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Nonlinear Dynamic Analysis of Fiber Movement

Danfeng Shen* and Guoming Ye

College of Mechanical Engineering, Dong Hua University, Shanghai 201620, People's Republic of China

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Abstract: This paper adopts nonlinear vibration method to analyze the fluctuation process of fiber movement. Based on Hamilton Principle, this paper establishes differential equation of fiber axial direction movement. Using variable-separating method, this paper separates time variable from space variable. By using the disperse movement equation of Galerkin method, this paper also discusses stable region of transition curve and points out those influencing factor and variation trend of fiber vibration.

Keywords: Model, Nonlinear vibration, Method of harmonic balance, Fiber, Chord

Introduction

During the pulling procedure in post processing of chemical fiber, fiber axial direction is a basic form of movement. This movement is analogous to the chord movement. Due to its high flexibility, light mass and low damp, fiber movement can easily evolve into wide range vibration. It is well known that using the method of linear dynamics to explain correctly fiber jumping phenomenon is very difficult [1-3]. However, this paper presents a study of fiber movement process by using non-linear method; our research results provide an approach for process engineers to have better understanding of the regular pattern of fiber movement, to develop better process specifications and to enhance product quality.

Basic Fiber Model

In order to simplify the study, we made the following basic premises:

1. Take no account of flexural rigidity, torsional rigidity and shearing rigidity.
2. Cross-section, which was vertical to fiber axes before deformation, will still be vertical to fiber axes after deformation.

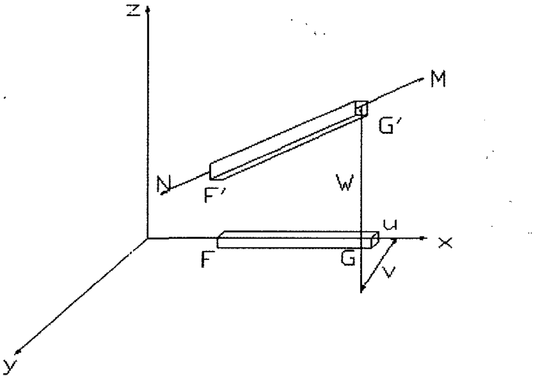

Figure 1. Model of short snippet fiber.

*Corresponding author: sdfdhu@sina.com

3. Regard gravity sag of fiber as a parabola.
4. Assume deformation constitutive relation of fiber to obey Hookes law and the force at each point is equal.

In Figure 1, we define F and G as the two endpoints of a infinitesimal section of the fiber which have not deformed, and define F' and G' as the two end points of a infinitesimal section of the fiber which have deformed.

Displacement of point F can be expressed as:

$$\Delta_F = u(x,t)i + w(x,t)k \tag{1}$$

Displacement of point G can be written as follow:

$$\Delta_G = \left(u + \frac{\partial u}{\partial x}dx\right)i + \left(w + \frac{\partial w}{\partial x}dx\right)k \tag{2}$$

According to Figure 1, it can be derived that: $\Delta_F + \Delta_{F'G'} = \Delta G + dxi$.

where $\Delta_{F'G'}$ is defined as the position vector of point G' relative to point F'.

Equation (3) gives deformable fiber length:

$$|\Delta_{F'G'}| = ds = \left[\left(1 + \frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2\right]^{1/2} dx \tag{3}$$

Equation (4) gives the unit vector that runs parallel to deformable fiber:

$$\frac{\Delta_{F'G'}}{|\Delta_{F'G'}|} = \delta = \left[\left(1 + \frac{\partial u}{\partial x}\right)i + \frac{\partial w}{\partial x}k\right] = \frac{dx}{ds} \tag{4}$$

This fiber has an initial tension N_0. During the movement, the length of the fiber changes and inner tension of the fiber also changes. The instantaneous tension value of the fiber is given by equation (5):

$$N = N_0 + \frac{EA(ds - dx)}{dx} \tag{5}$$

Mathematics Model of Fiber Vibration

In Figure 2, let us set up fiber model of axial direction movement, fiber moves along X direction between two fix

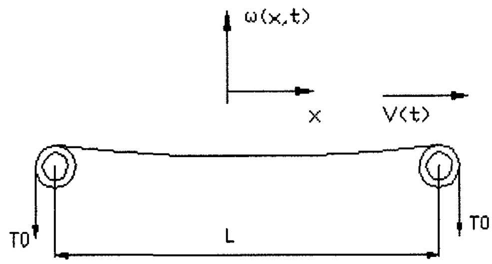

Figure 2. Fiber vibration model.

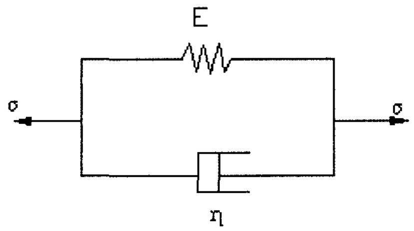

Figure 3. Kelvin model.

rollers with constant initial tension and speed $V(t)$. Now, we assume that cross direction vibration exists only at ω direction, and the vibration displacement is $\omega(x,t)$. The visco-elasticity fiber system is equivalent to Kelvin model in Figure 3. According to Newton Law and Hamilton Principle, we set up fiber vibration differential equation follows [4]:

$$c^2\frac{\partial^2\omega}{\partial x^2} = \frac{\partial^2\omega}{\partial t^2} + 2V\frac{\partial^2\omega}{\partial x\partial t} + V^2\frac{\partial^2\omega}{\partial x^2} + \frac{\partial V}{\partial t}\frac{\partial\omega}{\partial x} \tag{6}$$

where $c = \sqrt{T_0/\rho A}$ meaning the fiber wave velocity, T_0 is the constant of initial tension, ρ is the linear density, A is cross-sectional, L is the distance between two rollers. There is no cross direction vibration, the boundary condition is:

$$\omega(0,t) = 0, \quad \omega(L,t) = 0 \tag{7}$$

During the following analyses, we assume that the axial direction speed $V(t)$ is:

$$V(t) = V_0 + V_1\cos(\Omega t) \tag{8}$$

where V_0 is the average speed, V_1 and Ω represent amplitude value and frequency of simple harmonic wave. On substituting the above formulas into equation (6), we obtain:

$$\frac{\partial^2\omega}{\partial t^2} + (V_0^2 - c^2)\frac{\partial^2\omega}{\partial x^2} + 2V_0\frac{\partial^2\omega}{\partial x\partial t} + 2V_1\cos(\Omega t)\frac{\partial^2\omega}{\partial x\partial t}$$
$$+ 2V_0V_1\cos(\Omega t)\frac{\partial^2\omega}{\partial x^2} - \Omega V_1\sin(\Omega t)\frac{\partial\omega}{\partial x}$$
$$+ V_1^2\cos^2(\Omega t)\frac{\partial^2\omega}{\partial x^2} = 0 \tag{9}$$

We can assume that the fiber vibration modality is the standard chord vibration modality. Using Galerkin method and adopting separate variable method, we can separate time variable t from space variable x:

$$\omega(x,t) = \sum_{n=1}^{N} q_n(t)\sin\left(\frac{n\pi x}{L}\right) \tag{10}$$

where $q_n(t)$ is the fiber n^{th} order general displacement function and $\sin(n\pi x/L)$ is the vibration mode function.

In terminal excitation vibration of tensioning fiber, basic modality is dominant. We choose order 1 modality of equation (10), and substitute it into equation (9). We obtain general Mathieu equation as [5]:

$$\ddot{q}_1 + \frac{\pi^2}{L^2}(c^2 - V_0^2)q_1 - 2\frac{\pi^2}{L^2}V_0V_1\cos(\Omega t)q_1$$
$$- \frac{\pi^2}{L^2}V_1^2\cos^2(\Omega t)q_1 = 0 \tag{11}$$

Introducing nondimensional variables:

$$\tau = \frac{Lt}{\pi c},\ \gamma = \frac{V_0}{c},\ \alpha_1 = \frac{V_1}{V_0},\ \alpha = \frac{\alpha_1}{\varepsilon^2},\ \omega = \frac{\pi c\Omega}{L} \tag{12}$$

where ε is a small parameter, it satisfies $0 < \varepsilon \le 1$.

On substituting equation (12) into equation (11), we obtain:

$$\ddot{q}_1 + (1-\gamma^2)q_1 - \varepsilon^2 2\gamma^2\alpha\cos(\omega\tau)q_1$$
$$-\varepsilon^4\gamma^2\alpha^2\cos^2(\omega\tau)q_1 = 0 \tag{13}$$

Generally, axial direction speed $V(t)$ is far slower than wave speed c, let us set $\omega_0^2 = 1-\gamma^2$, substitute it into equation (13) and omit high order term:

$$\ddot{q}_1 + \omega_0^2 q_1 = \varepsilon^2 2\gamma^2\alpha\cos(\omega\tau)q_1 \tag{14}$$

Set $\delta = \omega^2, \varepsilon' = -\varepsilon^2\gamma^2\alpha, t = \omega\tau/2$, substitute them into equation (14), we obtain standard Mathieu equation as:

$$\ddot{q} + (\delta + 2\varepsilon'\cos 2t)q = 0 \tag{15}$$

Dynamics Unstable Region of Fiber Vibration

We analyze dynamics unstable region and feedback curve in main parameter resonant vibration using the method of harmonic balance and method of perturbation. When $\varepsilon = 0$, the period solution with 2π of the fork equation $\ddot{q} + \delta q = 0$ can be written as [5,6]:

$$q = a\cos(2n-1)t + b\sin(2n-1)t \quad (n = 1, 2, \ldots)$$

For $\delta = (2n-1)^2$

Let us discuss the situation when δ is close to 1 in equation (15). Set:

$$\delta = 1 + \varepsilon'\delta_1 + \varepsilon'^2\delta_2\ldots \tag{16}$$

Firstly, we assume approximate solution in equation (15) is:

$$q = A_1\cos t + A_3\cos 3t \tag{17}$$

As a result: $\ddot{q} = -A_1\cos t - 9A_3\cos 3t$ (18)

Set $A_1 = a,\ A_3 = A_{30} + \varepsilon' A_{31} + \varepsilon'^2 A_{32} + \ldots$ (19)

Substituting equations (16)~(19) into equation (15), we can obtain:

$$\begin{aligned}&[(\varepsilon' + \varepsilon'\delta_1 + \varepsilon'^2\delta_2)a + \varepsilon'(A_{30} + \varepsilon' A_{31})]\cos t\\ &+[(-8+\varepsilon'\delta_1+\varepsilon'^2\delta_2)(A_{30}+\varepsilon' A_{31}+\varepsilon'^2 A_{32})+\varepsilon' a]\cos 3t\\ &+\ldots = 0\end{aligned} \tag{20}$$

Knowing that the coefficients of $\cos t$ and $\cos 3t$ are equal to zero, we obtain:

$$\begin{aligned}&(\varepsilon'+\varepsilon'\delta_1+\varepsilon'^2\delta_2)a+\varepsilon'(A_{30}+\varepsilon' A_{31}) = 0\\ &(-8+\varepsilon'\delta_1+\varepsilon'^2\delta_2)(A_{30}+\varepsilon' A_{31}+\varepsilon'^2 A_{32})+\varepsilon' a = 0\end{aligned} \tag{21}$$

We may work out:

$$A_{30}=0,\ \delta_1 = -1,\ A_{31} = \frac{a}{8},\ \delta_2 = -\frac{1}{8}$$

Also we can obtain the period solution and the transition curve as:

$$\begin{aligned}q &= a\cos t + \frac{1}{8}\varepsilon' a\cos 3t\\ &= a\cos\frac{\Omega t}{2} + \frac{1}{8}\varepsilon^2\left(\frac{V_0}{c}\right)^2 \alpha\cos\frac{3\Omega t}{2}\end{aligned} \tag{22}$$

$$\delta = 1 - \varepsilon' - \frac{1}{8}\varepsilon'^2 = 1 + \frac{V_0 V_1}{c^2} - \frac{1}{8}\left(\frac{V_0 V_1}{c^2}\right)^2 \tag{23}$$

In a similar way, we assume that approximate solution in equation (15) is:

$$q = A_1\sin t + A_3\sin 3t$$

We can obtain period solution and transition curve as:

$$\begin{aligned}q &= a\sin t + \frac{1}{8}\varepsilon' a\sin 3t\\ &= a\sin\frac{\Omega t}{2} + \frac{1}{8}\varepsilon^2\left(\frac{V_0}{c}\right)^2 \alpha\sin\frac{3\Omega t}{2}\end{aligned} \tag{24}$$

$$\delta = 1 + \varepsilon' - \frac{1}{8}\varepsilon'^2 = 1 - \frac{V_0 V_1}{c^2} - \frac{1}{8}\left(\frac{V_0 V_1}{c^2}\right)^2 \tag{25}$$

To Mathieu equation, the value of period π and 2π come into curve on $\delta - \varepsilon'$ plane, so it makes up a boundary between stable solution and unstable solution. Here, we figure out the transition curve on Figure 4 and Figure 5.

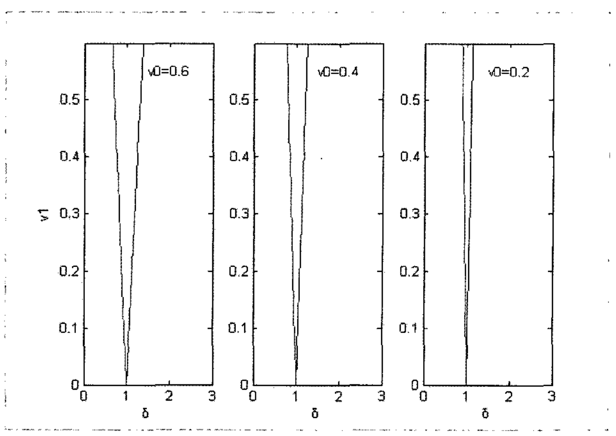

Figure 4. System stability chart with various values of parameter V_1.

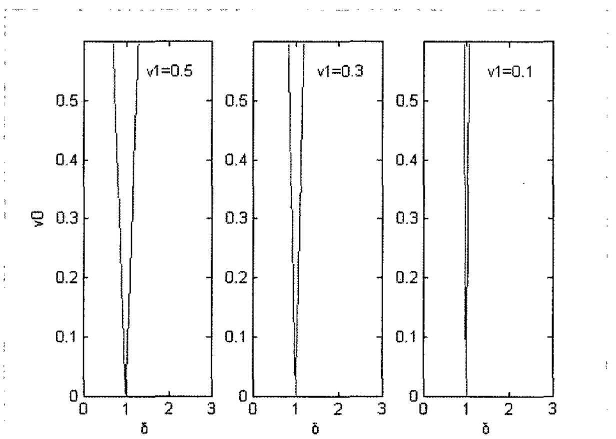

Figure 5. System stability chart with various values of parameter V_0.

Figure 4 shows the stable region variance diagram in which system in different parameter V_0 follows the changes of parameter V_1. Figure 5 shows the stable region variance diagram in which system in different parameter V_1 follows the changes of parameter V_0.

The above two diagrams show that unstable region increases with the increase of V_1 and V_0. Therefore, decreasing the values of V_1 and V_0 is beneficial to the stabilization of fiber movement.

Conclusions

This paper uses analytical method of nonlinear vibration to conduct researches on fiber axial direction movement in chemical fiber production. The researches show:

1. By increasing initial tension of fiber, the stable region of fiber movement can be increased.
2. By decreasing linear density of fiber, the stable region

of fiber movement can be increased.

3. By decreasing the speed of fiber, the stable region of fiber movement can be increased.
4. By decreasing fluctuation amplitude value of fiber speed, the stable region of fiber movement can be increased.
5. This paper assumes that δ is approximately equal to 1. If we choose even more order approximate solution, we can solve for multi-unstable regions. But the tendency is that the unstable region gets narrower when δ gets bigger.

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