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Estimations of Parameters in Multi-component Series Systems Using Masked Data

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Abstract. The exact cause of the system's failure is often unknown in the masked system lifetime data. In such type of data, there are two observable quantities, namely (i) the systems time to failure and (ii) the set of systems components that contains the component, which might cause the system to fail. Our objective in this paper is to use the maximum likelihood procedure in the presence of masked data to make inference for the reliability of the system's components. We assume a multi-component series system where each component has a constant failure rate. Different cases that permit for closed form solutions of point estimates are considered. The results obtained in this paper generalize other published results.

Key Words : *Exponential distribution, Constant failure rate, masked data, partial masked data, maximum likelihood procedure.*

1. INTRODUCTION

Reliability estimate of each component in a multi-component system based on life data from the system is an important problem to be attempted. Such estimates are extremely useful. The reason is that they reflect the components' reliability after their assembly into an operational system. These estimates can be used, under some certain conditions, to predict the reliability of a new configuration of components in a new system

One can derive the estimations of reliability components based on system life-test data by considering a series system assumption and applying a competing risk model (Moeschberger and David, 1971).

In competing risk model, the observable quantities are: 1) the systems' lifetime, and 2) the exact component caused the system to fail. The competing risk model is widely

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used to estimate the component life distribution parameters, see Lawless (1982) and Nelson (1982).

In many reliability situations and life testing, the exact cause of system to fail is often either unknown or isolated to a subset of the system components. The data collected under such environment are called masked system life test data (Usher and Hodgson, 1988). Masking is usually due to limited resources for diagnosing the causes of system failures as well as the modular nature of the system. For example, if a computer system fails, a repairman may isolate the cause of the system failure to a module of several components. Therefore he can replace the whole module with a new one to bring back the system into operation. In this case, the observable quantities for each system on the life test are: 1) the system lifetime, and 2) the set of components that includes the component actually caused the system to fail.

The objective of this approach is to make inference for the reliability of system's components where the system's reliability can be derived from the component's reliability and the system's configuration.

Several studies used masked system life test data to make inference for the reliability components and the component life distribution parameters in multi-component systems.

The problem of a two-component series system when the system's components have constant failure rates was studied by Miyakawa (1984). He derived closed form expressions for the maximum likelihood estimators for the parameters included based on masked data. Usher and Hodgson (1988) extended Miyakawa's results to a three-component series system using the same assumption that the failure rates of the components are constant.

Guess et al. (1991) extended and clarified the derivation of the likelihood function under the assumption that masking is independent of the exact failure cause.

Under the same assumption that the system components have constant failure rates Lin et al. (1993) in the case of three-component series system, derives the exact maximum likelihood estimates, using masked system data.

Maximum likelihood and Bayes estimates of the values of reliability of system's components are derived by Sarhan (2001) in the case of n component series system when the components have constant failure rates.

Iterative maximum likelihood procedure is used by Usher (1996) in the case of two-component series system when the system's components life times have Weibull distributions. He illustrated the approach with a simple numerical example.

The maximum likelihood estimators for the parameters included in the cases of two-component and three-component series systems are derived by Sarhan (2003) under the assumption that the lifetime distributions of the components are Weibull. He derived closed-form expressions for maximum likelihood estimates in some particular cases, which generalize the results obtained by Usher and Hodgson (1988). The maximum likelihood and Bayes estimators for the parameters included in a parallel system are derived by Sarhan and El-Bassiouny (2003) under the assumption that the life times of the system's components have complementary exponential distributions.

The aim of this paper is to derive the exact maximum likelihood estimates (mle's) of the component life distribution parameters and the values of reliability functions of the system and its components using masked system life test data. We consider a series system with J independent and non-identical components each has a constant failure rate. The likelihood equations of the unknown parameters are presented in the general case in sections 2. Different cases that permit for closed form solutions of the likelihood equations are studied in Section 3. Section 4 introduces numerical studies and conclusion.

The following notations and assumptions are considered throughout this paper:

Notations and Assumptions

1. The system is a series system with J independent components.
2. n independent and identical systems are put on the life test. The test is terminated when all systems failed (there is neither replacement nor repair).
3. X_{ij} denotes the lifetime of component j, $j = 1,2,\cdots,J$, in the system i, $i = 1,2,\cdots,n$.
4. $X_i = \min_{1\le j\le J}(X_{ij})$ is lifetime of the system i, $i = 1,2,\cdots,n$.
5. For a fixed j, $j = 1,2,\cdots,J$, the random variables $X_{1j}, X_{2j},\ldots,X_{nj}$ are independent and identically distributed each has exponential distribution with mean $E[X_{ij}] = \frac{1}{\lambda_j}$, $i = 1,2,\cdots,n$. That is, the probability density function of component j in system i $(i = 1,2,\ldots,n;\ j = 1,2,\ldots,J)$ is

$$f_j(x) = \lambda_j \exp\{-\lambda_j x\}, \lambda_j > 0, x \ge 0 \tag{1}$$

   The reliability function of component j in system i $(i = 1,2,\ldots,n;\ j = 1,2,\ldots,J)$ is

$$\overline{F}_j(x) = \exp\{-\lambda_j x\}, \lambda_j > 0, x \ge 0 \tag{2}$$

   The hazard rate function of component j in system i $(i = 1,2,\ldots,n;\ j = 1,2,\ldots,J)$ is

$$h_j(x) = \lambda_j \tag{3}$$

6. The observable quantities for system i on the life test are: (i) the system lifetime X_i and (ii) a set S_i of system's components that contains the component causes the system i to fail. Note that $S_i \subseteq \{1, 2,\cdots,J\}$.
7. Masking is s-independent of the cause of failure.

2. THE LIKELIHOOD FUNCTION AND MLE

The likelihood function becomes

$$L = \prod_{i=1}^{n}\left\{\sum_{j\in S_i} h_j(x_i)\prod_{l=1}^{J}\overline{F}_l(x_i)\right\} \tag{4}$$

Substituting from (2-3) into (4), we get

$$L=\prod_{i=1}^{n}\left\{\sum_{j\in S_i}\lambda_j\right\}\exp\left\{-\sum_{j=1}^{J}\lambda_j\sum_{i=1}^{n}x_i\right\} \tag{5}$$

Let $\tau=\sum_{i=1}^{n}x_i$, then L takes the following form

$$L=\prod_{i=1}^{n}\left\{\sum_{j\in S_i}\lambda_j\right\}\exp\left\{-\tau\sum_{j=1}^{J}\lambda_j\right\} \tag{6}$$

The log-likelihood function is

$$\ln L=\sum_{i=1}^{n}\ln\left\{\sum_{j\in S_i}\lambda_j\right\}-\tau\sum_{j=1}^{J}\lambda_j \tag{7}$$

The first partial derivatives of $\ln L$ with respect to λ_l $(l=1,2,\ldots,J)$ are

$$\frac{\partial\ln L}{\partial\lambda_l}=\sum_{i=1}^{n}\frac{\sum_{j\in S_i}\delta_{jl}}{\sum_{j\in S_i}\lambda_j}-\tau\sum_{l=1}^{J}\delta_{jl} \tag{8}$$

where

$$\delta_{jl}=\begin{cases}1 & if\ j=l,\\ 0 & if\ j\neq l.\end{cases}$$

Setting $\frac{\partial\ln L}{\partial\lambda_l}=0$, for $(l=1,2,\ldots,J)$, we get the likelihood equations as in the following form

$$\sum_{i=1}^{n}\frac{\sum_{j\in S_i}\delta_{jl}}{\sum_{j\in S_i}\lambda_j}-\tau\sum_{j=1}^{J}\delta_{jl}=0 \tag{9}$$

The mle's of the parameters λ_l $(l=1,2,\ldots,J)$ can be derived by solving the above system of non-linear equations (9) with respect to λ_l $(l=1,2,\ldots,J)$. As it seems such system of non-linear equations has no closed form solution in λ_l $(l=1,2,\ldots,J)$. Usher ... studied this problem under some restriction when $J=2$ and $J=3$. In what follows we present some special cases that permit closed form solutions when $J>3$ as well as when $J=2$ and $J=3$.

3. MAIN RESULTS

In that follows we study some special cases, which give closed form solutions of the maximum likelihood estimates of the unknown parameters.

3.1 Case 1

Assume in this case that there is neither partial nor completely masking. It means that the cause of system failure is known. Let n_j denote the number of observation when the component j causes the system failure, namely $S_i = \{j\}$, $(j = 1,2,...,J)$. That is, $n = \sum_{j=1}^{J} n_j$. The likelihood equations in this case become

$$\frac{n_j}{\lambda_j} - \tau = 0,\ j = 1,2,...,J .$$

Therefore, the mle's of the parameters λ_j $(j = 1,2,...,J)$ are given by the following forms:

$$\hat{\lambda}_j = \frac{n_j}{\tau},\ j = 1,2,...,J .$$

3.2 Case 2

In this case we assume that the data is either not masked or completely masked (no partial masking). To explain such case we need the following additional assumptions: Let n_j denote the number of observation when the cause of system failure is known, that is $S_i = \{j\}$, $(j = 1,2,...,J)$. Let also n' denote the number of observations when the cause of system failure is completely masked, that is $S_i = \{1,2,...,J\}$. That is, $n = n' + \sum_{j=1}^{J} n_j$. The likelihood equations in this case are reduced to the following form:

$$\frac{n_j}{\lambda_j} + \frac{n'}{\sum_{j=1}^{J} \lambda_j} - \tau = 0,\ j = 1,2,...,J . \tag{10}$$

Solving the above system one can deduce the mle's of the parameters λ_j $(j = 1,2,...,J)$ as in the following forms:

$$\hat{\lambda}_j = \frac{n_j}{\tau}\left[1 + \frac{n'}{\sum_{j=1}^{J} n_j}\right],\ j = 1,2,...,J . \tag{11}$$

The forms (11) generalize the results obtained by Usher and Hodgson (1988) and Lin et al. (1993), when J=2, 3. Note that the forms obtained in case 1 can be derived from (11) by setting $n' = 0$. It means that case 2 generalizes case 1.

3.3 Case 3

In this case we assume that the data is either not masked or partial masked when the cause of the system to fail is isolated to a set of two components, say components r and k.

In this case we need the following additional assumptions: Let n_j denote the number of observation when the cause of system failure is known, that is $S_i = \{j\}$, $(j = 1,2,...,J)$. Further, let n_{rk} denote the number of observation when the cause of system failure is isolated to be either component r or component k, that is $S_i = \{r,k\}$, $(r,k \in \{1,2,...,J\})$. That is, $n = n_{rk} + \sum_{j=1}^{J} n_j$.

The likelihood equations for this case is reduced to the following form:

$$\begin{aligned} &\frac{n_j}{\lambda_j} - \tau = 0, \; j = \{1,2,...,J\} \setminus \{r,k\}, \\ &\frac{n_j}{\lambda_j} + \frac{n_{rk}}{\lambda_r + \lambda_k} - \tau = 0, \quad j = r,k. \end{aligned} \tag{12}$$

One can solve the above system (12) to get the mle's of the parameters λ_j $(j = 1,2,...,J)$ as in the following forms:

$$\hat{\lambda}_j = \frac{n_j}{\tau}, \; j \in \{1,2,...,J\} \setminus \{r,k\} \tag{13}$$

$$\hat{\lambda}_j = \frac{n_j}{\tau}\left[1 + \frac{n_{rk}}{n_r + n_k}\right], \; j = r,k. \tag{14}$$

The forms (13) and (14) extend the results obtained by Lin et al. (1993) from the case of three component series system to the case of J çomponent series system such that J is greater than or equal 3.

3.4 Case 4

In this case we assume that the data is either not masked or partial masked or completely masked (no partial masking). In this case we need the following additional assumptions: Let n_j denote the number of observations when the cause of system failure is known, that is $S_i = \{j\}$, $(j = 1,2,...,J)$. Further, let n_{rk} denote the number of observations when the cause of system failure is isolated to be either component r or component k, that is $S_i = \{r,k\}$, $(r,k \in \{1,2,...,J\})$. Let also n' denote the number of observations when the cause of system failure is completely masked, that is $S_i = \{1,2,...,J\}$. That is, $n = n' + n_{rk} + \sum_{j=1}^{J} n_j$.

The likelihood equations for this case reduce to the following form

$$\frac{n_j}{\lambda_j}+\frac{n'}{\sum_{j=1}^{J}\lambda_j}-\tau=0,\ j=\{1,2,\ldots,J\}\setminus\{r,k\},$$
$$\frac{n_j}{\lambda_j}+\frac{n_{rk}}{\lambda_r+\lambda_k}+\frac{n'}{\sum_{j=1}^{J}\lambda_j}-\tau=0,\quad j=r,k. \tag{15}$$

One can solve the above system (15) to get the mle's of the parameters λ_j $(j=1,2,\ldots,J)$ as in the following forms

$$\hat{\lambda}_j=\frac{n_j}{\tau}\left[1+\frac{n'}{n_{rk}+\sum_{j=1}^{J}n_j}\right],\ j\in\{1,2,\ldots,J\}\setminus\{r,k\} \tag{16}$$

$$\hat{\lambda}_j=\frac{n_j}{\tau}\left[1+\frac{n_{rk}}{n_r+n_k}\right]\left[1+\frac{n'}{n_{rk}+\sum_{j=1}^{J}n_j}\right],\ j=r,k. \tag{17}$$

From (11), (16) and (17) one can find out that the relations (11) can be deduced as special case from (16) and (17) by setting $n_{rk}=0$, that is if there is no partial masked data. Further, setting $n'=0$ in (16) and (17), the results given by (13) and (14) can be derived. That is case 3 generalizes both cases 1 and 2. Also the forms (16) and (17) extend the results obtained by Lin et al. (1993) from the case of three-component series system to the case of J component series system such that J is greater than or equal 3.

3.5 Case 5

Let us assume now that the data is either not masked or partial masked to a set of three-components or completely masked. For this case, the following additional assumptions are needed: Let n_j denote the number of observation when the cause of system failure is known, that is $S_i=\{j\}$, $(j=1,2,\ldots,J)$. Further, let n_{rkm} denote the number of observation when the cause of system failure is isolated to one of the components included in the set $S_i=\{r,k,m\}$, $(r,k,m\in\{1,2,\ldots,J\})$. Let also n' denote the number of observations when the cause of system failure is completely masked, that is $S_i=\{1,2,\ldots,J\}$. That is, $n=n'+n_{rkm}+\sum_{j=1}^{J}n_j$.

The likelihood equations for this case is reduced to the following form:

$$\frac{n_j}{\lambda_j}+\frac{n'}{\sum_{j=1}^{J}\lambda_j}-\tau=0,\ j=\{1,2,\ldots,J\}\setminus\{r,k,m\},$$
$$\frac{n_j}{\lambda_j}+\frac{n_{rkm}}{\lambda_r+\lambda_k+\lambda_m}+\frac{n'}{\sum_{j=1}^{J}\lambda_j}-\tau=0,\quad j=r,k,m. \tag{18}$$

In this case the mle's of the parameters λ_j $(j = 1,2,...,J)$ become

$$\hat{\lambda}_j = \frac{n_j}{\tau}\left[1 + \frac{n'}{n_{rkm} + \sum_{j=1}^{J} n_j}\right], j \in \{1,2,...,J\} \setminus \{r,k,m\} \tag{19}$$

$$\hat{\lambda}_j = \frac{n_j}{\tau}\left[1 + \frac{n_{rkm}}{n_r + n_k + n_k}\right]\left[1 + \frac{n'}{n_{rkm} + \sum_{j=1}^{J} n_j}\right], \quad j = r,k,m. \tag{20}$$

Setting $n_{rkm} = 0$ in (19) and (20), one can derive the results given by (11). That is, the results obtained in this case generalize the results derived in case 1.

3.6 Case 6

In this case it assumed that there is not completely masked data. For this case, the following additional assumptions are needed: Let n_j denote the number of observation when the cause of system failure is known, that is $S_i = \{j\}$, $(j = 1,2,...,J)$. Further, let n_{rk} denote the number of observation when the cause of system failure is either component r or component k. It means that, there are n_{rk} observation such that $S_i = \{r,k\}$, $(r,k \in \{1,2,...,J\})$. Let also n_{rkm} denote the number of observation when the cause of system failure is isolated to one of the components included in the set $S_i = \{r,k,m\}$, $(r,k,m \in \{1,2,...,J\})$. It is obvious that $n = n_{rk} + n_{rkm} + \sum_{j=1}^{J} n_j$.
The likelihood equations for this case become

$$\begin{aligned}
&\frac{n_l}{\lambda_l} - \tau = 0, \; l \in \{1,2,...,J\} \setminus \{r,k,m\}, \\
&\frac{n_l}{\lambda_l} + \frac{n_{rk}}{\lambda_r + \lambda_k} + \frac{n_{rkm}}{\lambda_r + \lambda_k + \lambda_m} - \tau = 0, l \in \{r,k\}, \\
&\frac{n_m}{\lambda_m} + \frac{n_{rkm}}{\lambda_r + \lambda_k + \lambda_m} - \tau = 0.
\end{aligned} \tag{21}$$

Solving the above system of equations, one can obtain the maximum likelihood estimates of the parameters λ_j $(j = 1,2,...,J)$ as in the following forms:

$$\hat{\lambda}_j = \frac{n_j}{\tau}, j \in \{1,2,...,J\} \setminus \{r,k,m\} \tag{22}$$

$$\hat{\lambda}_j = \frac{n_j}{\tau}\left[1+\frac{n_{rk}}{n_r+n_k}\right]\left[1+\frac{n_{rkm}}{n_r+n_k+n_m+n_{rk}}\right], \quad j=r,k, \tag{23}$$

$$\hat{\lambda}_m = \frac{n_m}{\tau}\left[1+\frac{n_{rkm}}{n_r+n_k+n_m+n_{rk}}\right]. \tag{24}$$

Setting $n_{rkm}=0$ in (22 ~ 24), we shall get the results given by (11). That is, cases 5 generalize case 2. Also the results obtained in this case generalize the results presented by Lin et al. (1993).

Reliability estimations

Zahna (1966) presented the principle of invariance for the mle, which states that: if $u(\theta)$ is a real function of the unknown θ and $\hat{\theta}$ is the mle of a parameter θ, then the mle of $u(\theta)$ becomes $\hat{u}=u(\hat{\theta})$. Once we have obtained the mle's of the parameters λ_j, $j=1,2,\cdots,J$, we can derive the mle of the value of reliability functions of the system and its components at a specified time t_0 according to such principle. The mle of the value of reliability function of component j at time t_0, $\bar{F}_j(t_0)$, is

$$\hat{\bar{F}}_j(t_0) = \exp\left\{-\hat{\lambda}_j t_0\right\}.$$

Also the mle of the value of reliability function of the system at a specified time t_0, $\bar{F}_s(t_0)$, is given by

$$\hat{\bar{F}}_s(t_0) = \exp\left\{-\sum_{j=1}^{J}\hat{\lambda}_j t_0\right\}.$$

4. NUMERICAL RESULTS AND CONCLUSION

In this section, we use Monte Carlo simulation technique to simulate masked system life test data in the presence of masking. These data are employed to get estimations of the parameters included in the studied system. It is assumed here that thirty independent and identical systems are put on life test. The test is terminated when all systems are failed. When a system fails we record its time to failure and a set of system components that contains the component causes the system failure. Each system consists of four independent components with constant failure rate. It is assumed also in the simulation study that the exact values of the parameters are $\lambda_1=0.1$, $\lambda_2=0.14$, $\lambda_3=0.12$ and $\lambda_4=0.11$.

The exact values of reliability functions of the system and its components are: $\bar{F}_s(1.2) = 0.569$, $\bar{F}_1(1.2) = 0.866$, $\bar{F}_2(1.2) = 0.845$, $\bar{F}_3(1.2) = 0.887$ and $\bar{F}_4(1.2) = 0.876$.

Table 1 presents the data generated from a series system consists of four independent components in the presence of masking system life test and considering the cases previously mentioned. Based on the data generated from the simulation, the total time on test was $\tau = \sum_{i=1}^{30} x_i = 54.228$.

Table 1. The lifetimes and cause of system failure obtained from simulation.

i	X_i	S_i				
		Case 1	Case 3	Case 4	Case 5	Case 6
1	2.16	{2}	{2}	{2}	{2}	{2}
2	1.59	{4}	{4}	{4}	{4}	{4}
3	4.03	{4}	{4}	{4}	{4}	{4}
4	4.66	{3}	{3}	{3}	{3}	{3}
5	1.82	{2}	{2}	{2}	{2}	{2}
6	0.20	{4}	{4}	{4}	{4}	{4}
7	1.21	{1}	{1, 2}	{1, 2, 3, 4}	{1, 2, 3, 4}	{1, 2, 3}
8	3.12	{2}	{1, 2}	{1, 2}	{1, 2, 3}	{1, 2, 3}
9	0.22	{4}	{4}	{4}	{4}	{4}
10	2.35	{2}	{2}	{2}	{2}	{2}
11	0.15	{3}	{3}	{3}	{3}	{3}
12	0.57	{3}	{3}	{3}	{3}	{3}
13	1.37	{1}	{1, 2}	{1, 2}	{1}	{1, 2}
14	2.03	{3}	{3}	{3}	{3}	{3}
15	1.49	{2}	{1, 2}	{1, 2}	{2}	{1, 2}
16	0.16	{1}	{1}	{1}	{1}	{1}
17	8.26	{2}	{2}	{2}	{2}	{2}
18	0.07	{3}	{3}	{3}	{3}	{3}
19	2.92	{3}	{3}	{3}	{3}	{3}
20	5.02	{3}	{3}	{3}	{3}	{3}
21	0.74	{2}	{2}	{2}	{2}	{2}
22	0.72	{4}	{4}	{4}	{4}	{4}
23	1.40	{2}	{2}	{2}	{2}	{2}
24	0.32	{1}	{1}	{1}	{1}	{1}
25	0.38	{1}	{1}	{1}	{1}	{1}
26	0.81	{2}	{1, 2}	{1, 2}	{2}	{1, 2}
27	1.94	{3}	{3}	{3}	{1, 2, 3}	{1, 2, 3}
28	0.37	{4}	{4}	{4}	{4}	{4}
29	1.48	{2}	{2}	{2}	{2}	{2}
30	2.68	{3}	{3}	{3}	{3}	{3}

Also in Table 1, one can find the information required to calculate the MLE's needed. Table 2 presents such information.

Using these data we calculate the mle of the unknown parameters by the formulae obtained and by considering the cases presented in Section 3. For each case we calculated the mle of $\overline{F}_j(t_0)$ and $\overline{F}_s(t_0)$ when $t_0 = 1.2$ time unit.

Table 3 gives the values of the mle's of the parameters and the associated percentage errors. The percentage error, say PE, associated with the estimate $\hat{\theta}$ of the parameter θ is calculated according to the following formula

$$PE = \frac{\left|\hat{\theta} - \text{exact value of } \theta\right|}{\text{exact value of } \theta} \times 100\%$$

It must be pointed out that larger sample size of a particular unit causing the failure of the system will contribute to a better estimation. This improvement in the estimation will reduce the value of the percentage error associated to the estimate of the parameter of the life distribution of that unit. Also small value of the parameter, of the exponential life distribution leads to a larger mean and hence less possibility that unit causes the failure of the system and hence smaller sample size.

Table 2. Simulated data of components cause the system failure in some cases.

#Components causes the system failure	Case 1	Case 3	Case 4	Case 5	Case 6
n_1	5	3	3	4	3
n_2	10	7	7	9	7
n_3	9	9	9	8	8
n_4	6	6	6	6	6
n_{12}	0	5	4	0	3
$n_{13}, n_{14}, n_{23}, n_{24}, n_{34}$	0	0	0	0	0
n_{123}	0	0	0	2	3
$n_{124}, n_{134}, n_{234}$	0	0	0	0	0
n'	0	0	1	1	0

Our simulation method is run for the purpose of illustrating the use of the desired forms of estimators.

Note that for case 1 smaller values for λ_1 and λ_4 compared with the values for λ_2 and λ_3 has given larger sample size for λ_2 and λ_3. Since the total time on test τ is fixed the estimation values are expected to increase with the sample size.

In Table 2, one can easily notice that, units two and three have larger sample sizes compared with units one and four. The effect of masking, in case3, on {1, 2} has lead to less observations from the individuals masked set elements.

The difference of the present sample size compared with case 1 can be approximately referred to the sample size from the masked set i.e. n_{12}. In general, one can notice that including a unit in a masked set would diverge the estimation from the actual value and hence increase the percentage error. Similarly one can explain the sample size distribution among individual units and elements of masked sets.

Table 3. The mle's and the corresponding PE's of the parameters.

Parm.	Case1		Case3		Case 4		Case 5		Case 6	
	MLE	PE	MLE	PE	MLE	PE	MLE	PE	MLE	PE
λ_1	0.092	7.796	0.083	17.02	0.080	19.88	0.084	16.43	0.082	17.83
λ_2	0.184	31.72	0.194	38.31	0.187	33.54	0.188	34.32	0.192	36.96
λ_3	0.166	38.31	0.166	38.31	0.172	43.08	0.167	39.29	0.169	40.47
λ_4	0.111	0.586	0.111	0.586	0.114	04.06	0.114	04.06	0.111	0.586
$\overline{F}_j(t_0)$	0.895	3.417	0.905	4.54	0.908	4.917	0.904	4.415	0.906	4.666
$\overline{F}_j(t_0)$	0.802	5.143	0.792	6.275	0.799	5.484	0.798	5.597	0.794	6.049
$\overline{F}_j(t_0)$	0.819	7.614	0.819	7.614	0.814	8.277	0.818	7.725	0.816	7.946
$\overline{F}_j(t_0)$	0.875	0.120	0.875	0.120	0.872	0.479	0.872	0.479	0.875	0.120
$\overline{F}_j(t_0)$	0.515	9.480	0.514	9.589	0.515	9.480	0.515	9.480	0.514	9.589

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