Multirate Digital Control for Fuzzy Systems: LMI-Based Design and Stability Analysis

Do Wan Kim, Jin Bae Park*, Young Hoon Joo, and Sung Ho Kim

Abstract: This paper studies an intelligent digital control for nonlinear systems with multirate sampling. It is worth noting that the multirate control design is addressed for a given nonlinear system represented by Takagi-Sugeno (T-S) fuzzy models. The main features of the proposed method are that i) it is provided that the sufficient conditions for stabilization of the discrete-time T-S fuzzy system in the sense of Lyapunov stability criterion, which is can be formulated in the linear matrix inequalities (LMIs); and ii) the stability properties of the trivial solution of the digital control system can be deduced from that of the solution of its discretized versions. An example is provided for showing the feasibility of the proposed method.

Keywords: Linear matrix inequality, multirate digital control, stability analysis, Takagi-Sugeno fuzzy system.

1. INTRODUCTION

Many industrial control systems consist of an analog plant and a digital controller interconnected via A/D and D/A converters. Owing to the recent development of the microprocessor and its interfacing hardware, the digital controller has played an important role in controlling robot manipulator, chemical process, and aircraft attitude. However, such applications have often severe nonlinearities, which thus post additional difficulties to the digital control design. So far, various digital control techniques have been consistently pursued with tremendous effort by many researchers in the field. One of them is to synergetically merge the linear digital control scheme [1-3] and the Takagi-Sugeno (T-S) fuzzy-model-based control technology [4-12] which provides a way to achieve digital control [4-7,12] for nonlinear control systems.

Drawing upon recent progress in the T-S fuzzy-model-based digital control, it is observed that a number of important works have used a singlerate controller [4-7,12] to meet the stability requirements. The digital control problem was conducted as a stabilizing the discretized model of continuous-time T-S fuzzy plant in [4-6] and a stabilizing the jumped fuzzy system in [12]. However, strictly speaking, the problem reformulation is incorrect because their discretized model has the approximation error, which is directly proportional to the sampling time. One gets better exactly discretized model if one can make A/D and D/A conversions faster. But faster A/D and D/A conversions mean higher cost in implementation. In addition, the digital control system is hybrid system involving continuous-time and discrete-time, but their discussion in [4-6] only contained the stability of the digital control system in the discrete-time domain.

A multirate control approach [13-16] can be an alternative. Interestingly, advantages of applying faster A/D and D/A conversions are obtained by using A/D and D/A at different rates. Furthermore, in [15], stability analysis between the multirate digital control system and the discrete-time control system was well tackled. At this point, we attempt the multirate control for T-S fuzzy system that has not yet been fully tackled under this framework.

Motivated by the above observations, we develop an intelligent multirate control for a class of nonlinear systems under the high speed D/A converter. The main contribution of this paper is two-fold. First, we derive some sufficient conditions in terms of the linear matrix inequalities (LMIs), such that the equilibrium point is a globally asymptotically stable equilibrium point of the discrete-time fuzzy model derived by the fast discretization in the sense of Lyapunov stability criterion. Second, we show that if the discrete-time control system is globally asymptotically stable, so is

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the resulting digital control system.

This paper is organized as follows: In Section 2, we formulates the digital control problem of the fuzzy system with multirate-sampling. In Section 3, synthesis and analysis of multirate digital control system are provided. In Section 4, the chaotic Lorenz system is used to demonstrate the effectiveness of the proposed method. This paper is concluded in Section 5.

2. PROBLEM STATEMENT

In the following, let T and T' be the sampling period and the control update period, respectively. For convenience, we take $T'=T/N$ for a positive integer N, where N is an input multiplicity. Then, $t=kT+lT'$ for $k\in\mathbb{Z}_{\geq 0}$ and $l\in\mathbb{Z}_{[0,N\text{-}1]}$, where the indexes k and l indicate sampling and control update instants, respectively.

Consider a nonlinear digital control system described by

$$\dot{x}(t)=f(x(t),u_d(t)) \tag{1}$$

for $t\in[kT+lT',\ kT+lT'+T')$, $(k,l)\in\mathbb{Z}_{\geq 0}\times\mathbb{Z}_{[0,N-1]}$, where $x(t)\in\mathbb{R}^n$ is the state vector, and $u_d(t)=u_d(kT,lT')\in\mathbb{R}^m$ is the multirate digital control input. The control actions are switched with T' and N. Moreover, the digital control signals are fed into the plant with the ideal zero-order hold.

To facilitate the control design, we will develop a simplified model, which can represent the local linear input-output relations of the nonlinear system. This type of models is referred as T-S fuzzy models. The fuzzy dynamical model corresponding to (1) is described by the following IF-THEN rules [4-11]:

$$\begin{aligned}R_i:\ &\text{IF } z_1(t) \text{ is about } \Gamma_{i1} \text{ and } \cdots \text{ and } z_p(t) \text{ is about } \Gamma_{ip},\\ &\text{THEN } \dot{x}(t)=A_i x(t)+B_i u_d(t),\end{aligned} \tag{2}$$

where R_i, $i\in I_q=\{1,2,\ldots,q\}$, is the ith fuzzy rule, $z_h(t)$, $h\in I_p=\{1,2,\ldots,p\}$, is the hth premise variable, and $\Gamma_{ih},(i,h)\in I_q\times I_p$, is the fuzzy set. Then, given a pair $(x(t),\ u_d(t))$, using the center-average defuzzification, product inference, and singleton fuzzifier, the overall dynamics of (2) has the form

$$\dot{x}(t)=A(\theta(t))x(t)+B(\theta(t))u_d(t), \tag{3}$$

where $A(\theta(t))=\sum_{i=1}^{q}\theta_i(z(t))A_i, B(\theta(t))=\sum_{i=1}^{q}\theta_i(z(t))B_i$, $\theta_i(z(t))=\dfrac{w_i(z(t))}{\sum_{i=1}^{q}w_i(z(t))}, w_i(z(t))=\prod_{h=1}^{p}\Gamma_{ih}(z_h(t))$ and $\Gamma_{ih}(z_h(t))$ is the grade of membership of $z_h(t)$ in Γ_{ih}. The possibly time-varying parameter vector $\theta\in\mathbb{R}^q$ belongs to a convex polytope Θ, where

$$\Theta:=\left\{\sum_{i=1}^{q}\theta_i=1,\quad 0\leq\theta_i\leq 1\right\}.$$

It is clear that as θ varies inside Θ, $A(\theta(t))$ and $B(\theta(t))$ range over a matrix polytope

$$[A(\theta(t)),B(\theta(t))]\in\mathbf{Co}\{(A_i,B_i),i\in I_q\},$$

where $\mathbf{Co}$ denotes the convex hull. In this note, the stabilization of the polytopic model (3) is equivalent to the simultaneous stabilization of its vertices (A_i,B_i), $i\in I_q$.

The main problem in this paper is to design the multirate feedback controller such that the equilibrium point of (3) is a globally asymptotically stable in the sense of Lyapunov stability criterion. The system (3) is a hybrid system involving both continuous-time and discrete-time. This makes the traditional synthesis and analysis methodologies using purely discrete-time and continuous-time formulations difficult to apply. Because of these difficulties, we first derive the sufficient conditions to globally asymptotically stabilize the equilibrium point of the discritized version of (3), and then we show that the stability of the trivial solution of (3) with the multirate feedback controller.

3. MAIN RESULTS

3.1. Fast discretization of fuzzy system

To develop the discretized version of (3), we apply the fast discretization technique [14] to (3). In specific, we first derive a multirate discretized version of (3), and then we apply a discrete-time lifting technique to the multirate discrete-time model.

Connecting the fast-sampling operator and the fast-hold operator with $[kT+lT',\ kT+lT'+T')$, $(k,l)\in\mathbb{Z}_{\geq 0}\times\mathbb{Z}_{[0,N-1]}$, to (3) leads the multirate discrete-time plant model.

Assumption 1: Suppose that the firing strength $\theta_i(z(t))$ for $t\in[kT+lT',kT+lT'+T')$ is $\theta_i(z(kT+lT'))$. Then, the nonlinear matrices $\sum_{i=1}^{q}\theta_i(z(t))A_i$ and $\sum_{i=1}^{q}\theta_i(z(t))B_i$ of (3) can be approximated as the piecewise constant matrices $A(\theta(kT+lT'))$ and $B(\theta(kT+lT'))$, respectively. Obviously, we assume that $\theta_i(z(t))=\theta_i(z(kT+lT'))$ if T' is sufficiently small.

The next result presents the multirate discretized version of (3).

Proposition 1: The multirate discrete-time model of (3) can be approximate by

$$x(kT+lT'+T') \approx G(\theta(kT+lT'))x(kT+lT') + H(\theta(kT+lT'))u_d(kT+lT') \quad (4)$$

for $t \in [kT+lT', kT+lT'+T')$, $(k,l) \in \mathbb{Z}_{\geq 0} \times \mathbb{Z}_{[0,N-1]}$, where $G(\theta(kT+lT')) = \sum_{i=1}^{q} \theta_i(z(kT+lT'))G_i$, $H(\theta(kT+lT')) = \sum_{i=1}^{q} \theta_i(z(kT+lT'))H_i$, $G_i = \exp(A_i T')$, and $H_i = (G_i - I)A_i^{-1}B_i$.

Proof: The exact solution to (3) at $t = kT + lT' + T'$ is

$$\begin{aligned} &x(kT+lT'+T') \\ &= \Phi(kT+lT'+T', kT+lT')x(kT+lT') \\ &+ \int_{kT+lT'}^{kT+lT'+T'} \Phi(kT+lT'+T', \tau)B(\theta(\tau))u_d(\tau)d\tau. \end{aligned}$$

Under the Assumption 1, the state transition matrix $\Phi(\cdot,\cdot)$ satisfies $\frac{\partial}{\partial t}\Phi(t_0,t_0) = \sum_{i=1}^{q}\theta_i(z(kT))A_i$ and $\Phi(t,t_0) = \Phi(t,t_1)\Phi(t_1,t_0)$. Then, we get

$$\begin{aligned} &x(kT+lT'+T') \\ &= \exp(A(\theta(kT+lT'))T')x(kT+lT') \\ &+ \int_{kT+lT'}^{kT+lT'+T'} \exp(A(\theta(kT+lT'))(kT+lT'+T'-\tau)) \\ &\times B(\theta(kT+lT'))u_d(kT+lT')d\tau \qquad (5) \\ &= \exp(A(\theta(kT+lT'))T')x(kT+lT') \\ &+ \left(\exp(A(\theta(kT+lT'))T') - I\right) \\ &\times A^{-1}(\theta(kT+lT'))B(\theta(kT+lT'))u_d(kT+lT'). \end{aligned}$$

However, (5) is not represented in the polytopic structure unlike the general fuzzy system. For this reason, (5) is approximated as follows:

$$\begin{aligned} &x(kT+lT'+T') \\ &= \exp\left(\sum_{i=1}^{q}\theta_i(z(kT+lT'))A_iT'\right)x(kT+lT') \\ &+\left(\exp\left(\sum_{i=1}^{q}\theta_i(z(kT+lT'))A_iT'\right)-I\right) \\ &\times\left(\sum_{i=1}^{q}\theta_i(z(kT+lT'))A_i\right)^{-1}\sum_{i=1}^{q}\theta_i(z(kT+lT'))B_iu_d(kT+lT') \\ &\approx \sum_{i=1}^{q}\theta_i(z(kT+lT'))\exp(A_iT')x(kT+lT') \\ &+\sum_{i=1}^{q}\theta_i(z(kT+lT'))\left(\exp(A_iT')-I\right)A_i^{-1}B_iu_d(kT+lT'). \end{aligned} \quad (6)$$

Therefore, we obtain (4), the approximately discretized version of (3). □

In the proposed discretization method, two approximations are performed as follows:

$$\exp(A(\theta(kT+lT'))T') \approx G(\theta(kT+lT')), \quad (7)$$

$$\left(G(\theta(kT+lT')) - I\right)A^{-1}(\theta(kT+lT'))B(\theta(kT+lT')) \approx H(\theta(kT+lT')). \quad (8)$$

To analyze these, introduce approximation error defined by

$$e_1 = \left\|\exp(A(\theta(kT+lT'))T') - G(\theta(kT+lT'))\right\|_2.$$

Applying Taylor series expansion from the right-hand side gives

$$\begin{aligned} e_1 &= \frac{T^2}{n^2}\left\|\left(\frac{1}{2!}\left(\sum_{i=1}^{q}\theta_i(z(kT+lT'))A_i\right)^2 - \frac{1}{2!}\sum_{i=1}^{q}\theta_i(z(kT+lT'))A_i^2\right)+\cdots\right\|_2 \\ &= O\left(T'^2\right). \end{aligned} \quad (10)$$

In the same manner, approximation error in (8) is

$$\begin{aligned} e_2 &= \left\|\left(G(\theta(kT+lT')) - I\right)A^{-1}(\theta(kT+lT')) \times B(\theta(kT+lT')) - H(\theta(kT+lT'))\right\|_2 \\ &= O(T'). \end{aligned} \quad (11)$$

From (10) and (11), approximation error clearly goes to zero as T' approaches zero.

To transform the system (4) into the singlerate one, we invoke the discrete-time lifting.

Assumption 2: Suppose that the firing strength $\theta_i(z(t))$, for $t \in [kT, kT+T)$ is $\theta_i(z(kT))$. It is reasonable that $\theta_i(z(t)) = \theta_i(z(kT))$ if T is sufficiently small.

Proposition 2: Given the system (3) for $l \in \mathbb{Z}_{[0,N-1]}$ a lifted sampled input

$$\tilde{u}_d(kT) = \begin{bmatrix} u_d(kT) \\ u_d(kT+T') \\ \vdots \\ u_d(kT+NT'-T') \end{bmatrix} \in \mathbb{R}^{mN} \quad (12)$$

leads a lifted system

$$x(kT+T) \approx \widetilde{G}(\theta(kT))x(kT) + \widetilde{H}(\theta(kT))\widetilde{u}(kT) \quad (13)$$

for $t \in [kT, kT+T), k \in \mathbb{Z}_{\geq 0}$, where

$$\begin{aligned}\widetilde{G}(\theta(kT)) &= G^N(\theta(kT)) \in \mathbb{R}^{n\times n},\\ \widetilde{H}(\theta(kT)) &= [G^{N-1}(\theta(kT))H(\theta(kT)),\\ &\quad G^{N-2}(\theta(kT))H(\theta(kT)),\\ &\quad \cdots, H(\theta(kT))] \in \mathbb{R}^{n\times mN}.\end{aligned}$$

Proof: Under Assumption 2, the exact solution to (3) evaluated at $t = kT+T$ is

$$\begin{aligned}x(kT+T) &= \Phi(kT+T,kT)x(kT)\\ &+ \int_{kT}^{kT+T} \Phi(kT+T,\tau)B(\theta(kT))u_d(\tau)d\tau.\end{aligned} \quad (14)$$

Applying the discrete-time lifting, $\Phi(kT+T,kT)$ and $\int_{kT}^{kT+T}\Phi(kT+T,\tau)B(\theta(\tau))u_d(\tau)d\tau$ of (14) can be represented by

$$\begin{aligned}\Phi(kT+T,kT) &= \Phi(kT+NT',kT+NT'-T')\\ &\times \Phi(kT+NT'-T',kT+NT'-2T')\\ &\times\cdots\Phi(kT+T',kT)\\ &= \Phi^N(T')\end{aligned} \quad (15)$$

and

$$\begin{aligned}&\int_{kT}^{kT+T}\Phi(kT+T,\tau)B(\theta(\tau)) \quad u_d(\tau)d\tau\\ &= \int_{kT}^{kT+T'}\Phi(kT+NT',\tau)d\tau B(\theta(kT))u_d(kT)\\ &+ \int_{kT+T'}^{kT+2T'}\Phi(kT+NT',\tau)d\tau B(\theta(kT))u_d(kT,T')\\ &+\cdots+\int_{kT+NT'-T'}^{kT+NT'}\Phi(kT+NT',\tau)d\tau B(\theta(kT))\\ &\times u_d(kT,NT'-T')\\ &= \Phi^{N-1}(T')\int_0^{T'}\Phi(\alpha)d\alpha B(\theta(kT))u_d(kT)\\ &+\Phi^{N-2}(T')\int_0^{T'}\Phi(\alpha)d\alpha B(\theta(kT))u_d(kT+T')\\ &+\cdots+\int_0^{T'}\Phi(\alpha)d\alpha B(\theta(kT))u_d(kT+NT'-T')\end{aligned} \quad (16)$$

respectively. From Proposition 1, we know that

$$\Phi(T') \approx G(\theta(kT)), \quad (17)$$

$$\int_0^{T'}\Phi(\alpha)d\alpha B(\theta(kT)) \approx H(\theta(kT)). \quad (18)$$

Therefore, substituting (17) and (18) to (15) and (16), respectively, we can obtain the lifted system (13). □

3.2. An LMI approach to multirate digital control design

In this subsection, we convert the multirate digital control problem to the solvability of LMIs. For the system (13), we consider the following multirate feedback controller

$$u(kT) = K_l(\theta(kT))x(kT) \quad (19)$$

and have the lifted control input represented as

$$\widetilde{u}(kT) = \widetilde{K}(\theta(kT))x(kT), \quad (20)$$

where $\widetilde{K}(\theta(kT)) = [K_0^T(\theta(kT)),\ K_1^T(\theta(kT)), \cdots, K_{N-1}^T(\theta(kT))] \in \mathbb{R}^{mN\times n}$, $K_0(\theta(kT)) = \sum_{i=1}^{q}\theta_i(z(kT))K_{0i}$, and $K_l(\theta(kT)) = K_0(\theta(kT))(G(\theta(kT)) + H(\theta(kT))K_0(\theta(kT)))^l$. The closed-loop system with (13) and (20) is

$$\begin{aligned}x(kT+T,0) &\approx \left(\widetilde{G}(\theta(kT)) + \widetilde{H}(\theta(kT))\widetilde{K}(\theta(kT))\right)x(kT)\\ &= G_c^N(\theta(kT))x(kT),\end{aligned}$$

where $G_c = G(\theta(kT)) + H(\theta(kT))K_0(\theta(kT))$.

The next theorem provides the sufficient conditions for the stabilization in the sense of the Lyapunov asymptotic stability for (13).

Theorem 1: The given system (13) under (20) is globally asymptotically stable in the sense of Lyapunov stability criterion if there exist $Q = Q^T > 0$ and constant matrices F_i such that

$$\begin{bmatrix} -Q & * \\ G_iQ + H_iF_i & -Q\end{bmatrix} < 0, \quad i \in [1,q],$$

$$\begin{bmatrix} -Q & * \\ \dfrac{G_iQ + H_iF_j + G_jQ + H_jF_i}{2} & -Q\end{bmatrix} < 0, \quad i < j \in [1,q],$$

where $*$ denotes the transposed element in symmetric position.

Proof: For the system (13), choose the Lyapunov function as

$$V(x(kT)) = x^T(kT)Px(kT). \quad (24)$$

Then, the rate of increases of $V(x(kT))$ is

$$\begin{aligned}&\Delta V(x(kT))\\ &= V(x(kT+T,0)) - V(x(kT))\\ &= x(kT+T,0)^T Px(kT+T,0) - x(kT)^T Px(kT)\\ &= x^T(kT)\left(G_c^{NT}(\theta(kT))PG_c^N(\theta(kT)) - P\right)x(kT).\end{aligned} \quad (25)$$

Supposed that $G_c^T(\theta(kT))PG_c(\theta(kT)) - P < 0$, then (25) is obviously negative definite. Note that

$$G_c^T(\theta(kT))PG_c(\theta(kT)) - P$$

$$\le \frac{1}{4}\sum_{i=1}^{q}\sum_{j=1}^{q}\theta_i(z(kT))\theta_j(z(kT))\Big(\big(G_i+H_iK_{0j}+G_j+H_jK_{0i}\big)^T$$
$$\times P\big(G_i+H_iK_{0j}+G_j+H_jK_{0i}\big)-4P\Big)$$
$$=\sum_{i=1}^{q}\theta_i^2(z(kT))\Big(\big(G_i+H_iK_{0i}\big)^T P\big(G_i+H_iK_{0i}\big)-P\Big)$$
$$+2\sum_{i<j}^{q}\theta_i(z(kT))\theta_j(z(kT))\left(\left(\frac{G_i+H_iK_{0j}+G_j+H_jK_{0i}}{2}\right)^T P\right.$$
$$\left.\times\left(\frac{G_i+H_iK_{0j}+G_j+H_jK_{0i}}{2}\right)-P\right). \quad (26)$$

Thus, if the following two inequalities are negative definite, then the controlled system (21) is globally asymptotically stable.

$$\big(G_i+H_iK_{0i}\big)^T P\big(G_i+H_iK_{0i}\big)-P<0, \quad (27)$$

$$\left(\frac{G_i+H_iK_{0j}+G_j+H_jK_{0i}}{2}\right)^T P \times\left(\frac{G_i+H_iK_{0j}+G_j+H_jK_{0i}}{2}\right)-P<0. \quad (28)$$

Applying Schur complement and the congruence transformation with diag $\left[P^{-1},I\right]$ to (27) and (28), and letting $Q=P^{-1}$ and $F_i=K_{0i}P^{-1}$ yields the LMIs (22) and (23). Therefore, (21) is globally asymptotically stable. □

Remark 1: In Theorem 1, the singlerate control problem is a special case of the multirate one with $N=1$.

3.3. Stability analysis

The stability of (3) controlled by (19) can be determined solely by the information at the sampling and control instants. From that reason, we will analyze the global asymptotic stability of the closed-loop system of (3) and (19) at the control instants from stability at the sampling instants. Then, we will conclude that (3) controlled by (19) is globally asymptotically stable based on the stable properties at the sampling and control instants.

In a preparatory stage to analyze the stability, we need to show that (13) and (4) are locally controllable of which definition is as follows:

Definition 2: For the controller synthesis, it is assumed that the fuzzy system is locally controllable, that is, (A_i, B_i) are controllable.

The following modified results are extensions of Lemma 6 and Lemma 8 in [15].

Lemma 1: Assume that G_i satisfies that for every eigenvalue of G_i, none of $N-1$ points $\lambda_i \exp\left(j\frac{2\pi k}{N}\right)$, $k=1,2,\ldots,N-1$, is an eigenvalue of G_i. If the system (4) is locally controllable, so is (13).

Based on Lemma 1 and Lemma 2, the next theorem concludes global asymptotical stability for (4) controlled by (20) from global asymptotical stability of the closed-loop system with (13) and (20).

Theorem 2: If the lifted system (13) controlled by (20) is globally asymptotically stable, so is the multirate discrete-time system (4) with (20).

Proof: Equation (4) at $l=0$ is

$$x(kT+T')=G(\theta(kT))x(kT)+H(\theta(kT))u(kT).$$

Then, we compute (4) at $l=1$ as

$$\begin{aligned}x(kT+2T')&=G(\theta(kT+T'))G(\theta(kT))x(kT)\\&+G(\theta(kT+T'))H(\theta(kT))u(kT)\\&+H(\theta(kT+T'))u(kT+T').\end{aligned}$$

Proceeding forward, we can readily obtain (29) at the bottom of this page, for $l=N-1>2$. Therefore, for all $l\in\mathbb{Z}_{[0,N-1]}$, it follows that

$$\hat{x}(kT)=\widehat{G}(\theta)x(kT)+\widehat{H}(\theta)\tilde{u}(kT), \quad (30)$$

where

$$\hat{x}(kT)=\begin{bmatrix}x(kT+T')\\x(kT+2T')\\\vdots\\x(kT+NT')\end{bmatrix},$$

$$\widehat{G}(\theta)=\sum_{i_1=1}^{q}\sum_{i_2=1}^{q}\cdots\sum_{i_N=1}^{q}\theta_{i_1}(z(kT+NT'-T'))$$

$$\begin{aligned}&x(kT+NT')\\&=\underbrace{G(\theta(kT+NT'-T'))G(\theta(kT+NT'-2T'))\cdots G(\theta(kT))}_{N}x(kT)\\&+\underbrace{G(\theta(kT+NT'-T'))G(\theta(kT+NT'-2T'))\cdots G(\theta(kT+T'))}_{N-1}H(\theta(kT))u(kT)\\&+\underbrace{G(\theta(kT+NT'-T'))G(\theta(kT+NT'-2T'))\cdots G(\theta(kT+2T'))}_{N-2}H(\theta(kT+T'))u(kT+T')\\&+\cdots+H(\theta(kT+NT'-T'))u(kT+NT'-T')\end{aligned} \quad (29)$$

$$\times\theta_{i_2}(z(kT+NT'-2T'))\cdots\theta_{i_N}(z(kT))\widehat{G}_{i_1i_2\ldots i_n},$$

$$\widehat{H}(\theta)=\sum_{i_1=1}^{q}\sum_{i_2=1}^{q}\cdots\sum_{i_N=1}^{q}\theta_{i_1}(z(kT+NT'-T'))$$

$$\times\theta_{i_2}(z(kT+NT'-2T'))\cdots\theta_{i_N}(z(kT))\widehat{H}_{i_1i_2\ldots i_n},$$

$$\widehat{G}_{i_1i_2\ldots i_n}=\begin{bmatrix} G_{i_N} \\ G_{i_{N-1}}G_{i_N} \\ \vdots \\ \underbrace{G_{i_1}G_{i_2}\cdots G_{i_N}}_{N}\end{bmatrix},$$

$$\widehat{H}_{i_1i_2\ldots i_n}=$$

$$\begin{bmatrix} H_{i_N} & 0 & 0 & \cdots & 0 \\ G_{i_{N-1}}H_{i_N} & H_{i_{N-1}} & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \cdots & 0 \\ \underbrace{G_{i_1}G_{i_2}\cdots G_{i_{N-1}}}_{N-1}H_{i_N} & \underbrace{G_{i_1}G_{i_2}\cdots G_{i_{N-2}}}_{N-2}H_{i_{N-1}} & \cdots & \cdots & H_{i_1}\end{bmatrix} \tag{31}$$

in which $(i_1,i_2,\cdots,i_N)\in\underbrace{I_q\times\cdots\times I_q}_{N}$. Then, it follows from (30) and (20) that

$$\begin{aligned}\|\hat{x}(kT)\| &\le \left(\|\widehat{G}(\theta)\|+\|\widehat{H}(\theta)\widetilde{K}(\theta(kT))\|\right)\|x(kT)\| \\ &\le \underbrace{\sum_{i_1=1}^{q}\sum_{i_2=1}^{q}\cdots\sum_{i_N=1}^{q}}_{N}\sum_{j=1}^{q}\left(\|\widehat{G}_{i_1i_2\ldots i_n}\|\right. \\ &\quad \left.+\|\widehat{H}_{i_1i_2\ldots i_n}\widetilde{K}_j\|\right)\|x(kT)\|.\end{aligned} \tag{32}$$

Note that $\|\widehat{G}_{i_1i_2\ldots i_n}\|+\|\widehat{H}_{i_1i_2\ldots i_n}\widetilde{K}_j\|$ is independent of k. Therefore, $x(kT,\ lT')$ is globally asymptotically convergent at a control instant l in $[kT+T',\ kT+NT']$. □

This result shows that (3) controlled by (19) is globally asymptotically stable at every intersample points from stability of the closed-loop system of (13) and (20).

The next theorem deals with the stability of (3) controlled by (19) based on Lemma 2 and Theorem 2.

Theorem 3: If (13) controlled by (19) is globally asymptotically stable, so is the closed-loop system of (3) and (19).

Proof: It follows from (4) and (20), for $t\in[kT+lT',\ kT+lT'+T')$ that

$$\begin{aligned}\|x(t)\| &\le \|\exp(A(\theta(kT+lT'))(t-kT-lT'))x(kT+lT')\| \\ &\quad +\left\|\int_{kT+lT'}^{t}\exp(A(\theta(kT+lT'))(t-\tau))\right. \\ &\quad \left.\times B(\theta(kT+lT'))K_l(\theta(kT))x(kT)d\tau\right\| \\ &\le \exp\left(\|A(\theta(kT+lT'))\|T'\right)\|x(kT+lT')\| \\ &\quad +T'\exp(\|A(\theta(kT+lT'))\|T') \\ &\quad \times\|B(\theta(kT+lT'))K_l(\theta(kT))\|\|x(kT)\| \\ &\le \exp\left(\sum_{i=1}^{q}\|A_i\|T'\right)\|x(kT+lT')\| \\ &\quad +T'\exp\left(\sum_{i=1}^{q}\|A_i\|T'\right)\sum_{i=1}^{q}\sum_{j=1}^{q}\|B_iK_{lj}\|\|x(kT)\|.\end{aligned} \tag{33}$$

Note that $\exp\left(\sum_{i=1}^{q}\|A_i\|T'\right)$ and $T'\exp\left(\sum_{i=1}^{q}\|A_i\|T'\right)$ are independent of k and l. Therefore, we conclude that the closed-loop system of (3) and (19) is globally asymptotically stable. □

4. COMPUTER SIMULATIONS

Consider the Lorenz equation

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\\ \dot{x}_3(t)\end{bmatrix}=\begin{bmatrix}-\sigma x_1(t)+\sigma x_2(t)\\ rx_1(t)-x_2(t)-x_1(t)x_3(t)\\ x_1(t)x_2(t)-bx_3(t)\end{bmatrix}, \tag{34}$$

where $(\sigma,r,b)=(10,28,8/3)$. The fuzzy system corresponding to (34) is given by

$$\dot{x}(t)=A(\theta(t))x(t), \tag{35}$$

where the membership functions for all $x\in[x_{1min},\ x_{1max}]=[-20,30]$ are

$$\Gamma_1^1(x_1(t))=\frac{-x_1(t)+x_{1max}}{x_{1max}-x_{1min}},\quad \Gamma_1^2(x_2(t))=\frac{x_1(t)-x_{1min}}{x_{1max}-x_{1min}},$$

and the local system matrices are

$$A_1=\begin{bmatrix}\sigma & -\sigma & 0\\ r & -1 & -x_{1min}\\ 0 & x_{1min} & -b\end{bmatrix},\quad A_2=\begin{bmatrix}\sigma & -\sigma & 0\\ r & -1 & -x_{1max}\\ 0 & x_{1max} & -b\end{bmatrix}$$

Fig. 1 shows the trajectory of the fuzzy system (35).

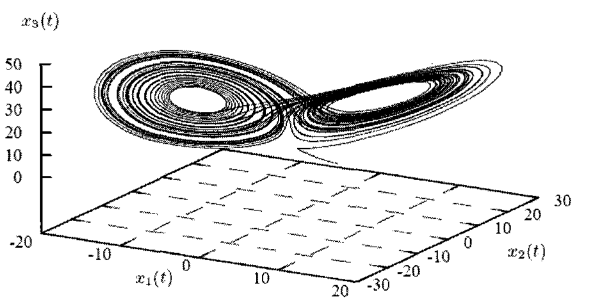

Fig. 1. Trajectory of the chaotic lorenz system.

Suppose $u_d(kT, lT') = K_l(x(kT))$ is the multirate digital control law that globally asymptotically stabilizes the equilibrium $x = [0]_{n\times 1}$ of the closed-loop system

$$\dot{x}(t) = A(\theta(t))x(t) + B(\theta(t))K_l(\theta(kT))x(kT), \quad (36)$$

where the input matrices preserving the local controllability of the system are arbitrarily chosen as

$$B_1 = B_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

To get a feel for the time response of the singlerate digital control system in the long sampling time, let us simulate the system (36) with $N = 1$. Applying Theorem 1 yields the digital gain matrices K_{0i}, as

$$K_{01} = [-50.6247 \quad -22.7771 \quad -6.5463]$$
$$K_{02} = [-50.4996 \quad -21.6944 \quad 9.1387]$$

for $T = 0.02$s,

$$K_{01} = [-28.9605 \quad -17.9937 \quad -10.0935]$$
$$K_{02} = [-28.2316 \quad -14.7501 \quad 13.5721]$$

for $T = 0.04$s, and

$$K_{01} = [-17.0271 \quad -8.0979 \quad -9.9826]$$
$$K_{02} = [-14.1617 \quad -1.8699 \quad 10.0034]$$

for $T = 0.08$s. Note that we cannot obtain the feasible digital gains in the case of $N = 1$ when $T > 0.09$s. Figs. 2 and 3 show the trajectory of the singlerate digital control system under state feedback. The initial conditions are $x_0 = [10 \quad -10 \quad -10]$. In fact, the singlerate digital control system cannot cover the intersample behavior and the discretization error. These two factors may destroy the stability of the system as T increases. Figs. 2 and 3 report the behavior of the singlerate digital control system, where the response under state feedback deviates from the equilibrium point as T increases. This is the impact of the both intersample behavior and discretization error.

We can overcome the both intersample response and discretization error by increasing N. When $N = 2$, the multirate digital control gains are taken as

$$K_{01} = [-97.9703 \quad -24.0258 \quad -3.4621]$$
$$K_{02} = [-97.9534 \quad -23.7372 \quad 4.8823]$$

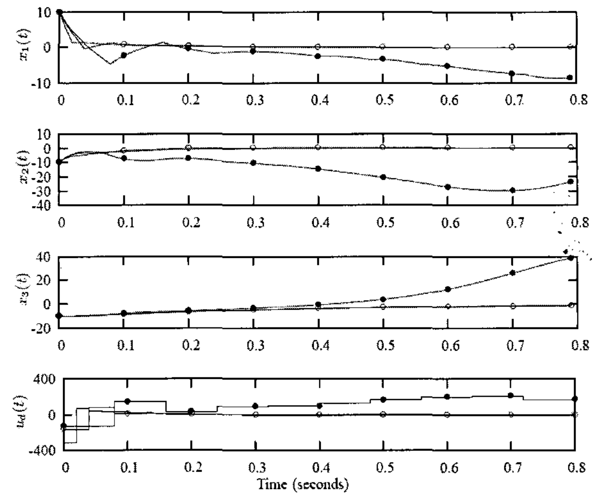

Fig. 2. Time responses of the closed-loop system under the singlerate digital controller for T = 0.02s (solid), T = 0.04s (solid with circle), and T = 0.08s (solid with bullet).

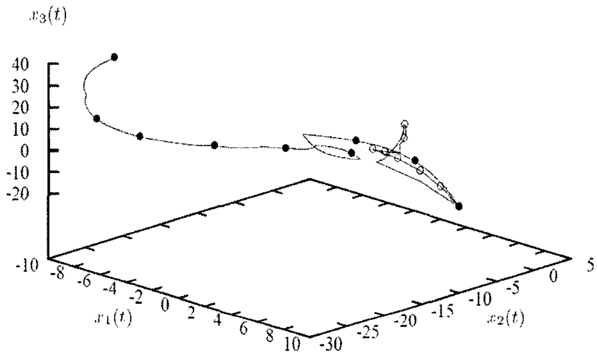

Fig. 3. Trajectories of the closed-loop system under the singlerate digital controller for T = 0.02s (solid), T = 0.04s (solid with circle), and T = 0.08s (solid with bullet).

for $T = 0.02$s,

$$K_{01} = [-50.6247 \quad -22.7771 \quad -6.5463]$$
$$K_{02} = [-50.4996 \quad -21.6944 \quad 9.1387]$$

for $T = 0.04$s, and

$$K_{01} = [-28.9605 \quad -17.9937 \quad -10.0935]$$
$$K_{02} = [-28.2316 \quad -14.7501 \quad 13.5721]$$

for $T = 0.08$s. Figs. 4 and 5 show the trajectories of the multirate digital control system under state feedback. The control $u_d(t)$ is shown on a shorter sampling period $T/2$. The same figures report that all trajectories for three different values of T are guided to the equilibrium points at origin. Note that we increase T to 0.12 where infeasibility is

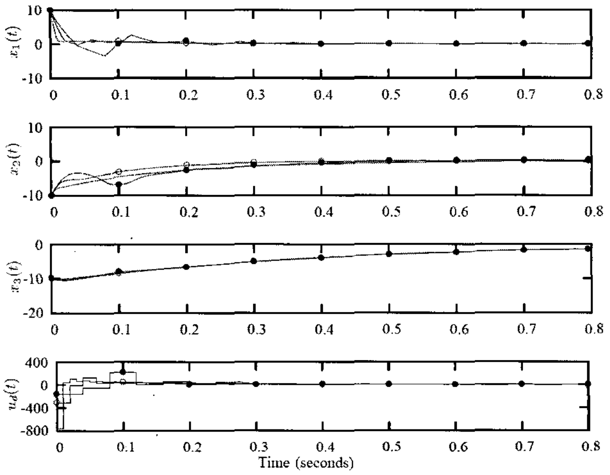

Fig. 4. Time responses of the closed-loop system under the multirate digital controller (N = 2) for T = 0.02s (solid), T = 0.04s (solid with circle), and T = 0.08s (solid with bullet).

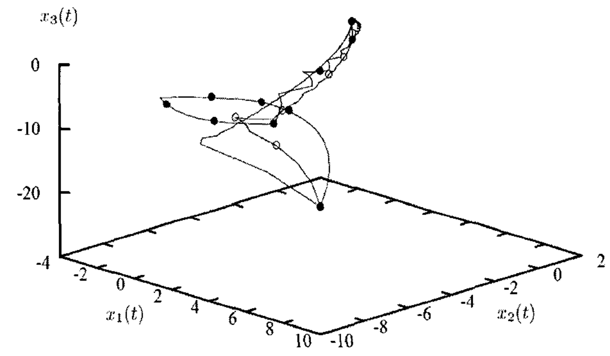

Fig. 5. Trajectories of the closed-loop system under the multirate digital controller ($N = 2$) for T = 0.02s (solid), T = 0.04s (solid with circle), and T = 0.08s (solid with bullet).

detected in the singlerate digital control. When $N = 10$, the multirate digital control gains are taken as

$$K_{01} = \begin{bmatrix} -81.8731 & -23.8708 & -4.1274 \end{bmatrix}$$
$$K_{02} = \begin{bmatrix} -81.8442 & -23.4583 & 5.8115 \end{bmatrix}$$

for $T = 0.12\text{s}$. Figs. 6 and 7 show the time responses and the trajectories of the multirate controlled systems. The stabilizability of the given system can be well guaranteed. It is noted that the proposed method guarantees the stability of the controlled system in much wider range of sampling period than the singlerate digital method in which may fail to stabilize the system especially for relatively longer sampling period, which is major advantage of the proposed method. This is because the proposed multirate control can reduce the impact of the both intersample behavior and discretization error as N increases.

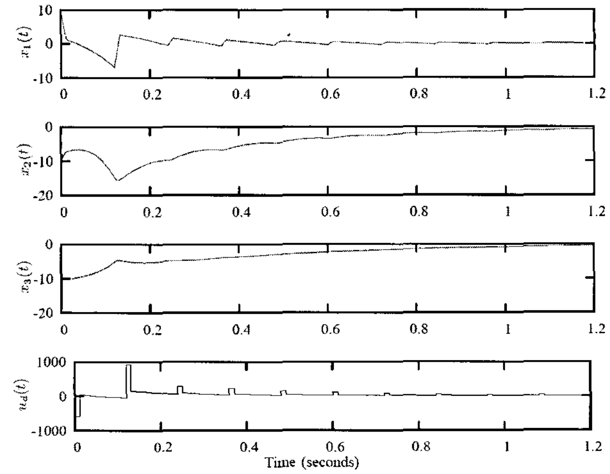

Fig. 6. Time responses of the closed-loop system under the multirate digital controller (N = 10) for T = 0.012s (solid).

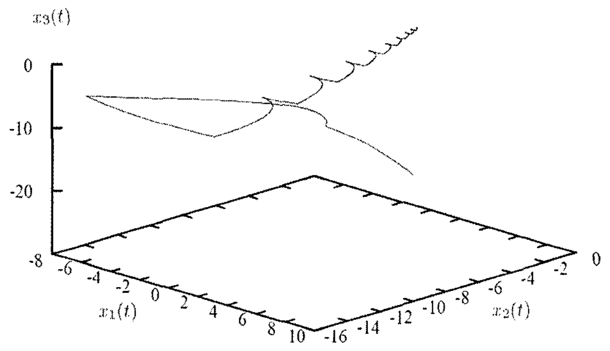

Fig. 7. Trajectories of the closed-loop system under the multirate digital controller (N = 10) for T = 0.012s (solid).

5. CLOSING REMARKS

This paper proposed the multirate control design using the LMI approach for the fuzzy system. Some sufficient conditions were derived for stabilization of the discretized model via the fast discretization. The stability of the digital control system was also guaranteed. The numerical example concretely demonstrated the advantage of the proposed multirate control method over the singlerate control method, which implies the potential of the proposed method for reliable digital industrial applications.

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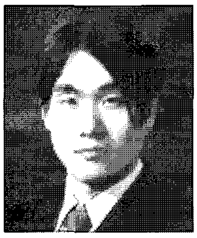

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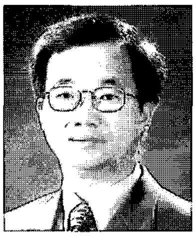

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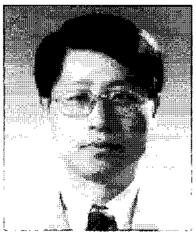

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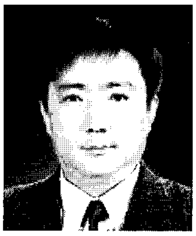

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