{X}}_b^v(n+1|n) + \mathbf{K}_b^v(n+1)v_b(n+1), \quad (35)$$

where the predicted states are determined by the previous combined updated state,

$$\hat{X}_w^v(n+1|n) = \phi(n+1)\hat{X}_w^v(n|n) + \varphi_\Delta \hat{u}_w(n),$$

$$\hat{X}_b^v(n+1|n) = \phi(n+1)\hat{X}_b^v(n|n) + \varphi_\Delta \hat{u}_b(n).$$

$K_w^v(n+1)$ and $K_b^v(n+1)$ express the Kalman gains for the warhead and body, for which $\hat{u}_w(n)$ and $\hat{u}_b(n)$ are the respective estimated inputs at $t=(n+1)\Delta t$. The problem then focus on the determination of β_w and β_b by means of Z_w and Z_b .

Define the i -th innovation in position with respect to (w.r.t.) the warhead and body at $t=(n+1)\Delta t$ as, respectively,

$$v_w^i(n+1) = H^i Z_w(n+1) - H^i \hat{X}_w^v(n+1|n), \quad (36)$$

$$i = 1, 2, 3$$

$$v_b^i(n+1) = H^i Z_b(n+1) - H^i \hat{X}_b^v(n+1|n), \quad (37)$$

$$i = 1, 2, 3$$

where H^i denotes the i -th row of H .

As previously mentioned, the warhead trajectory is closer to the original than the body. The innovation for the warhead is then less than for the body if a good trajectory estimation is provided. Since only two targets are detected the association probabilities corresponding to the warhead and body can be then defined by means of the normalized innovation as

$$\beta_w(n+1) = \frac{e_b}{e_b + e_w}, \quad (38)$$

$$\beta_b(n+1) = \frac{e_w}{e_b + e_w}, \quad (39)$$

where

$$e_w = \sum_{i=1}^3 \frac{[v_w^i(n+1)]^2}{P_w^{ii}},$$

$$e_b = \sum_{i=1}^3 \frac{[v_b^i(n+1)]^2}{P_b^{ii}},$$

where P_w^{ii} and P_b^{ii} are the ii -th elements of covariance matrices for the predicted states $\hat{X}_w^v(n+1|n)$ and $\hat{X}_b^v(n+1|n)$, respectively. Substituting (38) and (39) into (33) yields the combined updated state of the object in track at $t=(n+1)\Delta t$. (33)-(39) constitute the algorithm of the MPDAF. This trajectory should reach the warhead such that radar beam covers the warhead to maintain the track.

The flow chart of the proposed algorithm combining the extended Kalman filter with IE and MPDAF is depicted in Fig. 3.

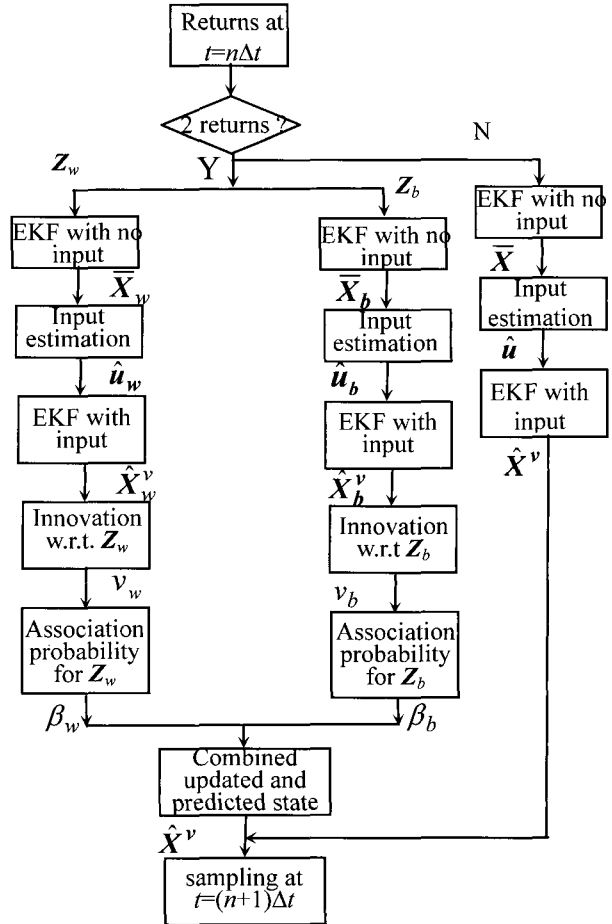


Fig. 3. Flow chart of the proposed algorithm.

5. SIMULATION ANALYSIS

The proposed algorithm performance is measured to determine the distance between the estimated and actual warhead trajectories. Let the estimation error w.r.t. the warhead signify the difference between the estimated and warhead trajectories. Similarly, the estimation error w.r.t. the body is the difference between the estimated and body trajectories. This section verifies the proposed algorithm in terms of estimation error w.r.t the wahead and body. The proposed algorithm should have a small estimation error w.r.t. the warhead to ensure the warhead is constantly tracked.

Case 1: Manuvering warhead

Consider an RV in reentry phase with $C = 2500$ kg/m² and initial values of $x(0)=300$ m, $y(0)=300$ m, $z(0)=45000$ m, $v(0)=1500$ m/s, $\gamma_1(0)=65^\circ$, and $\gamma_2(0)=15^\circ$. The RV splits into the warhead and body at $t_s = 5$ s and the warhead undertakes 3G lateral accelerations in three axes at $t=15$ s. The ballistic coefficients of the warhead and body are $C_w = 1500$ kg/m² and $C_b = 6000$ kg/m², respectively. (1)-(9) simulate the measured warhead and body trajectories

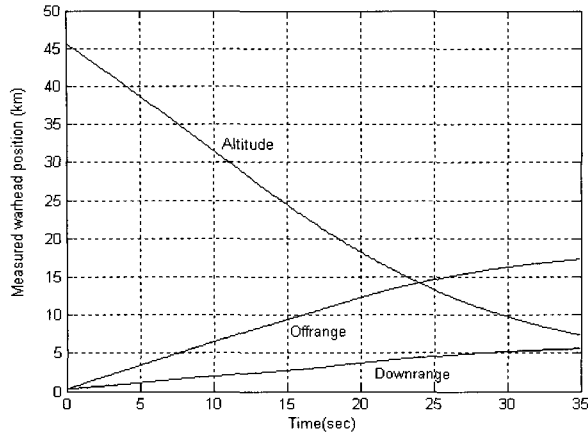


Fig. 4. The measured warhead position.

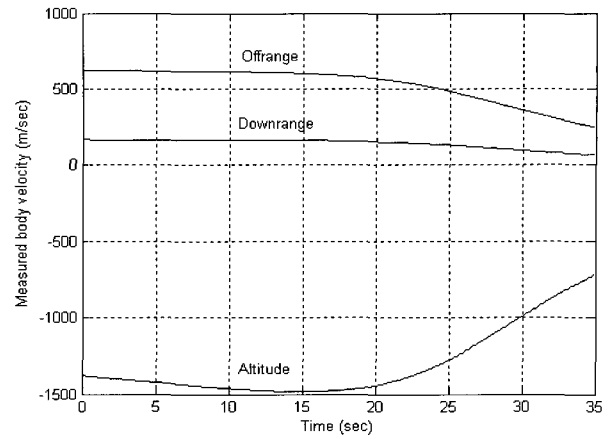


Fig. 7. The measured body velocity.

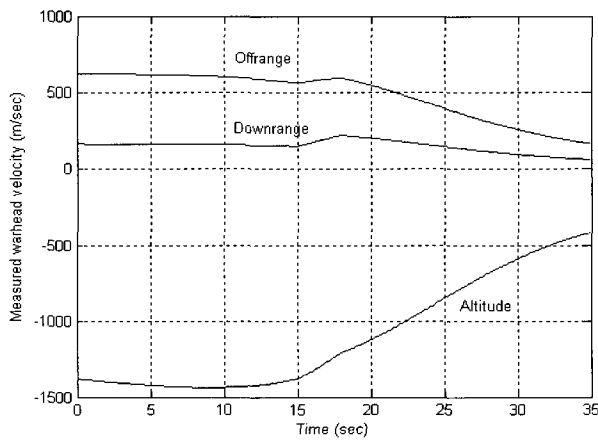


Fig. 5. The measured warhead velocity.

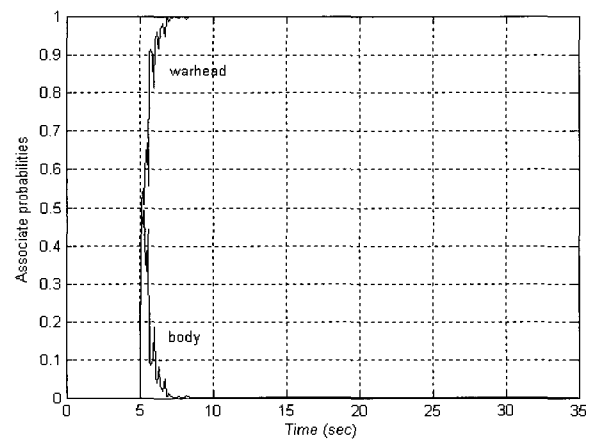


Fig. 8. The association probability evolution for warhead and body.

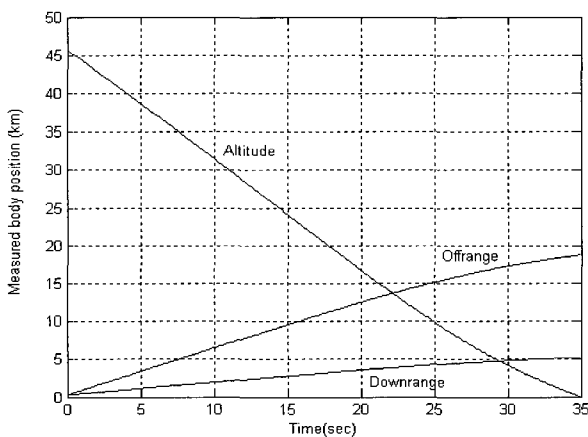


Fig. 6. The measured body position.

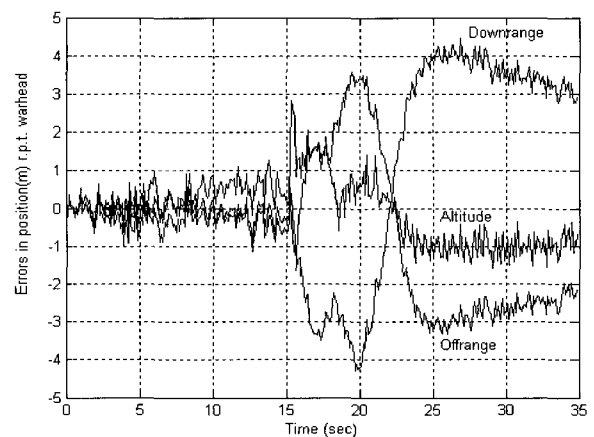


Fig. 9. The estimation errors w.r.t. the warhead in position.

with normally distributed noise. Figs. 4-7 illustrate the measured warhead and body trajectories.

Let the ballistic coefficient $C=2500\text{kg/m}^2$ in estimation. Fig. 8 demonstrates the corresponding association probability evolution. The association probability β_w approaches 1 within 3 seconds. Restated, the estimated trajectory is never influenced

by the measurement of the body after 3 seconds. Figs. 9-12 show the estimation errors w.r.t. the warhead and body. The errors w.r.t. the warhead, which are limited to $\pm 6\text{m}$ in position and $\pm 8\text{m/s}$ in velocity, are much smaller than those w.r.t. the body. Table 1 depicts the root mean squares (RMS) of estimation error w.r.t. the

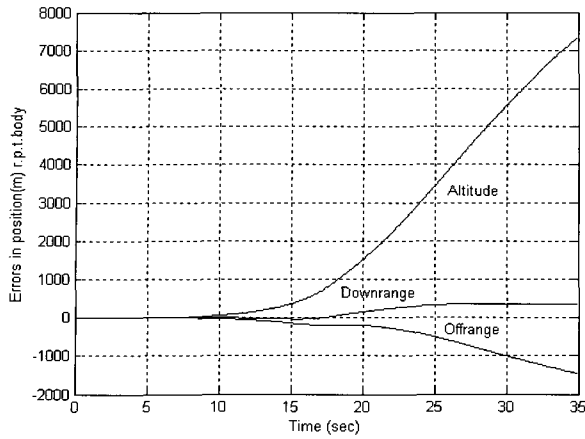


Fig. 10. The estimation errors w.r.t. the body in position.

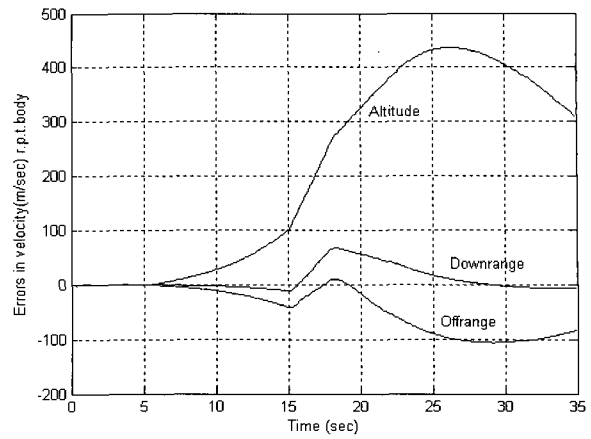


Fig. 12. The estimation errors w.r.t. the body in velocity.

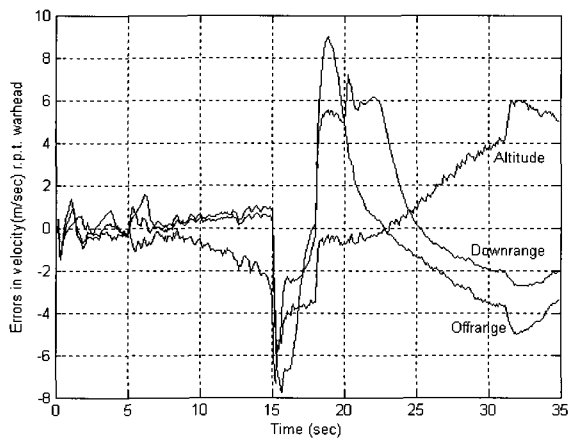


Fig. 11. The estimation errors w.r.t. the warhead in velocity.

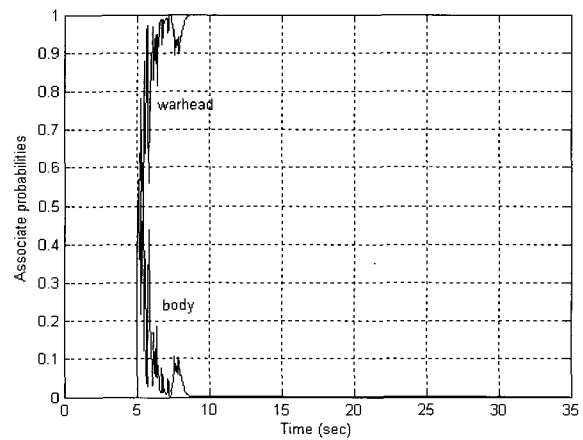


Fig. 13. The association probabilities for the warhead and body.

Table 1. Root mean square of the estimation error.

	position(m)			velocity(m/s)		
w.r.t.	X_R	Y_R	Z_R	X_R	Y_R	Z_R
warhead	3.47	2.29	1.29	2.68	2.91	3.15
body	245.33	580.1	2.28×10^3	23.26	52.21	175.82

warhead are much lower than those w.r.t. the body. The estimated trajectory follows the maneuvering warhead trajectory perfectly and provides an accurate data set to the radar beam former for tracking.

Consider that the RV undertakes the same lateral accelerations at 2 seconds after separation. Fig. 13 depicts the association probability propagation and shows β_w to be up to 1 in 4 seconds. It indicates that the proposed algorithm perform well disregarding the maneuvering undertaken by the warhead at the time near separation.

Case 2: Parameter uncertainty

The ballistic coefficient is the crucial parameter for obtaining the RV, warhead, and body trajectories from dynamic equations under some assumptions. The

Table 2. Root mean square of the estimation error in different ballistic coefficient C .

	position(m)			velocity(m/s)		
C	X_R	Y_R	Z_R	X_R	Y_R	Z_R
1500	3.52 (246)	2.26 (580)	1.41 (2.28×10^3)	2.86 (23.5)	3.53 (53.0)	4.17 (177)
3500	3.4 (245)	2.3 (580)	1.2 (2.28×10^3)	2.6 (23.2)	2.75 (51.9)	2.9 (175)
5000	2.34 (245)	2.26 (580)	1.21 (2.28×10^3)	2.52 (23.1)	2.66 (51.6)	2.72 (175)

* Value in (.) means the estimation error with respect to body otherwise with respect to warhead.

ballistic coefficient is a function of time and is hardly to measure for a defender. A constant is usually set in the entire estimation procedure. Table 2 presents the RMS' of estimation errors for different ballistic coefficients, which are almost unchanged as compared with Table 1, that the proposed method is robust to the parameter uncertainty.

Case 3: Variation of trajectories

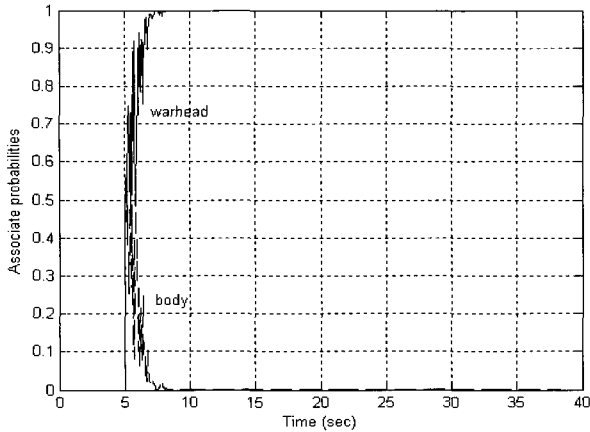


Fig. 14. The association probabilities ($C_w=1500\text{kg/m}^2$, $C_b=4000\text{kg/m}^2$).

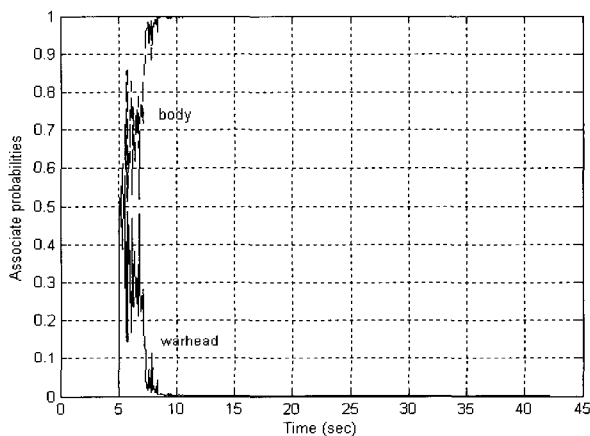


Fig. 15. The association probabilities ($C_w=2000\text{kg/m}^2$, $C_b=3000\text{kg/m}^2$).

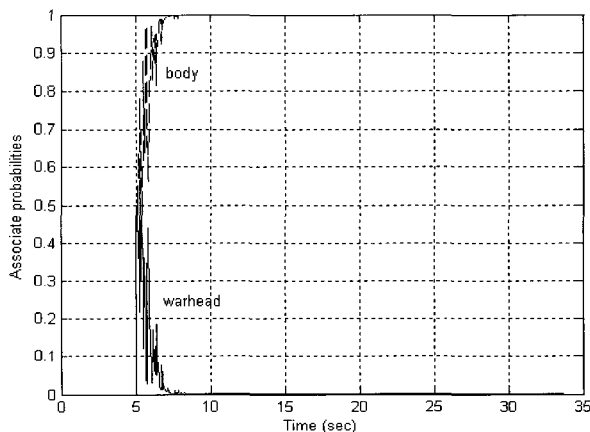


Fig. 16. The association probabilities ($C_w=7500\text{kg/m}^2$, $C_b=1500\text{kg/m}^2$).

The same trajectory is adopted but with different ballistic coefficients of the warhead and body. Fig. 14 depicts the association probabilities for the warhead and body with $C_w=1500\text{kg/m}^2$ and $C_b=4000\text{kg/m}^2$. The algorithm still works well because the trajectory

of the warhead is close to the original trajectory. With decreasing values of C_b , the body trajectory gradually approaches the original and the situations are changed. Figs. 15 and 16 illustrate the radar tracks the body clearly, indicating that the algorithm fails. This case demonstrates that the proposed filter uses the measurement originating from the object in track, and would lead the radar to lock to the object with the closest trajectory to the original.

6. CONCLUSIONS

This study presents a modified probabilistic data association filter to form an accurate filter for tracking a warhead that has separated from a reentry vehicle during the reentry phase. The proposed filter comprises an extended Kalman filter, an input estimator and a modified probabilistic data association filter. The extended Kalman filter combined with input estimation can accurately predict the trajectory and to produce the innovation reliably. The modified probabilistic data association filter with a well-defined association probability provides a precision combined updated state from the warhead and body measurements. Simulation results monitor the performance of the recommended algorithm by inspecting the estimation error corresponding to the warhead which should be kept in small. This investigation therefore concludes that the proposed algorithm is worthy of further study and application.

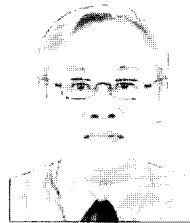
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