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호텔 사업의 수익 관리에 관계된 모든 수익을 고려한 호텔 객실의 전략적인 방 배분 문제†

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The Tactical-level Room Allocation Problem for Hotel Industry by Incorporating the Other Sources of Revenue into Overall Revenue Management System

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본 연구는 호텔에서 방 예약을 받는 시스템에서, 전술적으로 방의 등급을 정해 예약을 받는 방 배정 문제에서, 효과적인 절차를 총 수익에 관련되는 여러 가지 요소들을 고려하여, 총 수익을 최대화하는 알고리즘을 개발하였다. 이익을 최대화하고, 자원들의 효과적인 배분을 위해서 먼저 시장의 차별화가 어떻게 다르게 형성되는가를 분석하고, 이 결과를 전략적인 계층 방 분배 문제를 해결하는 데에 적용하였다. 하룻밤을 지내는 손님을 위한 예약에서는 동적인 모델링(dynamic model)이 사용되었고, 여러 밤을 지내는 손님들을 위해서는 정적인 모델링(static model)을 제시하였다.

Keywords : Hotel Industry, Revenue Management, Dynamic Programming, Room Allocation

1. Introduction

Effective revenue management (RM) in the hotel industry are getting more important than the past. Today's hotel executives are facing with a growth in market that shows insufficient rooms, an increment in demand, and a prosperous economic condition. Hotel managers adopted RM as an equating demand with supply to maximize potential revenue.

RM can be referred to as a yield management, which is a business practice that can maximize revenue. To maximize the profits, selling their products to the right customer at the right price at the right time is needed. This is mainly accomplished by targeting segmented micro-markets to maximize revenue.

RM is a revenue maximization technique which aims to increase net yield through the predicted allocation of available bedroom capacity to pre-determine market segments at optimum price.

RM for the hotel industry solved by a two-level room allocation problem [2]. The first level is tactical level and the second level is operational level. It has been proved that RM system increased sales [5]. A 5% decrease in sales expenses increases profits by 3%. Weatherford [7] addressed an additional 2.9% increase in revenue by incorporating guest mul-

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ti-night reservation requests.

Room mix problem was approached by monte carlo simulation method by Atul [1]. A monte carlo simulation is a way to perform hundreds or thousands of "what-if" analysis.

Kate and David [6] developed and tested a model that connects financial and marketing goals by reporting revenues, expenses, and profit by market segment. And it also showed that other sources of revenue can not be ignored in RM system.

Bonnie, Martin and Raymond [3] proposed that the other source of revenue has a large portion in the total revenues and costs.

Clearly the value of current RM system is their potential to enhance profit through analytical and systemized intelligence of customer base, market characteristics and hotel capacity. This is effectively achieved through detailed analysis of the internal and external functional parameters within which the hotel operates. Furthermore, this analysis must be based on historical data, current trends and projected levels of business.

Currently, there is no literature about tactical-level room allocation problem for hotel industry by incorporating the other sources of revenue into overall RM System.

2. Research Objective

The objective of this research is to develop an efficient procedure for solving the tactical-level room allocation problem for hotel industry by incorporating other sources of revenue into overall RM system. To maximize profits and allocate wisely available resources, first, determine how different market segments contributed. And apply this to an efficient procedure for solving tactical-level room allocation problem for the hotel industry by combining the dynamic model, which is not useful in solving multi-night reservation, for handling single-night reservation requests and the static model, not optimal, for handling multi-night reservation requests.

Markets must be segmented into more distinct components than the broad dichotomy of business and leisure, to establish a market fit product provision. Data has been collected from several journals based on market segment. The characteristics of each market such as quantifiable demand, price sensitivity, propensity to spend, booking patterns, booking lead times, duration of stay, check-in and departure patterns all need to be identified. Many hoteliers currently employ forecasting techniques; the needs of a RM system are somewhat more complex.

3. The Essential Elements of Revenue Management

Brief definitions of the elements of RM are introduced [4]. It is easier to understand the RM, if we reiterate the following definitions.

3.1 Dynamic Model Formulation

The tactical-level problem can be formulated with either the static to dynamic model. The static model assumes that parameters are constant over time. For this reason, a static model must be solved repeatedly to reflect changes in the parameters. A dynamic model allows the inclusion of changes in parameters by means of a time decomposition of the reservation period; therefore, the resulting solution is optimal over time.

3.2 Multi-night Stays

Most of the original RM systems used by the hotel industry assumed one-night stays. In practice, however, not all guests stay for just a single night. In fact, depending on the property, the percentage of stays for one night can be as low as 20%. However, the presence of multi-night reservation requests extremely complicates the ability to formulate and solve problems using the dynamic model. Instead, a heuristic based on integer programming formulation is developed to handle multi-night reservation requests separately from single-night reservation requests.

3.3 Nested Room Allocation

Using a non-nested room allocation approach, rooms are allocated to various booking classes, and the sum of allocation equals the booking capacity at the time the model is applied. A major drawback of this approach is the possibility that a request for a room in the highest-value booking class may be denied even though the total booking capacity may not have been reached. However, in a nested room allocation approach, rooms are allocated by nesting the booking classes according to their revenue values. That is, rooms allocated to booking classes with lower revenue values are made available to booking classes with higher revenue values. Additionally, rooms allocated to any particular booking class are "protected" from making them available to booking classes with lower revenue values.

3.4 Demand Patterns over the Reservation Period

Past researchers assumed a continuous probability distribution in order to describe the total demand for each booking class during the remaining booking periods. One major weakness of this approach is the inability to consider uncertainty in the underlying arrival pattern of demand for each booking class. In reality, for a set of booking limits, the total expected revenue that can be generated depend on both the demand levels for the various booking classes during the reservation period and the order in which these requests arrive at the reservation system. A non-homogeneous Poisson process was shown to be valid in literature for both the airline and hotel industries.

3.5 Overbooking

Although the total number of reservation made within each market segment is known to a hotel manager, the resulting number of reservations that turn into sales is a random variable because of no-shows. American Airlines reported that the benefits of its RM are mainly attributed to proper allocation of seats among various booking classes and to overbooking. The impact of overbooking is even more significant to the hotel industry where no-shows are much more common.

3.6 Group Reservations

Group reservations are another practical issue that exists in the hotel industry; therefore, it should be explicitly considered within the optimization model. The existence of group reservations has a significant effect on the total expected revenue. For example, by accepting a group reservation at a particular time, the optimization model may exclude larger group reservations into the optimization model since cancellations and no-shows of such reservations represents a large sum revenue loss.

3.7 Multiple Types of Rooms and Downgrading

This paper consider multiple types of the product (rooms) and downgrading. As a result, fewer customers are turned down for reservation requests. Hotels usually have different types of rooms, which differ in quality (size, furniture, services, etc.). Hence, there exists a natural order for the rooms, where any room can be substituted for all those that are better than it. Therefore, downgrading adds a new degree of flexibility to the room assignment process.

3.8 Booking Classes

This paper assumes predetermined prices for each booking class and that the demands of different booking classes are independent of each other, as a result of mutually independent market segmentation. Let F_i denote the booking price for a room in booking class i ($i = 1, 2 \cdots I$) and be indexed such that $F_1 > F_2 > F_3 > \cdots > F_I$. In a case with multiple room types, F_{ic} denotes the booking price booking class i for room type c.

4. Strategies for Tactical-Level Room Allocation Problems

4.1 Optimal Room Allocation Model for Single-Night Case

Discrete-time dynamic programming formulations are presented for obtaining optimal room allocation strategies for cases where customers stay at the hotel for a single night.

4.1.1 Model I : Single-Night Stay Ignoring Group Reservation and Downgrading

The optimal dynamic room allocation strategy corresponds to determining whether to accept or deny reservation requests for rooms in booking classes $i = 2, 3 \cdots I$ during decision period n. A set of customers generated as a result of some method of market segmentation is called a "booking class." Let f_s^n be the expected revenue that can be generated from decision period n onward until the actual stay night in consideration (i.e., $n = 1$) if there are s available rooms. Additionally, let P_i^n denote the request probability that a reservation request for booking class I will arrive during a decision period n.

$$f_s^n = \begin{cases} P_0^n f_s^{n-1} + P_1^n (F_1 + f_{s-1}^{n-1} + \sum_{i=2}^{I} P_i^n max \begin{cases} F_i + f_{s-1}^{n-1}, \\ f_s^{n-1} \end{cases} \\ \quad \text{for } s > 0, n > 0 \\ 0, \quad otherwise \end{cases} \quad \cdots\cdots (1)$$

The total expected revenue f_s^n that can be generated of a reservation request for a room in booking class i accepted during decision period n is given by $f_s^n = F_i + f_{s-1}^{n-1}$. If the reservation request is denied, the total expected revenue is given by f_s^{n-1}.

Therefore, a reservation request for booking class i during period n is accepting if and only if

$$F_i + f_{s-1}^{n-1} \geq f_s^{n-1} \quad \cdots\cdots (2)$$

Otherwise, the reservation request will be denied. With the expected marginal value, we can make optimal room allocation decisions and minimize data storage requirements.

$$f_s^n = \begin{cases} P_0^n f_s^{n-1} + \sum_{i=1}^{I} P_1^n \sum_{m=1}^{M_i} P_{im}^n \\ \qquad \max\begin{cases} mF_i + f_{s-m}^{n-1} \\ f_s^{n-1} \end{cases}, \text{for } s > 0, n > 0 \\ 0, \qquad otherwise \end{cases} \quad \cdots\cdots (3)$$

Let $\delta(n,s) = f_s^n - f_{s-1}^n$, $s = 1, 2\cdots$ be an expected marginal value of a room decision period n, given booking capacity s. For a given s, (n, s) is non-decreasing in n.

4.1.2 Model II : Single-Night Stay with Group Reservations

To determine optimal room allocation strategies for allowing group reservation requests, a similar technique used for developing Model I is used. Let P_{im}^n denote the probability that a reservation request for class i in decision period n is for m rooms, $m = 1, 2\cdots$M, where M is the maximum number of rooms allowed to be booked for each reservation request. It is assumed that a reservation request for m rooms is either accepted or denied in its entirety.

A reservation request for booking class i during decision period n will be accepted if and only if $mF_i + f_{s-m}^{n-1} \geq f_s^{n-1}$ Otherwise, the reservation will be denied.

Let $\delta_m(n,s) = \frac{1}{m}(f_s^n - f_{s-m}^n)$ be the expected marginal value of a room for reducing the room inventory of size s by m rooms simultaneously (as a group) in decision period n. For a given s and m, $\delta_m(n, s)$ is non-decreasing in n.

4.1.3 Model III : Single-Night Stay Considering Group Reservation and Downgrading

By assuming that the decision to downgrade to the higher-quality room type is based on the total expected demand during the remainder of the booking periods relative to the available capacity at the time the request for the room arrives at the CRS (Computer Reservation System). This time-dependent factor is called the demand the demand factor. Specifically, the demand factor is calculated by dividing the total demand by the fixed available rooms within a room type that is of a higher quality than one that is being requested during the remainder of the booking periods. The decision to downgrade is limited to neighboring higher-quality room types. This is because a lesser quality room type is not a proper substitute for the requested room type. By combing the strategies developed in Model II and this simple decision rule to downgrade, the decision to downgrade can be made efficiently.

4.2 Room Allocation Model for Multi-Night Case

- Heuristic for Handling Multi-Night Reservation Requests

A static mathematical programming formulation-based heuristic (SP) is proposed to handle multi-night stay problems. This heuristic is designed to allocate the number of rooms for multi-night reservation requests. Before describing the heuristic, the following notations are introduced :

H	: number of multi-night stays allowed by the CRS, $h = 1, 2\cdots H$
J	: total number of stay nights included in the rolling window, $j = 1, 2\cdots J$
K	: number of booking classes being offered, $i = 1, 2\cdots$I
C	: number of room types being considered, $c = 1, 2\cdots C$
$\lambda_{i,j,h}(t)$	: arrival rate of the i-th booking class requesting h nights, starting on the j-th night during the booking period t
$i_{i,j,h}$	: number of i-th booking class requests for h nights to accept, starting on the j-th night, and
$\psi(j)$	: number of available rooms to rent on the j-th night.

Step 1. For each j∈ J, compute the expected number of reservation requests for both single and multi-night stays.

$e_{i,j,h}$ = the expected number of i-th booking class arrivals during the remainder of the booking periods that request a room for h nights starting on the j-th stay night.

$$e_{i,j,h} = \int_{1}^{t} \lambda_{i,j,h}(\tau)d\tau \quad \text{for } all\ i \in I,\ j \in J,\ h \in H \cdots\cdots (4)$$

Step 2. Solve the following integer programming problem :

$$Max \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{h=1}^{H} h F_i x_{i,j,h} \cdots\cdots (5)$$

Subject to :

$$x_{i,j,h} \le e_{i,j,h} \quad \text{for } all\ i \in I,\ j \in J,\ h \in H \cdots\cdots (6)$$

$$\sum_{i=1}^{I}\sum_{h=1}^{H} x_{i,j,h} \le \psi(j) \quad \text{for } all\ j \in J \cdots\cdots (7)$$

$$\sum_{i=1}^{I}\sum_{h=2}^{H} x_{i,j,h} + \sum_{i=1}^{I}\sum_{h=1}^{H} x_{i,(j+1),h} \le \psi(j+1) \quad \text{for } all\ j \in J \cdots\cdots (8)$$

$$\sum_{i=1}^{I}\sum_{h=3}^{H} x_{i,j,h} + \sum_{i=1}^{I}\sum_{h=2}^{H} x_{i,(j+1),h} + \sum_{i=1}^{I}\sum_{h=1}^{H} x_{i,(j+2),h} \le \Psi(j+2) \quad \text{for } all\ j \in J \cdots\cdots (9)$$

$$\vdots$$

$$\sum_{i=1}^{I} x_{i,j,H} + \sum_{i=1}^{I}\sum_{h=H-1}^{H} x_{i,(j+1),h} + \cdots + \sum_{i=1}^{I}\sum_{j=1}^{H} x_{i,(j+H-1),h} \le \psi(j+H-1) \quad \text{for } all\ j \in J \cdots\cdots (10)$$

$$x \ge 0, \text{and} integer \quad \text{for } all\ i \in I,\ j \in J,\ h \in H \cdots\cdots (11)$$

In the above integer programming formulation, the objective function in Equation (5) maximizes the total revenue for those nights in consideration ; the constraint set in Equation (6) ensures that the number of rooms allocated will not exceed the expected number of arrivals; the constraint set in Equation (7), (8), (9), and (10) limits allocation of rooms to available rooms on each j night; and the constraint set in Equation (11) imposes the integrality of the decision variables.

Due to the static nature of the mathematical programming formulation, integer programming (IP) formulation must be resolved as often as possible with updated capacity. In the real-world applications, planning horizon is divided into k points called "checkpoints". The static model is solved repeatedly at these different checkpoints prior to the actual stay night in consideration. This implies that the foremost objective of the proposed IP formulation is to obtain integer solutions efficiently.

Fortunately, IP formulation developed for handling multi-night reservation requests corresponds to that of the transportation problem.

Although the transportation problem gets its name from a particular application, it should be viewed as a problem with a specific mathematical structure. A great variety of seemingly unrelated problems also exhibit this particular mathematical structure, such as one that has been developed for handling multi-night room allocation problems.

It can be can be observed that the input parameters of the constraint sets in Equation (6) of the proposed IP formulation do not correspond to integer values. This problem is easily handled by rounding down to the next integer value. Since all variables are required to have an integer value, this rounding down does not alter the optimal solution of the IP formulation.

5. Computational Results

To compare the performance of these heuristics to that of the optimal value, a statistical estimate of the UB (Upper Bound) of an optimal solution is obtained. This UB is computed based on perfect information about future reservation requests and customer arrivals.

The first heuristic, HEURI, combines the dynamic Model I that optimally allocates single-night single-room requests with an integer programming-based SP heuristic for handling multi-night reservation request. The second heuristic, HEURII, combines the dynamic Model II that optimally allocates single-night group requests with an integer programming-based SP heuristic for handling multi-night reservation requests. Lastly, the third heuristic, HEURIII, combines the dynamic Model III that allows downgrading between different room types with an integer programming-based SP heuristic for handling multi-night reservation requests.

All proposed heuristics require the SP heuristic to be solved, as needed (i.e. when the current strategy fails to accept current reservation requests). Also, in order to prevent unexpected situations where peak reservation requests are received by CRS during several successive booking periods, the SP heuristic is solved at least once at the end of each booking period (each day). Additionally, a base line heuristic, HEURIV, is also computed. This heuristic represents the simplest room allocation strategy that accepts reservation requests as long as capacity is available within the requested room type.

<Table 1> Scenarios and Corresponding Parameters

Scenario	No. of Room types	No. of Booking Class	Max. Group Size	Max. Multinight Request
Scenario	2	(2,3)	2	2

The booking prices used under Scenario 1 are $300 for the higher-quality room type and $109.47, $85.58, and $75.30 for the lower-quality room type, respectively.

In the scenario, there is demand factors(DF) that has nine different levels of total demand relative to capacity. It used to analyze the sensitivity of expected revenue gained by each heuristic.

$$DF = \frac{Total\ anticipated\ demand}{Total\ capacity\ of\ hotel}$$

These different demand factors are obtained by holding the total demand constant and varying the capacity of the hotel to generate nine different demand factors. These nine different demand factors will show the situations of proposed heuristics. If DF is much below 1.0, then there are more rooms than demand, it is anticipated that the proposed heuristics will give little revenue is anticipated from using the proposed heuristics. Additionally, by computing the performances of heuristics over various demand factors, the effects of overbooking can be observed. If the demand factor is above 1.0, then high revenue is expected.

The performances of proposed heuristics under Scenario are given in <Table 2> as percentages of the UB. The first column in <Table 2>. shows the total capacity of the hotel, and the second column contains the demand factor. Again, the ordering of the capacity vector is such that the capacity of the highest-quality room type is listed first. Column 3, 4, 5 and 6 show the performance of heuristics HEURI, HEURII, HEURIII, and HEURIV, respectively.

By using with the other source of revenue such that Food, Bar, Games and so on can clearly outperform better than just using room price. For the demand factors upper 1.76, the performance of HEURI and HEUR II clearly better than HEUR IV. Between HEURI and HEURII, HEURII performs better than HEURI. With this results, accepting single-night group requests is can increase the total revenue.

When demand factors range from 2.20 to 3.20 HEURIII has better performance than HEURIV. But, demand factor range is over 1.95, it is better use HEURIV. This shows that HEURIII can be applied only if demand factor is high. And also without considering the potential benefits of withholding rooms for the future customers for which room has higher value can not increase the overall revenue.

Considering demand factor below 1.46, it looks like HEURIV performance is the best. However, this conjecture can be occurred without noticing walk-ins. HEURI and HEURII are allow more rooms to be free to be used at the operational level. Hotels will always have walk-ins, and these unreserved rooms can be used to handle walk-ins during actual stay nights. Only if rooms can be sold at any upper price than the usual price, then the overall revenue can be obtained greater than the base line heuristic.

It can be observed from the Table that HEURIII has the worst performance when demand factor is below 1.06. The reason is HEURIII usually accepts reservation requests with the lesser booking piece charged to customers. Namely, HEURIII trying to accept reservation requests as long as rooms available in both rooms type.

6. The Results

For demand factors greater than 1.76, the increase in revenue from using advanced heuristics HEURI and HEURII are noticeably higher than using the base line heuristic HEURIV.

For demand factors lower than 1.0, need to be considered that hotels will always have walk-ins, and these unreserved rooms can be used to handle walk-ins during actual stay nights. Only if rooms can be sold at any upper price than the usual price, then the overall revenue can be obtained greater than the base line heuristic.

<Table 2> Revenue of Proposed Heuristics as a Percent of the Upper Bound under Scenario

Capacity (No. of rooms)	Demand Factor	HEUR I with UB (%)	HEUR II with UB (%)	HEUR III with UB (%)	HEUR IV with UB (%)
(10,100)	3.20	96.47	97.42	95.66	91.61
(30,130)	2.20	97.79	98.26	96.35	95.38
(40,140)	1.95	98.56	99.03	94.08	96.41
(50,150)	1.76	98.46	99.11	90.45	97.28
(70,170)	1.46	97.54	98.58	94.74	98.22
(90,190)	1.25	94.70	96.54	94.43	98.98
(115,215)	1.06	93.75	95.48	94.16	98.77
(140,240)	0.92	92.45	83.93	76.63	99.91
(170,270)	0.80	92.25	83.75	76.46	99.91

Downgrading can be applied when demand factor is higher than 2.20, then the overall revenue benefits from using advanced heuristics much higher than the revenue benefit from using the base line heuristic. And the performance of the heuristic HEURI is lower than HEURII or HEURII.

7. Future Research

The direction of the hotel's available capacity must be both tactical and strategic. Capacity management has been one further tactic adopted by hoteliers as a source of equating demand with supply. This can be seen as an effective strategy where the aim is to improve overall revenue. To improve overall revenue, analyzing the pattern of customer is needed. Finding a distribution of customers booking pattern will be helpful to predict allocation of available bedroom capacity to pre-determined market segment at optimum price.

RM is concerned with market sensitive pricing of fixed room capacity relative to specific market characteristics. Based on historical data and current trend information about hotel management, trying to show that reallocating room by the other sources of revenue will make more revenue than without using the other sources of revenue.

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