Robust Control Design for Flexible Joint Manipulators: Theory and Experimental Verification

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Abstract: A class of robust control for flexible joint manipulators with nonlinearity mismatched uncertainty is designed based on Lyapunov approach. The uncertainties are unknown but their values are within certain prescribed sets. No statistic information of the uncertainties is imposed. The control which utilizes state transformation via virtual control is proposed. The resulting robust control guarantees practical stability for the transformed system and later the stability for the original system is proved. The designed robust control is implemented by experiments in a 2-link flexible joint manipulator.

Keywords: Flexible joint manipulator, Lyapunov approach, mismatched uncertainty, robust, virtual control.

1. INTRODUCTION

The control design problem for flexible joint manipulators that are nonlinear and contain uncertainty, especially *mismatched* uncertainty is introduced. It has been shown that joint flexibility has a significant influence on system performance compared with rigid manipulators. The joint flexibility must be taken into account in both modeling and control design to achieve high performance. Gear elasticity, chain, and shaft wind up are common sources of joint flexibility. From modeling point of view internal deflection between the actuator and the driven link can be approximated by inputting a torsional spring at each joint.

One of the models of flexible joint manipulator was presented in [1] and we adopt this model in this paper. So far there have been various efforts devoted to the study of control for flexible joint manipulators. The references of these efforts are cited in [2]. These include exact model based approach, adaptive control, sliding modes [3,4], and robust control [5]. The exact model based approach includes singular perturbation [6], and feedback linearization scheme [7,8]. The control schemes which are designed using this approach require exact knowledge of the robot parameters. From all practical aspects, it is necessary to study the control design issue in the presence of uncertainty.

The use of adaptive control for flexible joint manipulators has been reported in numerous literatures. The control allows the existence of uncertainty in system models. The adaptive control schemes developed in [9,10]. In adaptive control the main concern has been the possible excessive transient response before adaptive parameter converges. Robust control and utilized feedback linearization is used in [11]. This approach uses a two-step estimation procedure. Stability analysis was not investigated in the closed-loop system when significant parameters are permitted in the open-loop system. In the past, numerous control strategies have been constructed based on a structural condition of the uncertainty, namely, the *matching condition* [12,13]. However, this condition does not hold in certain cases which include the flexible joint manipulator. For such a system we do not have a control input for each mode and therefore uncertainty is no longer matched. Several works have addressed the issue that the matching condition of a system does not hold. Other methods without using a two-stage algorithm are reported such as sliding mode control using a back stepping [14,15]. This control starts by taking the output as link angle, showing a convergence under the particular selection of the parameters bounds. Adaptive control using a backstepping [16] is also reported, however, yielding only convergence to some bound. This usually excludes uniform stability and uniform boundness which are crucial condition in real robot control.

Considering all aspects, we proposed a robust

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control using a backstepping algorithm under acceleration and jerk feedbacks which limits its application in some cases. The current approaches adopted in this paper are mainly inspired by the backstepping method, which is referenced by [17,18].

In this paper we consider the control problem for flexible joint manipulators. With robust control in the transformed system we can guarantee practical stability, later can also verify practical stability and good performance of the original system. By using a 2-link flexible joint manipulator, the robust control performance is verified through experiments.

2. FLEXIBLE JOINT MANIPULATORS

Consider an n serial link mechanical manipulator. The links are assumed rigid. The joints are however flexible. All joints are revolute or prismatic and are directly actuated by DC-electric motors. The dynamic equation of motion of the flexible joint manipulator can be expressed in terms of the partition of the generalized coordinates

$$\begin{bmatrix} D(q)_1 & 0 \\ 0 & j \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix}+\begin{bmatrix} C(q_1,\dot{q}_1)\dot{q}_1 \\ 0 \end{bmatrix}+\begin{bmatrix} G(q_1) \\ 0 \end{bmatrix} \tag{1}$$
$$+\begin{bmatrix} K(q_1-q_2) \\ -K(q_1-q_2) \end{bmatrix}=\begin{bmatrix} 0 \\ u \end{bmatrix}.$$

Here q is the n-vector of joint position. $D(q)$ is the link inertia matrix and J is a constant diagonal matrix representing the inertia of actuator. $C(q,\dot{q})\dot{q}$ represents the Coriolis and centrifugal force, $G(q)$ represents the gravitational force, and u denotes the input force from the actuators. Next, for the flexible joint robot define vectors $q_1 = [q^{(2)} q^{(4)} \ldots q^{(2n-2)} q^{(2n)}]^T$ and $q_2 = [q^{(1)} q^{(3)} \ldots q^{(2n-3)} q^{(2n-1)}]^T$, where $q^{(2)}$, $q^{(4)} \ldots$ are link angles and $q^{(1)}$; $q^{(3)} \ldots$ are joint angles.

$$\text{Let } q=\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \tag{2}$$

be the $2n$-vector of generalized coordinates for the system. We model the joint flexibility by a linear torsional spring at each joint and denote by K the diagonal matrix of joint stiffness. K is a constant diagonal matrix representing the torsional stiffness between links and joints (hence K^{-1} exists).

3. UNCERTAIN SYSTEM AND PRACTICAL STABILITY

We consider the following class of uncertain dynamical systems

$$\dot{\xi}(t)=f(\xi(t),t)+B(\xi(t),t)u(t), \tag{3}$$

where $t\in R$ is the time, $\xi(t) \in R^n$ is the state, $u(t) \in R^m$ is the control, $\sigma(t) \in R^0$ is the *uncertainty*, $f(\xi(t),t)$ is the system vector, and $B(\xi(t),\sigma(t),t)$ is the input matrix. From now on, unless otherwise stated, the norms in this paper are Euclidean.

Definition: A feedback control $u(t) = p(\xi(t),t)$ renders the uncertain dynamical system (5) *practically stable* if there exists constant $d_{\xi}>0$ such that for any initial time $t_0\in R$ and any initial state $\xi_0 \in R^n$, the following properties hold [19].

i) Existence and continuation of solutions
ii) Uniform boundedness
iii) Uniform ultimate boundedness
iv) Uniform stability

4. VIRTUAL ROBUST CONTROL DESIGN

The control design procedure is carried out successively. System (1) is considered. We first consider the system with constant uncertainty. Let $X_1=q_1$, $\sigma_2=\dot{q}_1$, $X_3=q_2$, and $X_4=\dot{q}_2$ also let $x_1=[X_1^T X_2^T]^T$, $x_2=[X_3^T X_4^T]^T$, and $x_2=[X_3^T X_4^T]^T$. Then we construct two subsystems as follows:

$$N_1 : \dot{x}_1(t)=f_1(x_1(t),\sigma_1)+B_1(x_1(t),\sigma_1)x_2(t), \tag{4}$$
$$N_2 : \dot{x}_2(t)=f_2(x(t),\sigma_2)+B_2(\sigma_2)u(t), \tag{5}$$

where

$$f_1(x_1,\sigma_1)=\begin{bmatrix} \dot{q}_1 \\ f_{11}(x_1,\sigma_1) \end{bmatrix},$$
$$f_{11}(x_1,\sigma_1)=-D^{-1}(q_1,\sigma_1)C(q_1,\dot{q}_1,\sigma_1)\dot{q}_1 - D^{-1}(q_1,\sigma_1)G(q_1,\sigma_1)-D^{-1}(q_1,\sigma_1)K(\sigma_1)q_1$$
$$f_2(x,\sigma_1)=\begin{bmatrix} \dot{q}_2 \\ -J^{-1}(\sigma_2)K(\sigma_1)q_2+J^{-1}(\sigma_2)K(\sigma_1)q_1 \end{bmatrix},$$
$$B_{11}(x_1,\sigma_1)=\begin{bmatrix} 0 & 0 \\ D^{-1}(q_1,\sigma_1)K(\sigma_1) & 0 \end{bmatrix},$$
$$B_2(\sigma_2)=\begin{bmatrix} 0 \\ J^{-1}(\sigma_2) \end{bmatrix}. \tag{6}$$

Here $\sigma_1\in R^{o_1}$ and $\sigma_2\in R^{o_2}$ are uncertain parameter vectors in N_1 and N_2, respectively.

Assumption 1: The parameter vectors are such that $\sigma_1\in \Sigma_1 \subset R^{o_1}$ and $\sigma_2\in \Sigma_2 \subset R^{o_2}$ where Σ_1, Σ_2 are prescribed and compact.

As seen from (6) f_1, f_2, B_2, and B_2 can not be decomposed by a nominal part and uncertain part (mismatched system). In other words, the total system (4)-(5) does not meet the matching condition [19]. Thus, the control design proposed by [14] does not apply directly. The control design here should tackle the system with nonlinearity and mismatched uncertainty. For a system with constant parameters we have the skew-symmetric property in matrix $\dot{D}(q_1,\sigma_1)$ $-2C(q_1,q_1,\sigma_1)$ [2]. This holds for flexible joint manipulators as well as rigid manipulators. The control problem is to design u which renders the system $a_{22}s_{21}$ practically stable. To design robust control, we propose a two-step procedure. Let us rewrite the first part of (1) as

$$\begin{aligned} &D(q_1)\ddot{q}_1 + C(q_1,\dot{q}_1)\dot{q}_1 + G(q_1) + Kq_1 \\ &= Ku_1 + K(q_{21} - u_1), \end{aligned} \tag{7}$$

where the "control" u_1 is implanted, called a virtual control. This does not affect the total dynamics in the subsystem N_1. The purpose of introducing u_1 is to design a controller for the first subsystem, and it is later embedded to the real control design. For given $S_1 = diag[S_{1i}]_{n\times n}, S_{1i} > 0,$ we choose a function $\overline{\rho}_1 : R^n \times R^n \to R+$ such that for all $\sigma_1 \in \Sigma_1$

$$\overline{\rho}_1(q_1,\dot{q}_1,\sigma_1) \ge \| \phi_1(q_1,\dot{q}_1,\sigma_1)\|, \tag{8}$$

where

$$\begin{aligned} \phi_1(q_1,\dot{q}_1,\sigma_1) &:= -G(q_1,\sigma_1) - K(\sigma_1)q_1 \\ &+ D(q_1,\sigma_1)S_1\dot{q}_1 + C(q_1,\dot{q}_1,\sigma_1)S_1 q_1. \end{aligned} \tag{9}$$

ϕ_1 comes from the mathematical derivation from the proof, which is shown in (37) in driving the derivative of Lyapunov function. If we define a different Lyapunov function ϕ_1 can take a different function. $\overline{\rho}_1$ is a bounding function of the defined ϕ_1. This function needs to be obtained as small as possible to avoid the possible over gain problem. S_1 is utilized for mathematical derivation from the defined Lyapunov function. Using this Lyapunov function, of course, the positive definite and decrescent condition satisfied. S_1 has a physical meaning in terms of weighting between Z_1 (link angle) and Z_2 (link angular velocity) and this also holds for the weighting between P gain $\overline{K}_{p1}$ and D gain $\overline{K}_{v1}$.

The stiffness matrix K can be decomposed into two parts, namely, the nominal and uncertain portions:

$$K = \overline{K} + \Delta K, \tag{10}$$

where $\overline{K}$ is the known nominal portion and ΔK is the uncertain portion. Furthermore, there exists a matrix $E \in R^{n\times n}$ such that

$$\Delta K = \overline{K}E. \tag{11}$$

Therefore we can express $K = \overline{K}(I + E)$, where I denotes the identity matrix.

Assumption 2: There exists a constant $\lambda_E > 0$ such that

$$\lambda_E := \min_{\sigma_1 \in \Sigma_1} \lambda_{\min}\left[I + \overline{K}E(\sigma_1)\overline{K}^{-1} \right] > 0. \tag{12}$$

Here, $\lambda_{\min}(\Pi)$, $\lambda_{\max}(\Pi)$ represents the minimum, and maximum eigenvalue respectively for the corresponding matrix Π. Since $\overline{K}$ and E are diagonal matrices, (15) is satisfied if E is such that

$$e_m := \min_i \{ \min_{\sigma_1 \in \Sigma_1} (e_i(\sigma_1))\} > -1, \quad i = 1,\cdots n, \tag{13}$$

where e_i is the diagonal component of E.

Assumption 3: The inertia matrix $D(q_1)$ is uniformly positive definite and uniformly bounded from above and below; that is, there exist positive scalar constants $\underline{\sigma}$ and $\overline{\sigma}$ such that

$$\underline{\sigma}I \le D(q_1) \le \overline{\sigma}I, \quad \forall q_1 \in R^n. \tag{14}$$

Remark 2: This assumption does not hold for arbitrary manipulators. Consider the manipulator with one revolute joint and one prismatic joint. There does not exist a finite $\overline{\sigma}$ for all $q \in R^2$.

Let the function $\sigma_1 : R^{2n} \to R_+$ be chosen such that it is $\|\mu_{1i}\| < \varepsilon_1$ (i.e., 2-times continuously differentiable) and

$$\rho_1 \ge (1+e_m)^{-1}\overline{\rho}_1(x_1). \tag{15}$$

For given scalar $\varepsilon_1 > 0$, choose virtual control u_1 such that

$$\begin{aligned} u_1(t) &= \overline{K}^{-1}(-K_{p1}q_1(t) - K_{v1}\dot{q}_1(t) + p_1(x_1(t),\rho_1(x_1(t)),\varepsilon_1)) \\ &-\overline{K}^{-1}\beta_1(\dot{q}_1(t) + S_1 q_1(t)), \end{aligned} \tag{16}$$

where

$$\begin{aligned} &K_{pl} := diag[k_{p1i}]_{n\times n}, k_{p1i>0}, \\ &K_{vl} := diag[k_{v1i}]_{n\times n}, k_{v1i>0}, \\ &\mu_1 = [\mu_{l1}\mu_{l2}\cdots\mu_{ln}]^T,\ p_1 = [p_{l1}p_{l2}\cdots p_{ln}]^T, \end{aligned} \tag{17}$$

p_{1i} is given by

$$p_{1i} = \begin{cases} -\dfrac{\mu_{1i}(x_1)}{\|\mu_{1i}(x_1)\|}\rho_1(x_1), & if \|\mu_{1i}(x_1)\| > \varepsilon_1 \\ -\sin\left(\dfrac{\pi\mu_{1i}(x_1)}{2\varepsilon_1}\right)\rho_1(x_1), & if \|\mu_{1i}(x_1)\| \le \varepsilon_1, i = 1,2,...n. \end{cases} \tag{18}$$

Note that

$$p_{1i} = \begin{cases} \le -\dfrac{\mu_{li}(x_1)}{\varepsilon_1}\rho_1(x_1), & if\ 0 \le \mu_{li}(x_1) \le \varepsilon_1 \\ \ge -\dfrac{\mu_{li}(x_1)}{\varepsilon_1}\rho_1(x_1), & if - \varepsilon_1 \le \mu_{li}(x_1) \le 0, \end{cases} \quad (19)$$

and $\|p_{1i}\| \le \rho_1$. $\beta_1 > 0$ is a scalar design parameter. The $\mu_1(\cdot)$ consists of PD control, and additionally $p_1(\cdot)$. The role of $p_1(\cdot)$ plays a role in compensating the uncertainty. The choice of β_1 will be made later. The proposed form yields the convergence region which makes it adjustable by assigning small ε_1, and ε_2 which will be shown later. Even though $\sin(\cdot)$ function is adapted to this control scheme,.still convergence region remains.

5. STATE TRANSFORMATION

We now transform the system (N_1, N_2) to a system $(\hat{N}_1, \hat{N}_2)$ as follows. First let

$$z_1 = \begin{bmatrix} Z_1^T & Z_2^T \end{bmatrix}^T, z_2 = \begin{bmatrix} Z_3^T & Z_4^T \end{bmatrix}^T, \text{and } z = \begin{bmatrix} z_1^T & z_2^T \end{bmatrix}^T,$$

where

$$Z_1 = q_1,\ Z_2 = \dot{q}_1,\ Z_3 = q_2 - u_1,\ Z_4 = \dot{q}_2 - \dot{u}_1. \quad (20)$$

This implies that $z_1 = x_1$ and $z_2 = x_2 - [u_1\ \dot{u}_1]^T$. From now on, we omit the argument on uncertainty in $D(q_1, \sigma_1)$, $C(q_1, \dot{q}_1, \sigma_1)$, and $K(\sigma_1)$ etc. if no confusion arises. Otherwise it will be denoted. Next, the dynamics of the manipulator are expressed in terms of z:

$$\hat{N}_1 : D(Z_1)\ddot{Z}_1 = -C(Z_1, \dot{Z}_1)\dot{Z}_1 - G(\dot{Z}_1) - KZ_1 + KZ_3 + Ku_1, \quad (21)$$

$$\hat{N}_2 : J\ddot{Z}_3 = -J\ddot{u}_1 - KZ_3 + KZ_1 - Ku_1 + u. \quad (22)$$

Therefore, the dynamics of $\hat{N}_1$ contains the virtual control u_1. As a sequel, the dynamics of $\hat{N}_2$ contains the information of u_1 and $\ddot{u}_1$.

We now propose the control design. For given $S_2 = diag[S_i]_{n\times n}$, $S_{2i}>0$, we choose a known scalar function $\rho_2 :\ R^{2n} \times R^n \times R^n \to R+$ such that

$$\rho_2(x_1, Z_3, Z_4) \ge \|\phi_2(x_1, Z_3, Z_4, \sigma_1, \sigma_2)\|, \quad (23)$$

where

$$\phi_2(x_1, Z_3, Z_4, \sigma_1, \sigma_2) \quad (24)$$

$$= -J(\sigma_1)u_1(x_1, Z_3, Z_4, \sigma_1, \sigma_1) - K(\sigma_1)Z_3 + K(\sigma_1)Z_1 - K(\sigma_1)u_1(x_1) + J(\sigma_2)S_2\dot{Z}_3.$$

Similar to ϕ_1 and ρ_1, which were explained before, ϕ_2 comes from the mathematical derivation from the proof, which is shown in (45) in driving the derivative of Lyapunov function $\dot{V}_2$. Based on the Lyapunov function V_2 which includes a positive definite matrix S_2, the second part of the second line in (45) can be defined by a function ϕ_2. The bounding function on ϕ_2 can be defined by ρ_2. Next, the control $u(t)$ deals with the bounding function ρ_2 by introducing p_2 in $u(t)$. S_2 is also utilized for mathematical derivation from the defined Lyapunov function V_2. Using this Lyapunov function, of course, the positive definite and decrescent condition are satisfied. S_2 has a physical meaning in terms of weighting between Z_3 (joint angle-virtual control u_1) and Z_4 (joint angular velocity-$\dot{u}_1$) and this holds for the weighting between P gain $\bar{K}_{p2}$ and D gain $\bar{K}_{v2}$.

For given scalar $\varepsilon_2 > 0$, choose the control input u as follows:

$$u(t) = -K_{p2}Z_3(t) - K_{v2}Z_3(t) + p_2(x_1(t), Z_3(t), Z_4(t), \varepsilon_2) \quad (25)$$

where

$$p_2(x_1, Z_3, Z_4, \varepsilon_2) = \begin{cases} -\dfrac{\mu_2(z_1, z_2)}{\|\mu_2(z_1, z_2)\|}\rho_2(x_1, Z_3, Z_4) & if\ \|\mu_2(z_1, z_2)\| > \varepsilon_2 \\ -\dfrac{\mu_2(z_1, z_2)}{\varepsilon_2}\rho_2(x_1, Z_3, Z_4) & if\ \|\mu_2(z_1, z_2)\| \le \varepsilon_2, \end{cases} \quad (26)$$

$$\begin{aligned} &\mu_2(z_1, z_2) = (Z_3 + S_2 Z_3)\rho_2(z_1, Z_3, Z_4), \\ &K_{p2} = diag[k_{p2i}]_{n\times n},\ k_{p2i} > 0, \\ &K_{v2} = diag[k_{v2i}]_{n\times n},\ k_{v2i} > 0. \end{aligned} \quad (27)$$

As stated before, the final control form of $u(\cdot)$ consists of PD control and $p_2(\cdot)$. The last term $p_2(\cdot)$ is also compensating the uncertainty and utilizing the virtual control $u_1(\cdot)$.

Remark 3: The control $u(t)$ relies on the acceleration and jerk signals. This can be undesirable due to possible noise or signal contamination, and it also occurs in case of sharp velocity or acceleration change However, in implementing the experiment, the acceleration or jerk can be adapted by computing the simple difference equation in a digital way as follows, which is excluding the complex computation. Then, the other remain terms can be put into the uncertainty terms, and eventually the robust control $u(t)$ counts for uncertainty by taking the bounding function. The experimental results shown later illustrate the justification of this method.

$$Z_3 = q_2 - u_1 = \frac{q_2(t+\Delta t) - u_1(t+\Delta t) - q_2(t) + u_1(t)}{\Delta t} \quad (28)$$

To avoid the differentiation of signals, a low pass filter design can be one of solutions. Also, we proposed a simple way of differentiating the signals, which seems to raise many questionable issues. Counting for acceleration feedback in the proposed controller, the possible spiky signals or noise can be embedded into the modified bounding function at $\phi_2(\cdot)$, which is modified as follows.

$$\begin{aligned} u(t) &= -K_{p2}Z_3 - K_{v2}\dot{Z}_3 + p_2 \\ &= -K_{p2}Z_3 - K_{v2}(\dot{q}_2 - \dot{u}_1)Z_3 + p_2 \\ &= -K_{p2}Z_3 - K_{v2}(\dot{q}_2 - u_1)_n \\ &\quad - K_{v2}(\dot{q}_2 - \dot{u}_1)_{un} + p_2, \end{aligned} \quad (29)$$

where $(\dot{q}_2 - \dot{u}_1)_n$ and $(\dot{q}_2 - \dot{u}_1)_{un}$ represent the nominal value calculated from the direct differentiation and noise-involved part, respectively. The $(\dot{q}_2 - \dot{u}_1)_{un}$ part can be put into the modified function $\rho_2(\cdot)$, yielding to a modified bounding function. However, there might be a question how we can estimate the bound of $(\dot{q}_2 - \dot{u}_1)_{un}$. In theoretical basis, this can be set by assigning a reasonable bounding function (affine or polynomial). If spiky signals crucially occur or noise are seriously affecting the control performance, then the controller $u(t)$ takes the computed values and the other values related to the noise can be confined by a bounding function $\|\Phi_2\| \le \rho_2$ as shown in (23) and (24). Selection of β_1 in (19) for u_1:

Let

$$\underline{\lambda}_2 = \min\{\lambda_{\min}(K_{v2}), \lambda_{\min}(S_2K_{p2})\}. \quad (30)$$

Step 1: Select λ_2 and compute proper value for λ_k where $\max_{\sigma_1 \in \Sigma_1} \|K\| \le \lambda_k$ and compute λ_E in (12).

Step 2: Choose ω_1 such that $\omega_1 > \dfrac{\lambda_k}{2\lambda_2}$.

Step 3: By ω_1 obtained from Step 2 choose β_1 such that $\beta_1 > \dfrac{\omega_1\lambda_k}{2\lambda_E}$.

Theorem 1: Subject to Assumptions 1-3, the system (21)-(22) is practically stable under the control (25). Furthermore, the size of the uniform ultimate boundedness region can be made arbitrary small by a suitable choice of ε_1 and ε_2.

Proof: Choose Lyapunov function candidates as follows

$$V(z_1, z_2) = V_1(z_1) + V_2(z_2), \quad (31)$$

where

$$V_1(z_1) = \frac{1}{2}(Z_2 + S_1Z_1)^T D(Z_2 + S_1Z_1) + \frac{1}{2}Z_1^T(K_{p1} + S_1K_{v1})Z_1, \quad (32)$$

$$V_2(z_2) = \frac{1}{2}(Z_4 + S_2Z_3)^T J(Z_4 + S_2Z_3) + \frac{1}{2}Z_3^T(K_{p2} + S_2K_{v2})Z_3, \quad (33)$$

and

$$\bar{K}_{p1} = K_{p1} + \bar{K}E\bar{K}^{-1}K_{p1}, \quad (34)$$

$$\bar{K}_{v1} = K_{v1} + \bar{K}E\bar{K}^{-1}K_{v1}. \quad (35)$$

From (34)-(35) we see that $\bar{K}_{p1}$ and $\bar{K}_{v1}$ are positive definite. To show that both V_1 and V_2 are legitimate Lyapunov function candidates, we shall prove that both V_1 and V_2 are positive definite and decrescent. The details are shown in [20].

The derivative of V_1 along the trajectory of the controlled system (21) is given by

$$\begin{aligned} \dot{V}_1 &= (\dot{Z}_1 + S_1Z_1)^T D(\ddot{Z}_1 + S_1\dot{Z}_1) \\ &\quad + \frac{1}{2}(\dot{Z}_1 + S_1Z_1)^T \dot{D}(\dot{Z}_1 + S_1Z_1) \\ &\quad + Z_1^T(\bar{K}_{p1} + S_1\bar{K}_{v1})\dot{Z}_1. \end{aligned} \quad (36)$$

From the skew-symmetric property in $\dot{D} - 2C$ it can be seen that

$$\begin{aligned} \dot{V}_1 &= (\dot{Z}_1 + S_1Z_1)^T(-G - KZ_1 + DS_1\dot{Z}_1 + CS_1Z_1 \\ &\quad + K u_1 + KZ_3) + Z_1^T(\bar{K}_{p1} + S_1\bar{K}_{v1})\dot{Z}_1. \end{aligned} \quad (37)$$

According to (9), it can be seen that

$$\begin{aligned} \dot{V}_1 &= (\dot{Z}_1 + S_1Z_1)^T(\phi_1 + Ku_1 + \bar{K}Eu_1) \\ &\quad + Z_1^T(\bar{K}_{p1} + S_1\bar{K}_{v1})\dot{Z}_1 + (\dot{Z}_1 + S_1Z_1)^T KZ_3. \end{aligned} \quad (38)$$

It follows from (16)

$$\begin{aligned} \dot{V}_1 &= (\dot{Z}_1 + S_1Z_1)^T \phi_1 \\ &\quad + (\dot{Z}_1 + S_1Z_1)^T(-K_{p1}Z_1 - K_{v1}\dot{Z}_1 + p_1 - \beta_1(\dot{Z}_1 + S_1Z_1)) \end{aligned}$$

$$
\begin{aligned}
&+\left(\dot{Z}_1+S_1Z_1\right)^T \bar{K}E\bar{K}^{-1} \\
&\quad\left(-K_{p1}Z_1-K_{v1}Z_1+p_1-\beta_1\left(\dot{Z}_1+S_1Z_1\right)\right) \\
&+Z_1^T\left(\bar{K}_{p1}+S_1\bar{K}_{v1}\right)\dot{Z}_1+\left(\dot{Z}_1+S_1Z_1\right)^T KZ_3 \\
&\le -\dot{Z}_1^T\bar{K}_{v1}\dot{Z}_1-Z_1^TS_1\bar{K}_{p1}Z_1+\left(\dot{Z}_1+S_1Z_1\right)^T\phi_1 \\
&\quad+\left(\dot{Z}_1+S_1Z_1\right)^T{}_{p1}+\left(\dot{Z}_1+S_1Z_1\right)^T\bar{K}E\bar{K}^{-1}p_1 \\
&\quad-\beta_1\lambda_E\left\|\dot{Z}_1+S_1Z_1\right\|^2+\left(\dot{Z}_1+S_1Z_1\right)^T KZ_3 \\
&\le -\underline{\lambda}_1\left\|z_1\right\|^2-\beta_1\lambda_E\left\|\dot{Z}_1+S_1Z_1\right\|^2 \\
&\quad+\left(\dot{Z}_1+S_1Z_1\right)^T\phi_1+\left(\dot{Z}_1+S_1Z_1\right)^T p_1 \\
&\quad+\left(\dot{Z}_1+S_1Z_1\right)^T p_1+\left(\dot{Z}_1+S_1Z_1\right)^T\bar{K}E\bar{K}^{-1}p_1 \qquad (39)\\
&\quad+\left(\dot{Z}_1+S_1Z_1\right)^T KZ_3,
\end{aligned}
$$

where

$$\underline{\lambda}_1=\min\left\{\lambda_{\min}\left(K_{v1}\right),\lambda_{\min}\left(S_1K_{pl}\right)\right\}. \qquad (40)$$

If $\left\|\mu_{1i}\right\|>\varepsilon_1,$

$$
\begin{aligned}
&\left(\dot{Z}_1+S_1Z_1\right)^T\phi_1+\left(\dot{Z}_1+S_1Z_1\right)^T p_1+\left(\dot{Z}_1+S_1Z_1\right)^T\bar{K}E\bar{K}^{-1}p_1 \\
&\le\sum_{i=1}^n\left\|\dot{Z}_{1i}+S_{1i}Z_{1i}\right\|\bar{\rho}_1+\sum_{i=1}^n\left(\dot{Z}_{1i}+S_{1i}Z_{1i}\right)p_{1i} \\
&\quad+\sum_{i=1}^n\left(\dot{Z}_{1i}+S_{1i}Z_{1i}\right)e_ip_{1i} \\
&\le\sum_{i=1}^n\left\|\dot{Z}_{1i}+S_{1i}Z_{1i}\right\|\left(1+e_m\right)\rho_1 \\
&\quad+\sum_{i=1}^n\left(\dot{Z}_{1i}+S_{1i}Z_{1i}\right)\left(-\frac{\dot{Z}_{1i}+S_{1i}Z_{1i}}{\left\|\dot{Z}_{1i}+S_{1i}Z_{1i}\right\|}\rho_1\right) \\
&\quad+\sum_{i=1}^n\left(\dot{Z}_{1i}+S_{1i}Z_{1i}\right)e_i\left(-\frac{\dot{Z}_{1i}+S_{1i}Z_{1i}}{\left\|\dot{Z}_{1i}+S_{1i}Z_{1i}\right\|}\rho_1\right) \\
&\le\sum_{i=1}^n\left\|\dot{Z}_{1i}+S_{1i}Z_{1i}\right\|\left(1+e_m\right)\rho_1 \qquad (41)\\
&\quad-\sum_{i=1}^n\left\|\dot{Z}_{1i}+S_{1i}Z_{1i}\right\|\rho_1-e_m\sum_{i=1}^n\left\|\dot{Z}_{1i}+S_{1i}Z_{1i}\right\|\rho_1=0.
\end{aligned}
$$

If $\left\|\mu_{1i}\right\|\le\varepsilon_1,$

$$
\begin{aligned}
&\left(\dot{Z}_1+S_1Z_1\right)^T\phi_1+\left(\dot{Z}_1+S_1Z_1\right)^T p_1+\left(\dot{Z}_1+S_1Z_1\right)^T\bar{K}E\bar{K}^{-1}p_1 \\
&\le\sum_{i=1}^n\left\|\dot{Z}_{1i}+S_{1i}Z_{1i}\right\|\rho_1+\sum_{i=1}^n\left\|\dot{Z}_{1i}+S_{1i}Z_{1i}\right\|^2\left(-\frac{\rho_1^2}{\varepsilon_1}\right) \\
&\quad-e_m\sum_{i=1}^n\left\|\dot{Z}_{1i}+S_{1i}Z_{1i}\right\|^2\frac{\rho_1^2}{\varepsilon_1} \\
&=\sum_{i=1}^n\left(1+e_m\right)\rho_1-\left(1+e_m\right)\sum_{i=1}^n\left\|\dot{Z}_{1i}+S_{1i}Z_{1i}\right\|^2\frac{\rho_1^2}{\varepsilon_1} \qquad (42)\\
&\le\frac{n\left(1+e_m\right)\varepsilon_1}{4}.
\end{aligned}
$$

Based on inequalities $ab\le\frac{1}{2}\left(a^2+b^2\right),a,b\in R,$ and $\left\|Z_3\right\|^2\le\left\|z_2\right\|^2$, we have that for any constant $\omega_1>0$,

$$
\begin{aligned}
&\left(\dot{Z}_1+S_1Z_1\right)^T KZ_3\le\left\|\dot{Z}_1+S_1Z_1\right\|\left\|Z_3\right\|\left\|K\right\| \\
&\le\left(\frac{1}{2}\omega_1\left\|\dot{Z}_1+S_1Z_1\right\|^2+\frac{1}{2}\omega_1^{-1}\left\|Z_3\right\|^2\right)\left\|K\right\| \\
&\le\left(\frac{1}{2}\omega_1\left\|\dot{Z}_1+S_1Z_1\right\|^2+\frac{1}{2}\omega_1^{-1}\left\|z_2\right\|^2\right)\left\|K\right\| \\
&\le\left(\frac{1}{2}\omega_1\left\|\dot{Z}_1+S_1Z_1\right\|^2+\frac{1}{2}\omega_1^{-1}\left\|z_2\right\|^2\right)\lambda_k, \qquad (43)
\end{aligned}
$$

This leads to

$$
\begin{aligned}
\dot{V}_1\le&-\underline{\lambda}_1\left\|z_1\right\|^2+\frac{n(1+e_m)\varepsilon_1}{4}-\beta_1\left(1+\lambda_E\right)\left\|\dot{Z}_1+S_1Z_1\right\|^2 \\
&+\left(\frac{1}{2}\omega_1\left\|\dot{Z}_1+S_1Z_1\right\|^2+\frac{1}{2}\omega_1^{-1}\left\|z_2\right\|^2\right)\lambda_k. \qquad (44)
\end{aligned}
$$

Next, the derivative of $V_2(z_2)$ along the trajectory of (22) is given by

$$
\begin{aligned}
\dot{V}_2&=\left(\dot{Z}_3+S_2Z_3\right)^T J\left(\ddot{Z}_3+S_2\dot{Z}_3\right) \\
&\quad+Z_3^T\left(K_{p2}+S_2K_{v2}\right)\dot{Z}_3 \\
&=\left(\dot{Z}_3+S_2Z_3\right)^T \qquad (45)\\
&\quad\left(-J\ddot{u}_1-KZ_3+KZ_1-KZ_1-Ku_1+u+JS_2\dot{Z}_3\right) \\
&\quad+Z_3^T\left(K_{p2}+S_2K_{v2}\right)\dot{Z}_3.
\end{aligned}
$$

It follows from (24), and (25)

$$
\begin{aligned}
\dot{V}_2&=\left(\dot{Z}_3+S_2Z_3\right)^T\left(\phi_2+u\right)+Z_3^T\left(K_{p2}+S_2K_{v2}\right)\dot{Z}_3 \\
&\le\left(\dot{Z}_3+S_2Z_3\right)^T p_2+\left(\dot{Z}_3+S_2Z_3\right)^T\phi_2-\underline{\lambda}_2\left\|z_2\right\|^2 \qquad (46)\\
&\le\left(\dot{Z}_3+S_2Z_3\right)^T p_2+\left\|\dot{Z}_3+S_2Z_3\right\|\rho_2-\lambda_2\left\|Z_2\right\|^2.
\end{aligned}
$$

If $\left\|\mu_2\right\|>\varepsilon_2$

$$
\begin{aligned}
&\left(\dot{Z}_3+S_2Z_3\right)^T p_2+\left\|\dot{Z}_3+S_2Z_3\right\|\rho_2 \\
&\le\left(\dot{Z}_3+S_2Z_3\right)^T\left(-\frac{\mu_2}{\left\|\mu_2\right\|}\rho_2\right)+\left\|\dot{Z}_3+S_2Z_3\right\|\rho_2
\end{aligned}
$$

$$\leq -\|\mu_2\| + \|\mu_2\| \\ = 0. \tag{47}$$

If $\|\mu_2\| \leq \varepsilon_2$

$$\left(\dot{Z}_3 + S_2 Z_3\right)^T p_2 + \left\|\dot{Z}_3 + S_2 Z_3\right\| \rho_2 \\ \leq \left(\dot{Z}_3 + S_2 Z_3\right)^T \left(-\frac{\dot{Z}_3 + S_2 Z_3}{\varepsilon_2} \rho_2{}^2\right) + \left\|\dot{Z}_3 + S_2 Z_3\right\| \rho_2 \\ \leq -\frac{\|\mu_2\|}{\varepsilon_2} + \|\mu_2\| \\ = \frac{\varepsilon_2}{4}. \tag{48}$$

Thus, $\dot{V}_2$ is upper bounded:

$$\dot{V}_2 \leq -\lambda_2 \|z_2\|^2 + \frac{\varepsilon_2}{4}. \tag{49}$$

Now, using (44) and (49),

$$\dot{V} = \dot{V}_1 + \dot{V}_2 \\ \leq -\lambda_1 \|z_1\|^2 + \frac{n(1+e_m)\varepsilon_1}{4} - \beta_1 \lambda_E \left\|\dot{Z}_1 + S_1 Z_1\right\|^2. \tag{50}$$

If $\underline{\lambda_2} - \frac{1}{2}\omega_1^{-1}\lambda_k > 0$ and $\beta_1 \lambda_E - \frac{1}{2}\omega_1 \lambda_k > 0$, then we have

$$\dot{V} \leq -\min\left(\underline{\lambda_1}, \underline{\lambda_2} - \frac{1}{2}\omega_1 \lambda_k\right) \|z\|^2 + \bar{k}, \tag{51}$$

where $\bar{k} := \frac{n(1+e_m)\varepsilon_1}{4} + \frac{\varepsilon_2}{4}$. Following (51) for $r_z \geq 0$, if $\|z_0\| \leq r_z$, we can satisfy the requirements of uniform boundedness, uniform ultimate boundedness and uniform stability [13]. It can be seen that $R_z \to 0$ as both ε_1 and $\varepsilon_2 \to 0$. Therefore if both ε_1 and $\varepsilon_2 \to 0$, then $\underline{d}_z$ converges to 0. Q.E.D.

Theorem 2: Subject to Assumptions 1-3, the control u given by (25) renders the original dynamic system N_1 and N_2 in (4)-(5) practically stable.

Proof: The practical stability has been shown by referring [20].

6. EXPERIMENTAL VERIFICATIONS

Consider a two-link flexible revolute joint manipulator which is modelled and manufactured in our research group (Figs. 1, 2). The designed two-link manipulator consists of link, timing belt and spring for generating or adjusting a stiffness between motor and link, electric motor, and motion controller. Here, a spring is introduced to generate a tension on the timing belt and pulley, which results in a torsional moment on the link and joint. The real two link manipulator with flexibility for experimental verification is illustrated in Fig. 2. The joint flexibility is installed at each space between link and motor. Let link angle vectors $q_1 = \left[q^{(2)} q^{(4)}\right]^T$ and joint angle vectors $q_2 = \left[q^{(1)} q^{(3)}\right]^T$. The uncertainty sets Σ_1, and Σ_2 comprise, $J(\sigma_1)$, $K(\sigma_1)$, $C(q_1, \dot{q}_1, \sigma_1)$ respectively, which are uncertain with parameter vectors σ_1 and σ_2. These are expressed as follows:

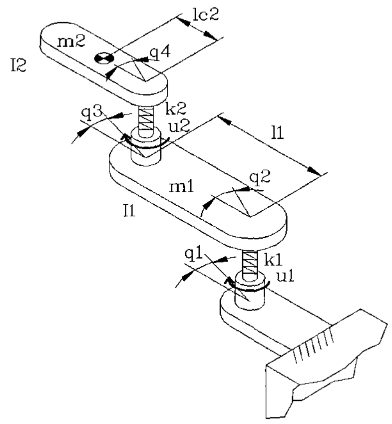

Fig. 1. A schematic diagram of two degrees of freedom manipulator with flexible joints.

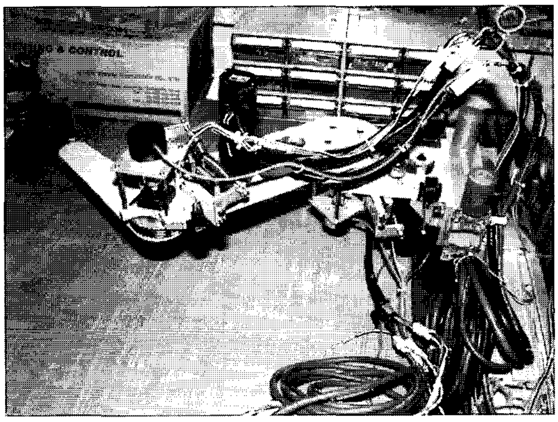

Fig. 2. Experimental setup for two degrees of freedom manipulator with flexible joints.

$$D(q_1) = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix},$$

$$C(q_1, \dot{q}_1) = \begin{bmatrix} -m_2 l_1 l_{c2} \sin q^{(4)} \dot{q}^{(4)} & -m_2 l_1 l_{c2} \sin q^{(4)} (\dot{q}^{(4)} + \dot{q}^{(2)}) \\ m_2 l_1 l_{c2} \sin q^{(4)} \dot{q}^{(2)} & 0 \end{bmatrix}$$

$$G(q_1) = \begin{bmatrix} (m_1 l_{c1} + m_2 l_1) g \sin q^{(2)} + m_2 l_{c2} g \sin(q^{(2)} + q^{(4)}) \\ m_2 l_{c2} g \sin(q^{(2)} + q^{(4)}) \end{bmatrix},$$

$$J=\begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix},$$

$$K=\begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix}=\begin{bmatrix} \bar{K}_1 & 0 \\ 0 & \bar{K}_2 \end{bmatrix}+\begin{bmatrix} \bar{K}_1 & 0 \\ 0 & \bar{K}_2 \end{bmatrix}\begin{bmatrix} e_1 & 0 \\ 0 & e_2 \end{bmatrix}, \quad (52)$$

where

$$\begin{aligned} a_{11} &= m_2 l_1 l_{c2}, \\ a_{12} &= m_2(l_1^2+l_{c2}^2)+m_1 l_{c1}^2+I_1+I_2, \\ a_{22} &= m_2 l_{c2}^2+I_2, \\ d_{11} &= 2a_{11}\cos(q^{(4)})+a_{12}, \\ d_{12} &= a_{11}\cos(q^{(4)})+a_{22}, \\ d_{21} &= d_{12}, \\ d_{22} &= a_{22}. \end{aligned} \quad (53)$$

The inertia matrix $D(q_1)$ entries are bounded with

$$|d_{11}| \le 2a_{11}+a_{12},\ |d_{12}| \le a_{11}+a_{22},\ |d_{22}| \le a_{22}. \quad (54)$$

$G(q_1)=[g_1\ g_2]^T$ entries are bounded with

$$\begin{aligned} &|g_1| \le g_{11}+g_{12},\ |g_2| \le g_{21}, \\ &g_{11}=(m_1 l_{c1}+m_2 l_1)g,\ g_{12}=m_2 l_{c2} g, \\ &g_{21}=g_{12}. \end{aligned} \quad (55)$$

We consider the system with constant uncertainty. Let q_d^2, q_d^4 be desired positions of links which are set by different values in this example. We want links to be placed to the desired position with keeping joint angle errors very close to 0. Let $\tilde{q}^2=q^2-q_d^2$, and $\tilde{q}^4=q^4-q_d^4$. The boundedness function ρ_1 is computed by

$$\rho_1=(\rho_{11}^2+\rho_{21}^2)^{\frac{1}{2}}, \quad (56)$$

where

$$\begin{aligned} \rho_{11} &= t_{11}+t_{21}\tilde{q}^{(2)2}+t_{31}\dot{\tilde{q}}^{(2)2}+t_{41}\tilde{q}^{(4)2}+t_{51}\dot{\tilde{q}}^{(4)2}, \\ \rho_{21} &= t_{12}+t_{22}\tilde{q}^{(2)2}+t_{32}\dot{\tilde{q}}^{(2)2}+t_{42}\tilde{q}^{(4)2}+t_{52}\dot{\tilde{q}}^{(4)2}, \end{aligned} \quad (57)$$

and

$$\begin{aligned} t_{11} &= g_{11}+g_{12}+\frac{K_1}{4}+K_1 q_d^2+\frac{1}{4}(2a_{11}+a_{12}) \\ &\quad +\frac{1}{4}(a_{11}+a_{22})s_{21}, \\ t_{21} &= K_1+\frac{a_{11}s_{11}}{2},\ t_{31}=(2a_{11}+a_{12})s_{11}+\frac{a_{11}s_{21}}{2}, \\ t_{41} &= a_{11}s_{21},\ t_{51}=(a_{11}+a_{22})s_{21}+\frac{a_{11}s_{11}}{2}+\frac{a_{11}s_{21}}{2}, \\ t_{12} &= g_{12}+\frac{K_2}{4}+K_2 q_d^4+\frac{1}{4}(a_{11}+a_{22})s_{11}+\frac{1}{4}a_{22}s_{21}, \\ t_{22} &= \frac{a_{11}s_{11}}{2},\ t_{32}=(a_{11}+a_{22})s_{11}+\frac{a_{11}s_{11}}{2}, \\ t_{42} &= K_2,\ t_{52}=a_{22}s_{21}. \end{aligned} \quad (58)$$

Now, we have the following control:

$$\begin{aligned} &u_1=\bar{K}^{-1}(-K_{p1}\tilde{q}_1-K_{v1}\dot{\tilde{q}}_1+p_1-\beta_1(\dot{\tilde{q}}_1+S_1\tilde{q}_1)), \\ &\tilde{q}_1=q_1-q_{1d},\ \dot{\tilde{q}}=\dot{q}_1-\dot{q}_{1d},\ q_{1d}=\begin{bmatrix} q_d^2 & q_d^4 \end{bmatrix}^T, \end{aligned} \quad (59)$$

where

$$\begin{aligned} &p_1=\begin{bmatrix} p_{11} & p_{12} \end{bmatrix}^T, \\ &p_{11}=\begin{cases} -\dfrac{\mu_{11}}{\|\mu_{11}\|}\rho_1, & if\ \|\mu_{11}\|>\varepsilon_1, \\ -\sin(\dfrac{\pi\mu_{11}}{2\varepsilon_1})\rho_1, & if\ \|\mu_{11}\|\le\varepsilon_1, \end{cases} \\ &p_{12}=\begin{cases} -\dfrac{\mu_{12}}{\|\mu_{12}\|}\rho_1, & if\ \|\mu_{12}\|>\varepsilon_1, \\ -\sin(\dfrac{\pi\mu_{12}}{2\varepsilon_1})\rho_1, & if\ \|\mu_{12}\|\le\varepsilon_1, \end{cases} \\ &\mu_{11}=(\dot{\tilde{q}}^{(2)}+S_{11}\tilde{q}^{(2)})\rho_1, \\ &\mu_{12}=(\dot{\tilde{q}}^{(4)}+S_{21}\tilde{q}^{(4)})\rho_1. \end{aligned} \quad (60)$$

We have robust control u:

$$\begin{aligned} &u=-K_{p2}(q_2-u_1)-K_{v2}(\dot{q}_2-\dot{u}_1)+p_2, \\ &p_2=\begin{cases} -\dfrac{\mu_2}{\|\mu_2\|}\rho_2, & if\ \|\mu_2\|>\varepsilon_2, \\ -\dfrac{\mu_2}{\varepsilon_2}\rho_2, & if\ \|\mu_2\|\le\varepsilon_2, \end{cases} \\ &\mu_2=(\dot{q}_2-\dot{u}_1+S_2(q_2-u_1))\rho_2, \\ &\phi_2=-J\ddot{u}_1-KZ_3+KZ_1-Ku_1+JS_2\dot{Z}_3, \\ &\|\phi_2\|\le\rho_2. \end{aligned} \quad (61)$$

For experiments, we choose $\bar{m}_1$ =2.8kg, $\bar{m}_2$ =0.3kg l_1 =0.25m, l_{c1} =0.2m, l_{c2} =0.1m(known parameters) and $\bar{K}_1$ = 3.0kgfm, $\bar{K}_2$ = 1.5kgfm, $\bar{I}_1$ = 0.112kgm^2, $\bar{I}_2$ = 0.003kgm^2, J_1=J_2=0.001kgm^2, g = 9.8m/s^2, k_{p11} = k_{p21} = 20, k_{p12} = k_{p22} = 10, k_{v11} = k_{v21} = 10, k_{v12} = k_{v22} = 5, s_{11} = s_{12} = 1, s_{21} = s_{22} = 1, ε_1 = 5, and ε_2 = 5. $\beta_1 = 5$ is chosen according to the selection of β_1 based on above parameters. The control parameters including PID gains and other things as shown above are not simply selected ad hoc but chosen after several tunings; through several experiments. Fig. 3 shows the

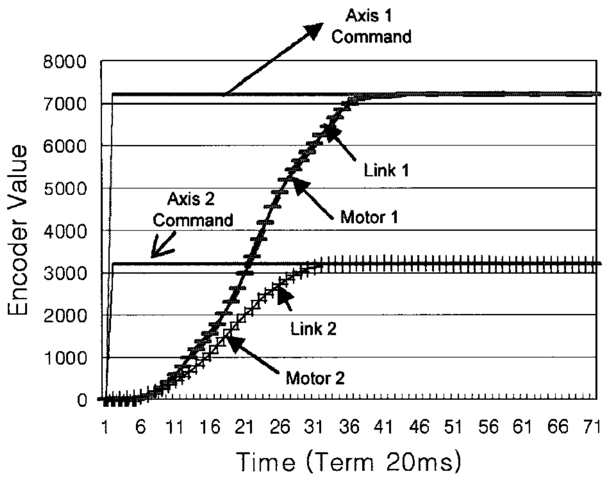

Fig. 3. Response history of motor and link angles with PID control (hard torsion case: around 1.2 kgf·cm).

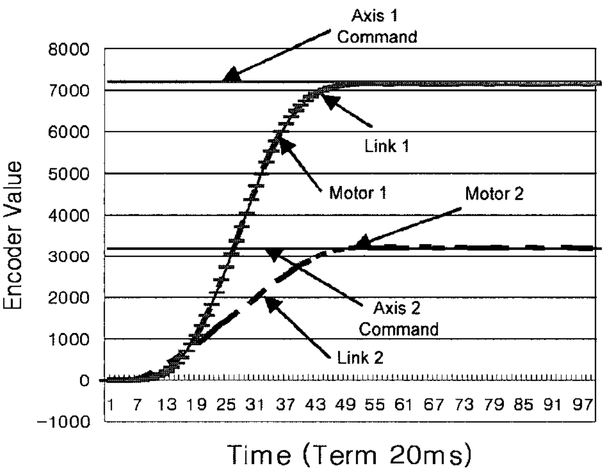

Fig. 4. Response history of motor and link angles with robust control (hard torsion case: around 1.2 kgf·cm).

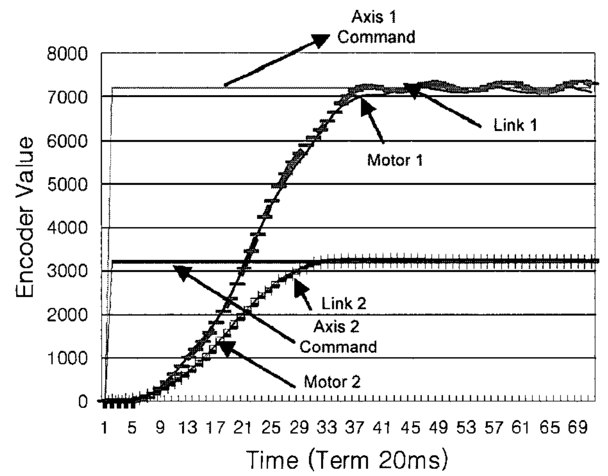

Fig. 5. Response history of motor and link angles with PID control (soft torsion case: around 0.7 kgf·cm).

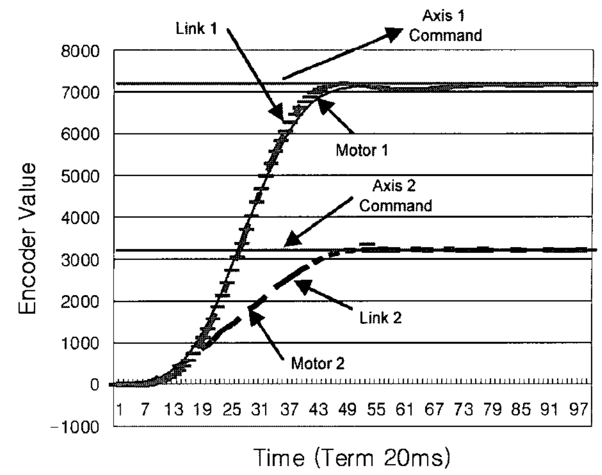

Fig. 6. Response history of motor and link angles with robust control (soft torsion case: around 0.2 kgf·cm).

experimental result under PID control with installing a stiff torsional spring at joints, which is close to a rigid joint manipulator. As we expect, the control performance is desirable in terms of small steady state error and fast rising time. Of course, the PID gains are well tuned by several trials. The PID gains for each link are assigned as P gain=250, D gain=10, I gain=20 for the link 1, and P gain=200, D gain= 20, I gain=30 for the link 2. Compared with the robust control (Fig. 4), the performance does not have much discrepancy between them. The vertical axis for each figure shows number of the encoder pulses for the joint angle, and the encoder has 10,000 pulses for one rotation. Next, when a soft torsional stiffness is installed at joints, the previous setting does not guarantee nice quality. The PID control makes the links and motors fluctuate at after certain time, continuously fluctuating (Fig. 5). The behavior explains how instability arises in a flexible joint manipulator with a wrongly designed controller.

However, this PID control does not show any robustness, which is set by setting appropriate gains for any system. On the other hand, robust control for step input is employed, which shows a desirable outcome. The gains selection is based on the appropriate tuning in the real experiments. Fig. 6 shows the step response under the proposed robust control, which shows enhanced performance compared with the previous control schemes.

Next, consider the control performance for given sinusoidal references as the joint stiffness is weak enough. Then there are big difference between PID and robust controls (Figs. 7 and 8). The robustness under the proposed controller is verified viewing the results of soft joint and hard joint, which is implicative that the control overcomes the parameter uncertainty in a desirable fashion.

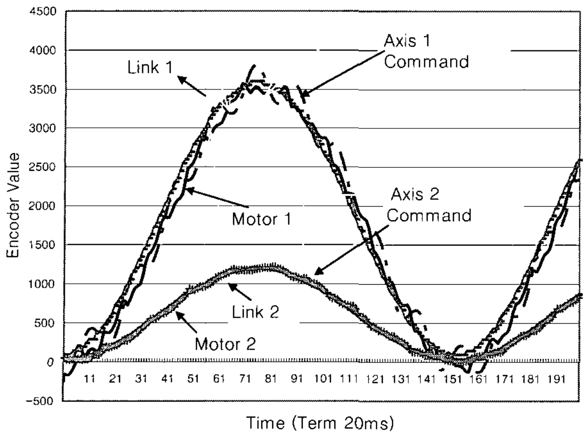

Fig. 7. Response history of motor and link angles with PID control (soft torsion case: around 0.2 kgf·cm, period: 3sec sinusoidal reference).

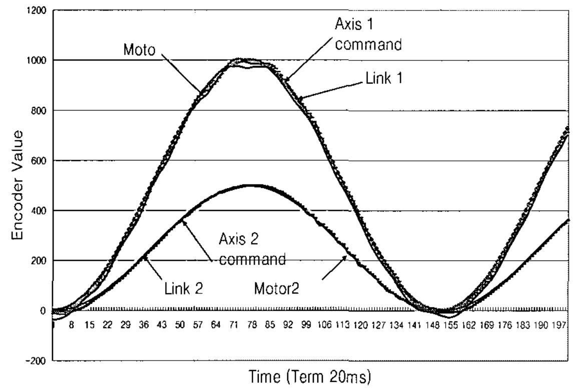

Fig. 8. Response history of motor and link angles with robust control (soft torsion case: around 0.2 kgf·cm, period: 3 sec sinusoidal reference).

In a summary, when the stiffness between motor and link is small and soft, whether the reference input is given by a step or a sinusoidal form, the system performance is not satisfactory due to a large steady state error and fluctuation, which implies the motor motion does not much deliver to the link due to a large coupling between those. On the other hand, with the use of the robust control, an improved system performance with respect to smaller settling time and steady state error is achieved in comparing with the PID control.

7. CONCLUSION

A framework under the design of robust control for flexible joint manipulators which are uncertain and mismatched is developed. After dividing the total system into two subsystems, we first implant a virtual (or implanted) control for the first subsystem. Next, we introduce a state transformation via virtual control. Based on this transformed system a control scheme is designed. This approach leads to overcoming the difficulty of control design in mismatched system. The control renders practical stability for the transformed system and later the practical stability for the original system is also investigated. The control is applicable to both the constant uncertainty and time-varying uncertainty. Furthermore, the size of the uniform ultimate boundedness ball and uniform stability ball can be made arbitrary small for the transformed system and for the original system.

The transformation is only based on the possible bound of uncertainty. Actually, this scheme shows a major development in controlling flexible joint manipulators with mismatched uncertainty and nonlinearity. Several experiments for two- link flexible joint manipulator are carried out to verify the control performance under the proposed robust control. The robust control possesses most enhanced control performance compared with the others.

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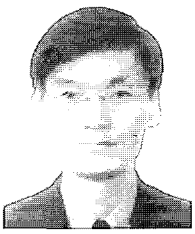

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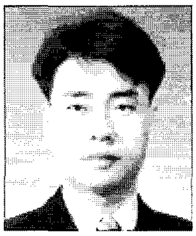

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