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A note for a classroom activity — Predicting German Tank Production during World War II

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During World War II there was a statistical analysis conducted by the Allied analysts to estimate the German war productions, including their tank productions. This article revisits the analysis of the tank productions as a classroom activity format. Various reformed ideas are proposed in order to enhance students' perspectives of the point estimation. Comprehensive simulation works and actual classroom discussions will be provided along with the theoretical investigations.

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1. INTRODUCTION

In the early years of World War II, the allied analysts tried to estimate the number of the tanks manufactured by the German. The analysts realized that each German tank has some unique items built in. On some types of equipment, such as tire molds and tank gear boxes, the items had unique numbers that are sequential like 1, 2, 3, 4, and so on. The analysts used these serial numbers to estimate the total production of the tanks made. The analysts assumed that the serial numbers denoted by Y_1, Y_2, …, Y_n are distributed according the Uniform distribution $U(0,\ \theta]$. Here, the parameter θ represents the total production of the items or the tanks manufactured as of the date of the analysis. The challenge was to find the best mathematical form of the data that estimates θ.

In junior or senior level mathematical statistics classes, the point estimation is usually not so well appreciated beyond the sample mean and the sample standard deviation as the estimates of μ and σ, respectively. One of the reasons is the lack of good examples that capture students' attention. This article exercises and manipulates the idea of the actual analysis of the German Tank production so that the value of the point estimation can be delivered to students in an engaging way. This article is based on the real classroom activities conducted in calculus-based undergraduate statistics classes. Students in this activity had some background in Order Statistics. All the simulation results were obtained through the TI-83/89 calculators.

2. THEORY

Suppose that Y_1, Y_2, $\ldots$, Y_n are randomly collected serial numbers from the uniform distribution on the interval $(0, \theta]$. In reality, the serial numbers in manufactured items would be labeled with integers, instead of real numbers. However, for convenience we assume that the serial numbers are continuously distributed on the domain. The upper bound θ in the interval is considered to be the total number of the tanks or the items manufactured.

Eighteen students were grouped into threes and were asked to propose their best formation of the data $Y_1, Y_2, \ldots, Y_n$ to estimate θ. After some discussion, the groups came up with a few, including $2\bar{Y}$, $Y_{(n)}$, $Y_{(n)} + \frac{1}{\theta}$, and $Y_{(n)} + Y_{(1)}$.

The first estimator, $2\bar{Y}$, is not so difficult consider as an estimate if one understands that the mean $\bar{Y}$ is the balancing point in the uniform distribution. By doubling the balancing center value in the uniform distribution, the upper bound can be reached. As we shall study later, this is not a wise choice because it does not treat the special message in the maximum order statistic $Y_{(n)}$ as to where the upper bound θ is.

The second choice, the maximum order statistic $Y_{(n)}$, is a natural choice because the largest value is usually a mirror of the upper bound. This estimator, however, lacks the property of the unbiasedness necessary for an estimator to be good. On average, the estimator $Y_{(n)}$ underestimates θ because $E(Y_{(n)}) = \frac{n}{n+1}\,\theta$. Because of this reason, the estimator $Y_{(n)}$ was not kept in consideration. Instead, it was modified into the form of $\frac{n+1}{n}\,\theta$, which provides unbiasedness.

The third estimator $Y_{(n)}+\frac{1}{\theta}$ chosen by a group was an interesting pick. In a sense, this group's choice was better than either $2\bar{Y}$ or the maximum order statistic $Y_{(n)}$, because they understood that $Y_{(n)}$ would underestimate θ and they had to think about how to close the gap between $Y_{(n)}$ and θ. They, however, were not aware that $Y_{(n)} + \frac{1}{\theta}$ can not possibly be an estimator because it involves the unknown parameter θ itself. The estimator $Y_{(n)} + Y_{(1)}$ was a very unique and clever form to make up the gap between $Y_{(n)}$ and the upper bound θ. For educational purposes, a dummy estimator $(n+1)Y_{(1)}$ was added to the list. As we shall see in Lemma 1, the estimator $(n+1)Y_{(1)}$ is an unbiased estimator. This, however, should not be a very good one, because it does not make sense to look at only the smallest value in order to estimate the upper bound.

In this article, we will be using the estimator $(n+1)Y_{(1)}$ to create some contrasts to the other estimators.

In summary, we consider the four estimators of

$$\theta : \ \hat{\theta}_1 = 2\bar{Y},\ \hat{\theta}_2 = \frac{n+1}{n}Y_{(n)},\ \hat{\theta}_3 = Y_{(n)} + Y_{(1)},\ \hat{\theta}_4 = (n+1)Y_{(1)}.$$

In Lemma 1 we first show that these four estimators are all unbiased.

Lemma 1. $\hat{\theta}_1 = 2\bar{Y}$, $\hat{\theta}_2 = \frac{n+1}{n}Y_{(n)}$, $\hat{\theta}_3 = Y_{(n)} + Y_{(1)}$, $\hat{\theta}_4 = (n+1)Y_{(1)}$ *are unbiased estimators of* θ.

Proof. The probability density functions of $Y_{(1)}$ and $Y_{(n)}$ are

$$f_{(1)}(y) = n[1-F(y)]^{n-1}f(y) \quad \text{and} \quad f_{(n)}(y) = n[F(y)]^{n-1}f(y),$$

respectively, where $f(y) = \frac{1}{\theta}$ and $F(y) = \frac{y}{\theta}$. These yield that

$$f_{(1)}(y) = n\left(1-\frac{y}{\theta}\right)^{n-1}\frac{1}{\theta} \quad \text{and} \quad f_{(n)}(y) = n\left(\frac{y}{\theta}\right)^{n-1}\frac{1}{\theta}.$$

Then, we have

$$E(Y_{(1)}) = \int_0^\theta yn\left(1-\frac{y}{\theta}\right)^{n-1}\frac{1}{\theta}\,dy = \frac{1}{n+1}\,\theta$$

and

$$E(Y_{(n)}) = \int_0^\theta yn\left(\frac{y}{\theta}\right)^{n-1}\frac{1}{\theta}\,dy = \frac{n}{n+1}\,\theta.$$

Now, we are ready to evaluate the expected values of the four estimators.

$$E(\hat{\theta}_1) = E(2\bar{Y}) = 2E(\bar{Y}) = 2\cdot\frac{\theta}{2} = \theta,$$

$$E(\hat{\theta}_2) = E\left(\frac{n+1}{n}Y_{(n)}\right) = \frac{n+1}{n}\cdot E(Y_{(n)}) = \frac{n+1}{n}\cdot\frac{n}{n+1}\,\theta = \theta,$$

$$E(\hat{\theta}_3) = E\left(Y_{(1)} + Y_{(n)}\right) = E\left(Y_{(1)}\right) + E(Y_{(n)}) = \frac{n}{n+1}\theta + \frac{1}{n+1}\theta = \theta,$$

and

$$E(\hat{\theta}_4) = E\left((n+1)Y_{(1)}\right) = (n+1)\frac{1}{n+1}\theta = \theta. \qquad \square$$

In Lemma 1 we verified that the four proposed estimators of θ are all unbiased. In order to evaluate the efficiencies among them, in Lemma 2 we consider the variance structure of the four estimators.

Lemma 2. *The variances of* θ_1, θ_2, θ_3, *and* θ_4 *are the following.*

(a) $V(\hat{\theta}_1) = \dfrac{\theta^2}{3n}$

(b) $V(\hat{\theta}_2) = \dfrac{\theta^2}{n(n+2)}$

(c) $V(\hat{\theta}_3) = \dfrac{2\theta^2}{(n+1)(n+2)}$

(d) $V(\hat{\theta}_4) = \dfrac{n\theta^2}{n+2}$

Proof. (a) We have

$$V(\hat{\theta}_1) = V(2\bar{Y}) = 4 \cdot V(\bar{Y}) = \frac{4}{n}\sigma^2 = \frac{4}{n} \cdot \frac{\theta^2}{12} = \frac{\theta^2}{3n}.$$

(b) We have

$$V(\hat{\theta}_2) = V(\frac{n+1}{n}Y_{(n)}) = \left(\frac{n+1}{n}\right)^2 V(Y_{(n)})$$
$$= \left(\frac{n+1}{n}\right)^2 \frac{n}{(n+2)(n+1)^2}\theta^2 = \frac{1}{n(n+2)}\theta^2.$$

Here,

$$V(Y_{(n)}) = E(Y_{(n)}^2) - E^2(Y_{(n)}),$$

where

$$E(Y_{(n)}^2) = \int_0^\theta y^2 f_{Y_{(n)}}(y)dy = \int_0^\theta y^2 \cdot \frac{ny^{n-1}}{\theta^n}\,dy = \frac{n}{n+2}\theta^2$$

and

$$E(Y_{(n)}) = \frac{n}{n+1}\theta.$$

(c) The probability density functions of $Y_{(1)}$ and $Y_{(n)}$ are

$$f_{Y_{(1)}}(y) = \left(1 - \frac{y}{\theta}\right)^{(n-1)}\frac{1}{\theta}$$

and

$$f_{Y_{(n)}}(y) = \frac{ny^{n-1}}{\theta^n}.$$

These two functions allow us to have

$$E(Y_{(1)}^2) = \frac{2\theta^2}{(n+1)(n+2)}$$

and

$$E(Y_{(n)}^2) = \frac{n}{(n+2)}\theta^2.$$

The following joint density function of $Y_{(1)}$ and $Y_{(n)}$ can be driven using the *multinomial* probability distribution and can be found in most mathematical statistics books.

$$f_{Y_{(1)},Y_{(n)}}(y_1, y_n) = \frac{n(n-1)}{\theta^n}(y_1 - y_n)^{n-2}$$

where $0 \leq y_1 \leq y_n \leq \theta$.

Then, we have

$$\begin{aligned} E(Y_{(1)}Y_{(n)}) &= \int_0^\theta \int_0^{y_n} y_1 y_n f_{Y_{(1)},Y_{(n)}}(y_1, y_n) dy_1 dy_n \\ &= \int_0^\theta \int_0^{y_n} y_1 y_n \frac{n(n-1)}{\theta^n}(y_1 - y_n)^{n-2} dy_1 dy_n \\ &= \frac{n(n-1)}{\theta^n} \int_0^\theta y_n \int_0^{y_n} y_1 (y_n - y_1)^{n-2} dy_1 dy_n \\ &= \frac{n(n-1)}{\theta^n} \int_0^\theta y_n \cdot \frac{y_n^n}{n(n-1)} dy_n \\ &= \frac{\theta^2}{n+2}. \end{aligned}$$

Therefore,

$$\begin{aligned} V(\hat{\theta}_3) &= E(Y_{(1)} + Y_{(n)})^2 - E^2(Y_{(1)} + Y_{(n)}) \\ &= E(Y_{(1)} + Y_{(n)})^2 - \theta^2 \\ &= EY_{(1)}^2 + EY_{(n)}^2 + 2E(Y_{(1)}Y_{(n)}) - \theta^2 \\ &= \frac{2\theta^2}{(n+1)(n+2)} + \frac{n}{(n+2)}\theta^2 + \frac{2\theta^2}{n+2} - \theta^2 \\ &= \frac{2\theta^2}{(n+1)(n+2)}. \end{aligned}$$

(d) We have

$$V(\hat{\theta}_4) = V((n+1)Y_{(1)}) = (n+1)^2 V(Y_{(1)}).$$

Then,

$$E(Y_{(1)}^2) = \int_0^\theta y^2 f_{Y_{(1)}}(y)dy = \int_0^\theta y^2 \cdot \frac{n}{\theta}(1-\frac{y}{\theta})^{n-1}dy = \frac{2\theta^2}{(n+1)(n+2)}.$$

So,

$$V(Y_{(1)}) = E(Y_{(1)}^2) - E^2(Y_{(1)}) = \frac{2\theta^2}{(n+1)(n+2)} - \left(\frac{\theta}{n+1}\right)^2 = \frac{n}{(n+1)^2(n+2)}\theta^2.$$

Therefore,

$$V(\hat{\theta}_4) = (n+1)^2 \cdot \frac{n}{(n+1)^2(n+2)}\theta^2 = \frac{n}{n+2}\theta^2 \qquad \square$$

In Lemma 1 we verified that the four estimators are all good in the sense of unbiasedness. Lemma 2, however, indicates that the four unbiased estimators perform unequally due to the different amounts of variability existing within the structure of the four estimators. The results in Lemma 2 imply that $V(\hat{\theta}_1) = O\left(\frac{1}{n}\right)$, $V(\hat{\theta}_2) = O\left(\frac{1}{n^2}\right)$, $V(\hat{\theta}_3) = O\left(\frac{1}{n^2}\right)$, and $V(\hat{\theta}_4) = O(1)$. As we shall see in the full discussion section later, the efficiencies of the four estimators are ranked from the highest to the lowest in the order of $\hat{\theta}_2$, $\hat{\theta}_3$, $\hat{\theta}_1$, and $\hat{\theta}_4$. In order to enrich our forthcoming discussions of the four estimators, we begin to consider some simulations in order to collect data.

3. SIMULATION RESULTS

The purpose of the simulation activity is to have students experience how theory behaves in practice. The idea is to draw a random sample from a $U(0,\theta]$-distribution, where θ is pre-specified integer. Then we evaluate the four estimators and their standard deviations based on the sample. The observations will be compared to the facts obtained in Lemma 1 and Lemma 2. In practice, θ is an unknown parameter that is to be estimated. For simulation purposes we assume that θ is 1000. The following data are obtained from the actual class activities conducted in the first authors' mathematical statistics class in 2004. The TI-83/89 function `RandInt` function was used to generate random integer numbers on the interval $[1, 1000]$ interval. Each of the 18 students who were participating the activity obtained his or her own sample of size n. We considered two different values of n. The case I deals with $n = 10$, and Case II with $n = 70$.

Case I: $\theta = 1000$ and $n = 10$

Table 1. $y_{(1)}$ and $y_{(n)}$ of the 18 samples

samples	1	2	3	4	5	6	7	8	9
$y_{(1)}$	126	45	145	55	74	69	76	295	68
$y_{(n)}$	911	887	860	858	865	899	993	954	586
$\bar{y}$	547.4	533.3	481.8	479.2	515.8	461.7	439.1	567.0	304.8
samples	10	11	12	13	14	15	16	17	18
$y_{(1)}$	11	9	17	264	203	6	8	63	97
$y_{(n)}$	796	946	939	767	896	884	975	859	961
$\bar{y}$	474.3	383.1	391.1	565.4	591.7	308.8	478.5	519.7	492.3

Table 2. $\hat{\theta}_1$, $\hat{\theta}_2$, $\hat{\theta}_3$, and $\hat{\theta}_4$ of the 18 samples

samples	1	2	3	4	5	6	7	8	9	10
$\hat{\theta}_1$	1095	1067	964	958	1032	923	878	1134	610	949
$\hat{\theta}_2$	1002	976	946	944	952	989	1092	1049	645	876
$\hat{\theta}_3$	1037	932	1005	913	939	968	1069	1249	654	807
$\hat{\theta}_4$	1386	495	1595	605	814	759	836	3245	748	121
samples	11	12	13	14	15	16	17	18	Mean	SD
$\hat{\theta}_1$	766	782	1131	1183	618	957	1039	985	948	166
$\hat{\theta}_2$	1041	1033	844	986	972	1073	945	1057	968	104
$\hat{\theta}_3$	955	956	1031	1099	890	983	922	1058	970	124
$\hat{\theta}_4$	99	187	2904	2233	66	88	693	1067	997	950

Case II: $\theta = 1000$ and $n = 70$

Table 3. $y_{(1)}$ and $y_{(n)}$ of the 18 samples

samples	1	2	3	4	5	6	7	8	9	10
$y_{(1)}$	9	13	33	1	15	13	45	16	6	13
$y_{(n)}$	974	994	979	993	989	979	961	998	978	986
$\bar{y}$	502	509	530	569	485	440	520	472	470	550
samples	11	12	13	14	15	16	17	18		
$y_{(1)}$	22	1	9	11	30	36	18	20		
$y_{(n)}$	992	988	997	993	965	990	973	974		
$\bar{y}$	551	498	526	540	502	550	461	453		

Table 4. Evaluations of $\hat{\theta}_1$, $\hat{\theta}_2$, $\hat{\theta}_3$, and $\hat{\theta}_4$ of the 18 samples

samples	1	2	3	4	5	6	7	8	9	10
$\hat{\theta}_1$	1005	1019	1060	1138	969	881	1040	944	940	1100
$\hat{\theta}_2$	988	1008	993	1007	1003	993	975	1012	992	1000
$\hat{\theta}_3$	983	1007	1012	994	1004	992	1006	1014	984	999
$\hat{\theta}_4$	639	923	2343	71	1065	923	3195	1136	426	923
samples	11	12	13	14	15	16	17	18	**Mean**	**SD**
$\hat{\theta}_1$	1101	997	1053	1081	1005	1100	922	905	1014	75.7
$\hat{\theta}_2$	1006	1002	1011	1007	979	1004	987	988	998	11.2
$\hat{\theta}_3$	1014	989	1006	1004	995	1026	991	994	1001	11.5
$\hat{\theta}_4$	1562	71	639	781	2130	2556	1278	1420	1227	853.0

4. DISCUSSION

1. $\hat{\theta}_4$, the very bad: The estimator $\hat{\theta}_4 = (n+1)Y_{(1)}$ is very bad due to the fact that $V(\hat{\theta}_4) = O(1)$. This means that the variability of $\hat{\theta}_4$ does not decrease as n increases, but rather it is proportional to the size of θ because $\text{Var}(\hat{\theta}_4) = \frac{n}{n+2}\theta^2 \approx \theta^2$. This is not so surprising since the smallest value alone cannot bring up enough information as to what the upper bound might be. In the simulation of Case I, where $\theta = 1000$ and $n = 10$, the standard deviation of the eighteen values of $\hat{\theta}_4$ is 950. In Case II, where $\theta = 1000$ and $n = 70$, the standard deviation is 75.7. Both of these simulations revealed that the standard deviation of $\hat{\theta}_4$ is almost the size of θ and highly unreliable, even though it is an unbiased estimator of θ.

2. $\hat{\theta}_1$, the not-so-good-not-so-bad: The estimator $\hat{\theta}_1 = 2\bar{Y}$ seems to be a convenient way to estimate θ because $\bar{Y}$ is expected to be located half way between 0 and θ. What is not so helpful is that it does not utilize the maximum order statistic $Y_{(n)}$ to its maximum degree while $Y_{(n)}$ carries the most information as to what the upper bound θ might be. As Lemma 2 shows, $\text{Var}(\hat{\theta}) = \frac{1}{3n}\theta^2$, which reduces to zero in the order of n, whereas the variances of the other two estimators, $\hat{\theta}_2$ and $\hat{\theta}_3$, reduce to zero in the order of n^2. This puts the rank of $\hat{\theta}_1$ in terms of its goodness as an estimate of θ ahead of $\hat{\theta}_4$, but behind $\hat{\theta}_2$ and $\hat{\theta}_3$. In the simulations of Cases I and II, the sample standard deviations of $\hat{\theta}_1$ are 166 and 75.7, respectively. These are all the third largest among the four estimates in each case.

3. $\hat{\theta}_2$ or $\hat{\theta}_3$, the two good estimators: Lemma 2(b) and (c) yields that the variances of $\hat{\theta}_2$ or $\hat{\theta}_3$ reduce to zero in the order of n^2, which puts $\hat{\theta}_2$ or $\hat{\theta}_3$ ahead of the other two estimators $\hat{\theta}_4$ or $\hat{\theta}_1$. The standard deviations of $\hat{\theta}_2$ and $\hat{\theta}_3$ (*resp.*) are

$$\mathrm{SD}(\hat{\theta}_2) = \frac{\theta}{\sqrt{n(n+2)}} \quad \text{and} \quad \mathrm{SD}(\hat{\theta}_3) = \sqrt{\frac{2}{(n+1)(n+2)}}\,\theta$$

(*resp.*). This implies

$$\frac{\mathrm{SD}(\hat{\theta}_3)}{\mathrm{SD}(\hat{\theta}_2)} = \sqrt{\frac{2n}{n+1}}.$$

This ratio yields that the standard deviation of $\hat{\theta}_3$ is approximately 1.4 times larger than the standard deviation of $\hat{\theta}_2$. We conclude that $\hat{\theta}_2$ is superior to $\hat{\theta}_3$.

In Case I we have $\theta = 1000$ and $n = 10$. Then, we expect to have

$$\frac{\mathrm{SD}(\hat{\theta}_3)}{\mathrm{SD}(\hat{\theta}_2)} = \sqrt{\frac{2n}{n+1}} = \sqrt{\frac{2 \cdot 10}{10+1}} = 1.35.$$

The sample standard deviations of the 18 observed values of $\hat{\theta}_2$ and $\hat{\theta}_3$, in Case I, are 104 and 124, respectively, where the ratio of 124 to 104 is 1.192.

In Case II, where $\theta = 1000$ and $n = 70$, the expected theoretical ratio

$$\frac{\mathrm{SD}(\hat{\theta}_3)}{\mathrm{SD}(\hat{\theta}_2)} = \sqrt{\frac{2n}{n+1}} = \sqrt{\frac{2 \cdot 70}{1+70}} = 1.40.$$

The ratio of the two sample standard deviations of $\hat{\theta}_2$ and $\hat{\theta}_3$ turns out to be $11.5/11.2 = 1.03$.

In the class activities, the number of independent samples was 18 because there were 18 students. To enhance the accuracy of the simulation result, a TI program was written to obtain 500 random samples, where $\theta = 1000$ and the size of each sample $n = 70$. Some statistics obtained are as follows: The sample means of $\hat{\theta}_2$ and $\hat{\theta}_3$ are 999 and 1002, respectively, and the sample standard deviations of $\hat{\theta}_2$ and $\hat{\theta}_3$ are 49.3 and 70.3, respectively. The ratio of 70.3 to 49.3 is 1.43, which is quite close to what we have expected from the theoretical ratio

$$\frac{\mathrm{SD}(\hat{\theta}_3)}{\mathrm{SD}(\hat{\theta}_2)} = \sqrt{\frac{2n}{n+1}} = \sqrt{\frac{2 \cdot 500}{1+500}} = 1.41.$$

4. $\hat{\theta}_2$, the winner: The winner $\hat{\theta}_2 = \frac{nY_{(n)}}{(n+1)}$ has many titles. It goes like this: $\hat{\theta}_2 = \frac{nY_{(n)}}{(n+1)}$ is the "complete minimal sufficient uniformly minimum variance unbiased estimator" of θ. A very brief explanation of this lengthy title is that $\hat{\theta}_2 = \frac{nY_{(n)}}{(n+1)}$ is the best and the simplest estimate of θ. The exact theoretical notions of the terms are beyond the scope of this article. Those can be clarified in some advanced mathematical statistics books such as Lehmann(1983).

5. SUMMARY

The purpose of this article is to articulate the values obtained in point estimation as a statistical method. The article also indicated how students used Order Statistics in relation to point estimation processes. Linking to the historical analysis conducted during the World War II would make students more interested in and focused in the topic. After the theoretical foundation of the unbiasedness and the variance structures of some chosen estimators, simulations and discussions provided students with an experience of how experimental results fit theory. The contents of this article can be adopted and manipulated so that it can fit into each classroom situation when the theme is either point estimation or order statistic.

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