An Analysis on the State-Dependent Nature of DS/SSMA Unslotted ALOHA

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Abstract: **In this paper, we present a novel approach to analyze the throughput of direct-sequence spread spectrum multiple access (DS/SSMA) unslotted ALOHA system. In the unslotted system, the departure rate of interfering transmissions is proportional to the number of current *interferers* that can be regarded as the system state. In order to model this state-dependency, we introduce a two-dimensional state transition model that describes the state transition of the system. This model provides a more rigorous analysis tool for the DS/SSMA unslotted ALOHA systems with both fixed and variable packet lengths. Numerical results reveal that this analysis yields an accurate system performance that coincides with the simulation results. Throughout the analysis we have discovered that the state-dependency of the departure rate causes *interference averaging effect* in the unslotted system and that this effect yields a higher throughput for the unslotted system than for the slotted system when supported by a strong channel coding.**

Index Terms: **Direct-sequence spread spectrum multiple access (DS/SSMA), interference averaging effect, performance analysis, unslotted ALOHA.**

I. INTRODUCTION

Direct-sequence code-division multiple access (DS/CDMA) or spread spectrum multiple access (DS/SSMA) has attracted significant research interests as an efficient multiple access method for wireless networks [1]–[10]. DS/SSMA is advantageous in that it yields a higher radio capacity and has the capability of flexible data transmission. Especially, simple random access schemes such as DS/SSMA ALOHA have been shown to be efficient for non-real time applications in packet data access systems in wireless environment [1], [2]. In this point, DS/SSMA systems appear to be a good candidate for future wireless access networks.

A DS/SSMA ALOHA system can be either slotted or unslotted. The throughput can be easily analyzed in the case of the slotted systems [3]–[5] but is very challenging in the case of unslotted systems. The main difficulty stems from the fluctuation of the number of *interferers* or *interfering packets*.[1] In spite of this difficulty, there are several reasons that necessitate the analysis of the unslotted ALOHA system: In various types of wireless packet networks such as wireless sensor networks that inherently have a distributed structure, it becomes difficult to synchronize the overall network for the slot-based operation [11]. In addition, as will become clear through numerical results in Section V, the unslotted system outperforms the slotted system in terms of the maximum achievable throughput when strong error correction coding is supported.

There have been reported several works that tried to analyze DS/SSMA unslotted ALOHA system [6]–[10]: Yin *et al.* [6] used the approximation that the bit error probability of each bit in the transmission packets is 1 if the interference level exceeds the threshold and 0 otherwise. Storey *et al.* [7] and Joseph *et al.* [8] analyzed the system model with the variable packet length and the constant packet departure rate which is exponentially distributed.[2] Sato *et al.* [9] presented an analysis method that is applicable to the unslotted systems with fixed packet length by modeling the fluctuation of interference levels bit by bit and calculating the bit error probability depending on the state of Markov chain. So *et al.* [10] extended this analysis to the variable-length message case. However, in all the above works the state-dependency of the packet departure rate, which is an inherent nature of the unslotted DS/SSMA system, was not taken into account. To be more specific, the probability that an interfering packet finishes its transmission is proportional to the number of current interferers in the case of the unslotted system but the above analyses assumed a uniform departure rate for every bit of the target packet, ignoring this state-dependency. Such a state-independent assumption necessarily leads to an inaccurate performance estimation.

In this paper, we present a new performance analysis of the DS/SSMA unslotted ALOHA system that admits the state-dependency in the departure rate. In order to yield a more accurate analysis, we establish a system model that allows the departure rate to change bit by bit. We first apply this model to the case of fixed-length messages and then extend it to encompass the case of variable-length messages. As will become clear in Section V, this analysis closely matches with the simulation results, while the above analyses exhibit some deviations and, furthermore, it helps to discover the *interference averaging effect* that the unslotted system suffers less from uncorrectably severe interference than the slotted system does, which causes the unslotted ALOHA to outperform the slotted system when supported by a strong error-correction coding.

The paper is organized as follows: We first describe the system model of the unslotted DS/SSMA ALOHA system in Section II. Then we analyze the performance of the unslotted system for fixed and variable length messages in Sections III and IV, respectively. Finally, we confirm the accuracy of the proposed analysis through simulation in Section V.

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[1]The *interferers* or the *interfering packets* mean the packets which interfere with the target packet during its transmission.

[2]The departure rate is the average number of the packets that end transmission during unit interval (e.g., a bit time).

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II. SYSTEM MODEL

We consider a DS/SSMA unslotted ALOHA network in which each station has the same frequency of packet generation. Each station sends out messages at any time using a common spreading sequence and the receiver can separate them out if the receiving time offset is sufficiently large among the received packets [12]. We assume that the messages arrive at the network by Poisson process with the rate λ. We analyze the system for the fixed-length messages of length T_p and also for the variable-length messages, with each message fragmented into a multiple of the fixed length T_p (i.e., $T_p, 2T_p, \cdots$).[3]

In order to protect packets from interference, we use a t-error correcting code for forward error correction (FEC), which can correct up to t-bit errors regardless of the correlation of errors in a packet. We adopt the Gilbert-Varshamov lower bound [13] as the code rate R_c, i.e.,

$$R_c = 1 + A\log_2 A + (1-A)\log_2(1-A), \quad A \equiv \frac{2t+1}{L} \tag{1}$$

where L denotes the number of bits in a packet. We assume that every packet is received with equal power and the errors are caused only by the multiple access interference (MAI) and additive white Gaussian noise (AWGN). Then the bit error probability of an asynchronous DS/SSMA system is given by [14]

$$\begin{aligned} P_b(k) \approx\ & \frac{2}{3}Q\left[\left(\frac{k}{3N} + \frac{N_0}{2E_b}\right)^{-0.5}\right] \\ & + \frac{1}{6}Q\left[\left(\frac{k(N/3)+\sqrt{3}\sigma}{N^2} + \frac{N_0}{2E_b}\right)^{-0.5}\right] \\ & + \frac{1}{6}Q\left[\left(\frac{k(N/3)-\sqrt{3}\sigma}{N^2} + \frac{N_0}{2E_b}\right)^{-0.5}\right] \end{aligned} \tag{2}$$

with

$$\sigma^2 = k\left[N^2\frac{23}{360} + N\left(\frac{1}{20} + \frac{k-1}{36}\right) - \frac{1}{20} - \frac{k-1}{36}\right]$$

where k is the number of interfering users, N the number of chips per bit, E_b the energy per bit, N_0 the two-sided power spectral density of AWGN, and $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-u^2/2}du$.

Fig. 1 illustrates the fluctuations of the interference level, k: The first one is for the case in which all messages have the fixed length T_p, and the second is for the case of the variable-length messages of length T_p, $2T_p, \cdots$.

III. PERFORMANCE ANALYSIS OF FIXED-LENGTH MESSAGES

As well known, the throughput of the spread ALOHA system is $S = GQ_s$ for the offered load G and the probability of packet success Q_s.[4] To calculate Q_s, we use the probability conditioned by the number of interfering packets at the start of a target packet transmission, k. Then the throughput S may be written by

$$S = G\sum_{k=0}^{\infty} P_0(k)P_s(k) \tag{3}$$

where $P_0(k)$ denotes the probability that the number of interferers at the starting point of the target packet is k; and $P_s(k)$ denotes the packet success probability with k initial interfering packets. Noting that throughput S increases by using an error correcting code but the effective user information in each packet decreases by the ratio of R_c, we define the *effective throughput* S_e by[5]

$$S_e = SR_c. \tag{4}$$

In the case of the slotted spread ALOHA system, the number of interfering packets is constant throughout the packet transmission duration. In this case, $P_s(k)$ is given by

$$P_s(k) = \sum_{s=0}^{t}\binom{L}{s}P_b(k)^s(1-P_b(k))^{L-s} \tag{5}$$

for a bit error probability $P_b(k)$ in (2). In contrast, the number of interfering packets varies in time in the case of the unslotted spread ALOHA. This makes the bit error probability fluctuate with the number of interfering packets. For this reason, the analysis of the unslotted system becomes much more complicated

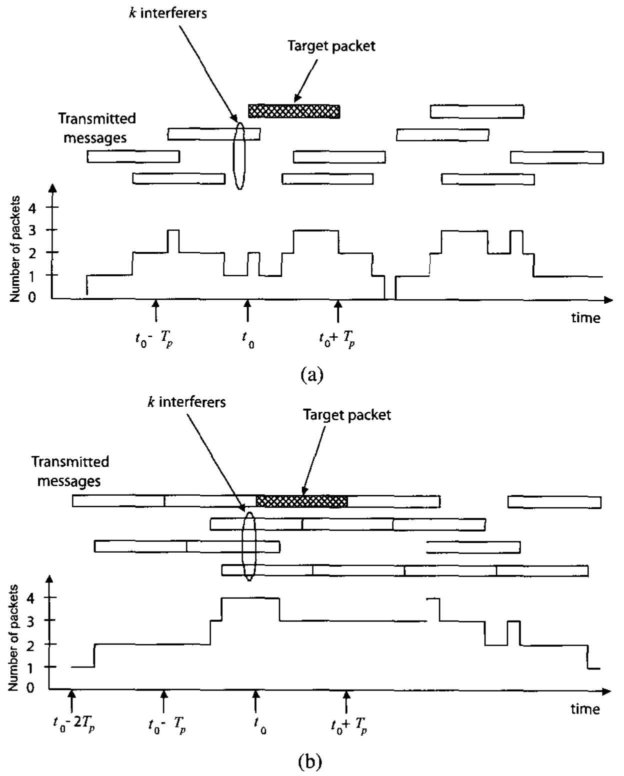

Fig. 1. Illustration of the fluctuation of interference level during the packet transmission for (a) fixed-length messages and (b) variable-length messages.

[3]As the wireless channel is subject to various different types of interference, we handle the variable-length messages in a fragmented form composed of multiple fixed-length packets.

[4]The offered load G refers to the average number of packets generated during the packet duration T_p, i.e., $G = \lambda T_p$.

[5]The effective throughput is calculated based on the Gilbert-Varshamov lower bound. Thus, (4) sets a lower bound on the effective throughput.

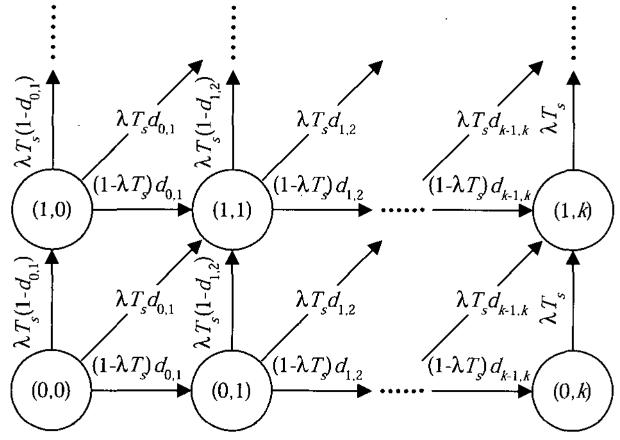

Fig. 2. State transition diagram of channel state vector.

than that of the slotted system. In order to consider this time varying property, we model the channel state by using a first order two-dimensional state transition diagram in which the *departure probability* depends on the current state. We define the *departure probability* as the probability that one or more interfering packets end transmission during one bit time. By solving this state transition model, we can analyze the state-dependent nature of the unslotted system accurately.

For each packet of length L (bits) and the packet transmission time T_p (seconds), the bit duration time is given by $T_s = T_p/L$. We assume that there are k interfering packets at the start of the target packet transmission, t_0, which means that k interfering packets have arrived in the interval $(t_0 - T_p, t_0]$. Let the arrival process be Poisson with the rate λ. Then the departure of the k interfering packets is uniformly distributed in the interval $(t_0, t_0 + T_p]$.[6]

As in the existing analyses, we consider that state changes would happen not chip-time based but bit-time based. We define the bit error event and the arrival-departure event for each bit time, assuming that the bit error event occurs during the bit time and the arrival-departure event occurs at the very end of each bit time. In addition, for simplicity of analysis, we assume that the probability that two or more arrivals happen during one bit duration is negligible. This assumption is valid if the processing gain N is much less than the packet length L.[7]

Let the number of arrivals and the number of departures in the interval $(t_0, T]$ be a and b, respectively. Then the channel state at time T is uniquely determined by the channel state vector (a, b). The channel state transition diagram from the n-th bit to the $(n+1)$-th bit takes the form depicted in Fig. 2. In the figure, we put the probability that no Poisson process packet arrives during one bit time T_s by $1-\lambda T_s$, not $\exp(-\lambda T_s)$, because $\exp(-\lambda T_s)$ approximates to $1 - \lambda T_s$ for $\lambda T_s = G/L \ll 1$. Thus the probability of one or more arrivals during T_s can be approximated to λT_s. Additionally, we ignore the probability of multiple arrivals and assume only the transitions between adjacent states. d_i denotes the departure probability that an interfering packet departs from the state $b = i$, where $0 \le i \le k-1$. By the uniform property addressed above, the departure probability of each interfering packet during the $L-n$ bit interval is $1/(L-n)$. At the n-th bit, the departure probability that an interfering packet departs among the $(k-i)$ remaining interferers is given by

$$d_i(n) = \binom{k-i}{1}\left(\frac{1}{L-n+1}\right)\left(1-\frac{1}{L-n+1}\right)^{k-i-1}. \tag{6}$$

For $G \ll L$ and $k-i \ge 2$, we can assume that $1/(L-n+1)$ is much less than 1, so (6) gets approximated to

$$\begin{cases} d_i(n) \simeq \frac{k-i}{L-n+1} & \text{for } k-i \ge 2, \\ d_i(n) = \frac{1}{L-n+1} & \text{for } k-i = 1. \end{cases} \tag{7}$$

This describes how the departure probability depends on the state of the system, that is, the number of departures and the bit position.

Let $\pi^k_{n,(a,b)}(s)$ denote the state probability at the start of the n-th bit with s bit errors conditioned by k initial interferers. We define the state probability vector by

$$\boldsymbol{\pi}^k_n(s) = [\pi^k_{n,(0,0)}(s) \ \cdots \ \pi^k_{n,(0,k)}(s) \ \pi^k_{n,(1,0)}(s) \ \cdots]^T. \tag{8}$$

Since the time instants of bit errors and the arrival-departure events are assumed separable, we can write the state transition equation in the form

$$\boldsymbol{\pi}^k_{n+1}(s) = \boldsymbol{A}^k_n \tilde{\boldsymbol{\pi}}^k_n(s) \tag{9}$$

where $\boldsymbol{A}^k_n$ denotes the state transition matrix and $\tilde{\boldsymbol{\pi}}^k_n(s)$ the state probability vector at the end of the n-th bit time. $\boldsymbol{\pi}^k_n(s)$ and $\tilde{\boldsymbol{\pi}}^k_n(s)$ are the state probability vectors that reflect the bit error events by the $(n-1)$-th and the n-th bit times, respectively. We define by $\boldsymbol{C}^k_n \equiv [c_{ij}(n)]^k$, a $(k+1)\times(k+1)$ departure probability matrix consisting of the elements

$$c_{ij}(n) = \begin{cases} 1-d_j(n), & \text{if } i=j, \\ d_j(n), & \text{if } i=j+1, \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

where $0 \le i \le k$ and $0 \le j \le k$. Then from Fig. 2 and (7), we can get the state transition matrix

$$\boldsymbol{A}^k_n = \begin{bmatrix} (1-\lambda T_s)\boldsymbol{C}^k_n & \boldsymbol{0} & \cdots \\ \lambda T_s \boldsymbol{C}^k_n & (1-\lambda T_s)\boldsymbol{C}^k_n & \cdots \\ \boldsymbol{0} & \lambda T_s \boldsymbol{C}^k_n & \cdots \\ \boldsymbol{0} & \boldsymbol{0} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \tag{11}$$

where $\boldsymbol{0}$ denotes a $(k+1)\times(k+1)$ zero matrix.

Now, we consider the relation between $\boldsymbol{\pi}^k_n(s)$ and $\tilde{\boldsymbol{\pi}}^k_n(s)$, which is governed by the bit error event. We find from (8) that the relations $a = \lfloor \frac{i}{k+1} \rfloor$ and $b = \langle i \rangle_{k+1}$ hold for the i-th row

[6]Note that when k arrivals of a Poisson process have occurred in an interval $(0, T]$, the k arrival times $\tau_1 < \tau_2 < \cdots < \tau_k$ are distributed as the order statistics of k uniform random variables on $(0, T]$ [15].

[7]In reality, the probability that two or more arrivals would happen during one bit duration, is given by $\sum_{k=2}^{\infty} \frac{(\lambda T_s)^k}{k!}\exp(-\lambda T_s) = 1 - \exp(-\frac{G}{L}) - \frac{G}{L}\exp(-\frac{G}{L})$ and the offered load G is not larger than the processing gain N in usual spread ALOHA systems. For example, for the values $N = 32$, $L = 1000$, and $G = N$, this probability drops below 0.05%, which demonstrates the validity of the assumption.

element of $\boldsymbol{\pi}_n^k(s)$.[8] Since the number of interfering packets is $k+a-b$ for the state $\pi_{n,(a,b)}^k(s)$, we get

$$\tilde{\pi}_{n,(a,b)}^k(s) = \Big(1 - P_b\big(k+a-b\big)\Big)\pi_{n,(a,b)}^k(s) + P_b\big(k+a-b\big)\pi_{n,(a,b)}^k(s-1). \tag{12}$$

We define the error probability matrix $\boldsymbol{P}_b^k = [p_{ij}]$ by

$$p_{ij} = \begin{cases} P_b\left(k + \lfloor \frac{i}{k+1} \rfloor - \langle i \rangle_{k+1}\right), & \text{if } i = j, \\ 0, & \text{otherwise} \end{cases} \tag{13}$$

where $i \geq 0$ and $j \geq 0$. Applying (12) and (13) to (9), we get

$$\boldsymbol{\pi}_{n+1}^k(s) = \boldsymbol{A}_n^k \left\{ (\boldsymbol{I} - \boldsymbol{P}_b^k)\boldsymbol{\pi}_n^k(s) + \boldsymbol{P}_b^k \boldsymbol{\pi}_n^k(s-1) \right\} \tag{14}$$

for the initial state probability vector

$$\boldsymbol{\pi}_1^k(s) = \begin{cases} [1 \ \mathbf{0}]^T, & \text{if } s = 0, \\ \mathbf{0}, & \text{otherwise} \end{cases} \tag{15}$$

where $\mathbf{0}$ denotes a zero row vector.[9]

With a t-error correcting capability, the probability of successful packet transmission is given by

$$P_s(k) = sum\left(\sum_{s=0}^{t} \boldsymbol{\pi}_{L+1}^k(s)\right) \tag{16}$$

where $sum(\boldsymbol{x})$ denotes the summation of all the elements of vector $\boldsymbol{x}$. Since arrivals are assumed Poisson with the rate λ, the probability that k initial interferers exist is given by

$$P_0(k) = \frac{G^k}{k!}\exp(-G). \tag{17}$$

By incorporating (14) and (15) into (16) and then applying (16) and (17) to (3), we can finally determine the throughput S of the DS/SSMA unslotted ALOHA system for the fixed-length message case. Since the dimension of the state vector is not finite, we need to bound it for the calculation. For the Poisson arrival process with the rate λ, we may approximate the maximum number of the arrivals during T_p by $G + 3\sqrt{G}$ since the average and the variance of the number of arrivals during T_p are both G, or λT_p. We define the approximation of the maximum number of arrivals as[10]

$$K(G) = \left\lceil G + 3\sqrt{G} \right\rceil. \tag{18}$$

Then, the dimension of the state vector $\boldsymbol{\pi}_n^k(s)$ is $(k+1) \times \big(K(G)+1\big)$. We can also bound the dimensions of the state transition matrix and the error probability matrix in a similar way. Since the number of arrivals during T_p is bounded by $K(G)$, the number of the initial interferers is also bounded by $K(G)$. Thus (3) may be rewritten as

$$S \simeq G \sum_{k=0}^{K(G)} P_0(k)P_s(k). \tag{19}$$

[8]The notation $\lfloor x \rfloor$ denotes the largest integer smaller than or equal to x and the notation $\langle x \rangle_y$ denotes the remainder of x divided by y.

[9]Note that $\mathbf{0}$'s in (11) and (15) both denote a zero matrix though their dimensions are different.

[10]The notation $\lceil x \rceil$ denotes the smallest integer larger than or equal to x.

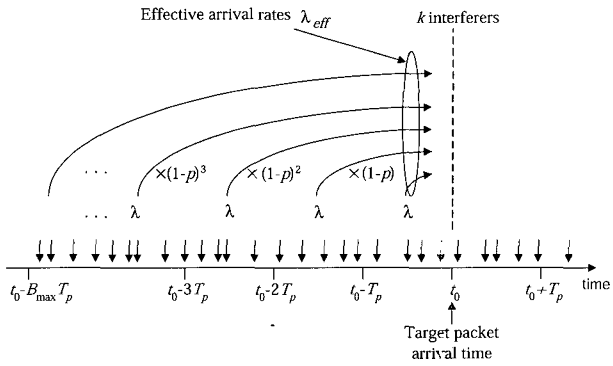

Fig. 3. Illustration of the effective arrivals of interfering packets.

IV. EXTENSION TO VARIABLE-LENGTH MESSAGES

Now we extend the above analysis to encompass the variable-length messages. As in the case of the fixed-length messages, we assume the message arrival of Poisson process with the rate λ. We deal with each variable-length message as a composite of B fixed-length packets of length L each. In order to effectively represent the variable lengths, we assume that B is a random variable with the probability distribution

$$P(B=x) = \begin{cases} p_g(1-p_g)^{x-1}, & \text{if } x < B_{\max}, \\ 1 - \sum_{x=1}^{B_{\max}-1} p_g(1-p_g)^{x-1}, & \text{if } x = B_{\max}, \\ 0, & \text{otherwise} \end{cases} \tag{20}$$

for a geometric distribution parameter p_g and the maximum value of B, $B_{\max}$. This distribution is not a geometric distribution in rigorous sense as it has a bounded maximum value but it can be approximated so because the error caused by this approximation is very small if $B_{\max}$ is greater than the average value of B by four times.[11] Under this approximation, (20) can be simplified to

$$P(B=x) = \begin{cases} p_g(1-p_g)^{x-1}, & \text{if } x < B_{\max}, \\ (1-p_g)^{B_{\max}-1}, & \text{if } x = B_{\max}, \\ 0, & \text{otherwise} \end{cases} \tag{21}$$

and the average number of packets for the messages is calculated to

$$\bar{B} = \sum_{x=1}^{\infty} kP(B=x) = \frac{1}{p_g}\{1 - (1-p_g)^{B_{\max}}\}. \tag{22}$$

As illustrated in Fig. 1(b), it is probable that a message that has arrived before the time instant $(t_0 - T_p)$ interferes with the target message arriving at the time instant t_0. In order to duly reflect this property in modeling, we adopt the concept of effective arrivals which regards all the past message arrivals that affect the target message as effectively occurred at the packet interval immediately before the target message arrival time. Fig. 3 illustrates this. We denote by $P(t_0, k)$ the probability that k interferers exist in the channel at time t_0, that is, the number of

[11]In this case, the distribution in (20) deviates from a geometric distribution by less than 2% (refer to the related discussions in Section V).

the effective arrivals at time t_0 is k. Then $P(t_0, k)$ becomes a Poisson distribution with the effective rate

$$\lambda_{eff} = p_g^{-1}(1 - (1 - p_g)^{B_{\max}})\lambda \tag{23}$$

which can be proved as follows:

We denote by k_x the number of messages that have arrived during the time interval $(t_0 - (x+1)T_p, t_0 - xT_p]$ and last until the time t_0. Then, for $P_x \equiv P(B > x)$ the relation

$$\begin{aligned} P(k_0 + k_1 + \cdots + k_{x'} = k) &= \frac{\{(P_0 + P_1 + \cdots + P_{x'})\lambda T_p\}^k}{k!} \\ &\times e^{-(P_0+P_1+\cdots+P_{x'})\lambda T_p} \end{aligned} \tag{24}$$

holds for $x' = 0, 1, \cdots, B_{\max} - 1$. We can prove this by the mathematical induction, as is done in the appendix. If we insert (21) and the value $x' = B_{\max} - 1$ to (24), we get

$$\begin{aligned} P(t_0, k) = &\frac{\{p_g^{-1}(1 - (1 - p_g)^{B_{\max}})\lambda T_p\}^k}{k!} \\ &\times e^{-p_g^{-1}(1-(1-p_g)^{B_{\max}})\lambda T_p}. \end{aligned} \tag{25}$$

This indicates that $P(t_0, k)$ is a Poisson distribution with the effective rate in (23). Therefore we can determine the distribution of the initial interferers $P_0(k)$ using (17) with the traffic load G replaced with

$$G_{eff} = \lambda_{eff} T_p. \tag{26}$$

Now, we consider the state transition matrix. Due to the memoryless property of the geometric distribution, all the departure probabilities of messages are p_g at the end of each packet transmission. So, if there were k initial interferers and j messages among those departed until the n-th bit, the departure rate at each bit time is given by $(k - j)p_g/L$. We redefine the departure matrix in (10) to be $\hat{C}_n^k \equiv [\hat{c}_{ij}]^k$ for

$$\hat{c}_{ij} = \begin{cases} 1 - (k-j)p_g/L, & \text{if } i = j, \\ (k-j)p_g/L, & \text{if } i = j+1, \\ 0, & \text{otherwise} \end{cases} \tag{27}$$

and apply it to (11) in place of C_n^k. Then we can obtain the state transition matrix for the variable-length message case. We observe from (27) that the departure rate $\hat{c}_{ij}$ depends on the departure state $d = j$ but not on the bit position n, differently from the fixed-length message case. In fact, all the initial interfering packets depart during the transmission time of the target packet $[t_0, t_0 + T_p]$ in the case of the fixed-length messages but, in the variable-length message case, some messages may not depart by the end of the target packet transmission time $(t_0 + T_p)$, due to the memoryless departure process. This is why the departure rate does not depend on the change of the bit position n and the state transition matrix maintains a uniform departure rate at any instance.

Once we get the state transition matrix as above, we can complete the analysis of the variable-length message system by taking the same procedure as for the fixed-length message case with the departure matrix in (11) replaced with that in (27). The packet success probability, $P_s(k)$, takes the same form as in (16) and the distribution of the initial interferers is also given by (17) for the effective traffic load G_{eff} in (26). Consequently we can determine the throughput of the variable-length unslotted ALOHA system by applying (16), (17), and (19) to (26).

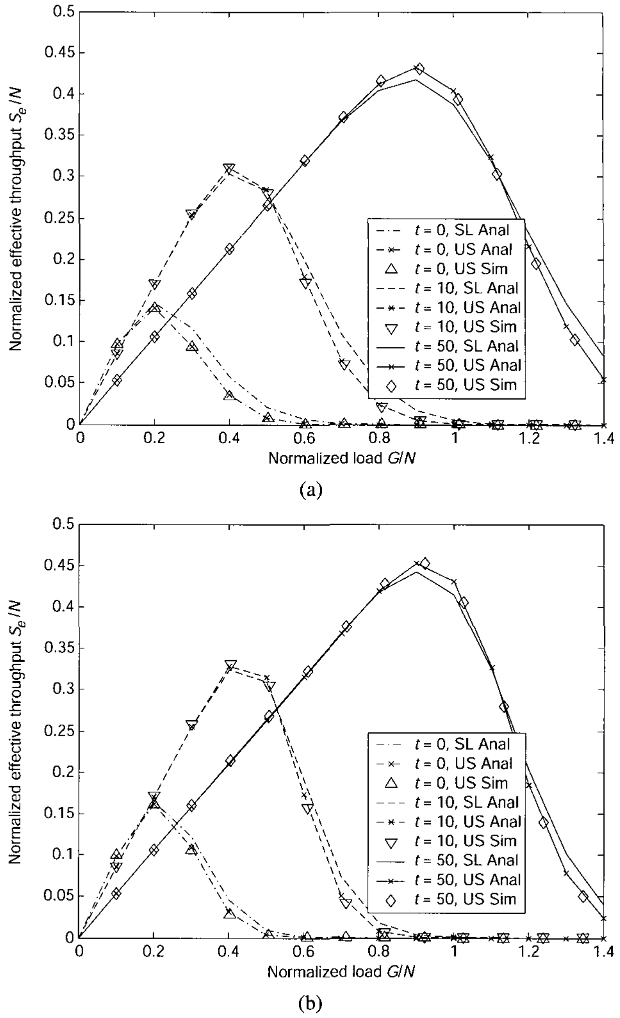

Fig. 4. Normalized effective throughputs for fixed-length messages for (a) $N = 32$ and (b) $N = 64$ (SL: Slotted, US: Unslotted, Anal: Analysis, Sim: Simulation).

V. NUMERICAL RESULTS AND DISCUSSIONS

We examine the numerical results for the above analyses in comparison with other existing analyses and computer simulations. For the analyses and simulations, we considered the DS/SSMA unslotted ALOHA system with the processing gain N of 32 or 64 and the packet length L of $1,000$ bits. We assumed that MAI is the only source of errors (i.e., the effect of thermal noise is assumed negligible). According to Morrow *et al.* [3], the optimal value of error correcting capability, t, is about 50 for the case $L = 1,000$, so we set t to 50 (except for the variable-t case of Fig. 4).

In order to check the accuracy of the proposed analysis, we first compare in Fig. 4 the analytical results of the unslotted DS/SSMA ALOHA system with the results obtained via com-

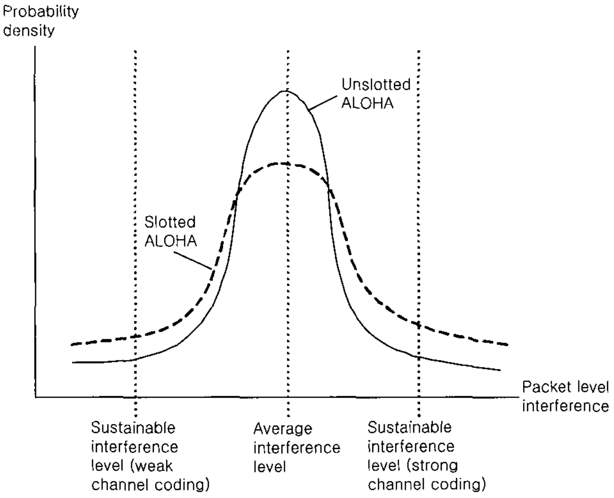

Fig. 5. Illustration of the interference averaging effect in the unslotted ALOHA system.

puter simulations for the two cases $N = 32$ and 64. For the simulation model of the fixed-length messages, we use $M/D/\infty/\infty$ model with Poisson arrivals and packet length of 1, 000. Generated Packets interfere with each other and thus interference levels vary bit-by-bit. According to each interference level, we generate random bit errors with the bit error probability in (2). At the end of each packet transmission, we compare the number of bit errors with the error correction capability t and determine whether or not the transmission is successful. The simulation time of each trial is 10^4 packet duration time and we present the average value of the results of ten trials as the simulation result. We observe that the presented analysis perfectly matches with the simulation results for various different values of error correction capability t and processing gain N. Based on this result, we may claim that the proposed analysis provides accurate performances.

Fig. 4 also compares the normalized effective throughputs (normalized by processing gain N) of the slotted and the unslotted systems. We observe that, without error correcting capability (i.e., for $t = 0$), the performance of the slotted system is slightly better than that of the unslotted system, which is consistent with the results of previous works. However, we observe that the maximum throughput of the unslotted system becomes higher than that of the slotted system as the error correcting capability gets stronger. We may interpret this phenomenon by introducing the concept of the *interference averaging effect* of the unslotted system as follows: In the slotted system, the interference level does not change during the transmission time, so the packet level interference (i.e., the overall interference level that a packet suffers from) equals the initial interference. In contrast, in the case of the unslotted system the level of interference varies in time, so the employed error correction code may possibly help to overcome a high level of initial interference if the number of packet arrivals during the packet transmission is small. Due to the state dependent nature of the departure rate in the unslotted system, the high interference level at the start of packet transmission tends to decrease during the packet transmission. In other words, a high interference level results in a high departure rate but the arrival rate is constant over the entire packet transmission, thus a packet with high initial interference can expect that the interference level would decrease during the transmission time. As a consequence, the variance of the packet level interference decreases, exhibiting an interference averaging effect for the unslotted system. Fig. 5 describes this interference averaging effect by illustrating the probability density of the packet level interference for both the unslotted and the slotted systems. As shown in the figure, the slotted system has a heavier tail distribution than the unslotted system, whereas the average interference level is the same for both systems as it is determined by the traffic load.

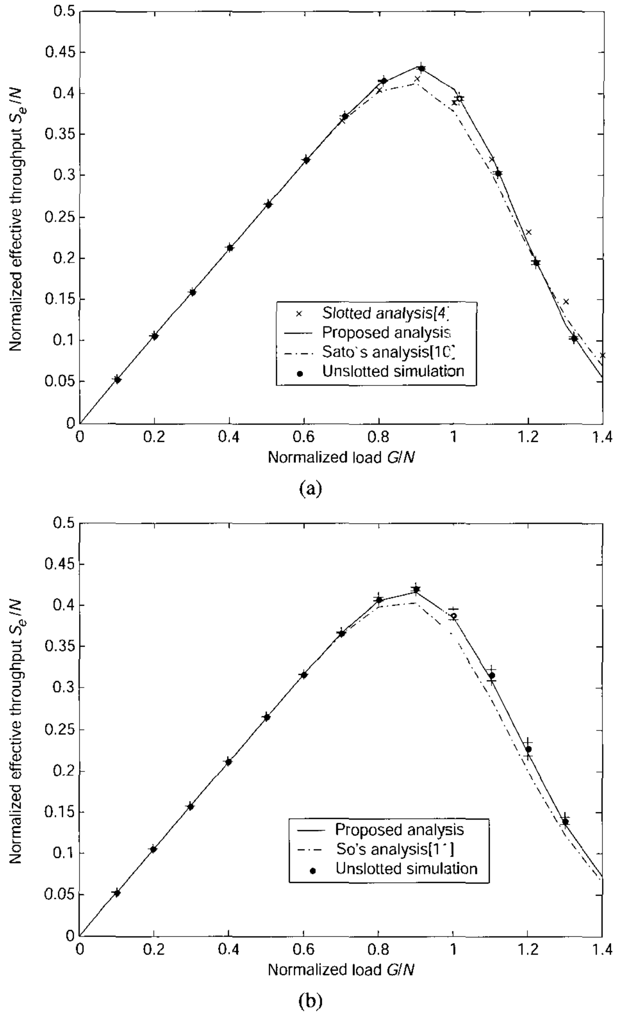

Fig. 6. Normalized effective throughputs of the presented analysis in comparison with that of the Sato's analysis for (a) fixed-length messages ($N = 32$, $t = 50$) and (b) variable-length messages ($N = 32$, $t = 50$, $p_g = 0.2$).

Figs. 6(a) and 6(b) show the normalized effective throughputs of the presented analysis in comparison with that of the analysis given by Sato *et al.* [9] and its extension by So *et al.* [10] for the values $N = 32$ and $t = 50$. For each of the simulation values presented we determined a 99% confidence interval and employed the t-Student distribution to assess the reliability of the data. We observe the following aspects from the two figures: First, the presented analysis exhibits a higher value

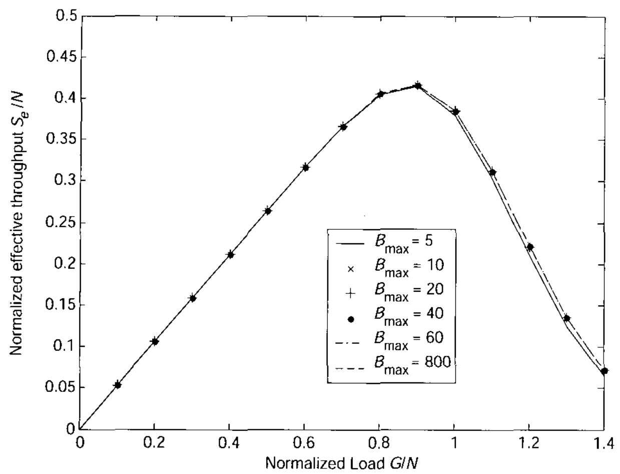

Fig. 7. Normalized effective throughputs for variable-length messages with different maximum block size $B_{\max}$ ($N = 32,\ t = 50,\ p_g = 0.2$).

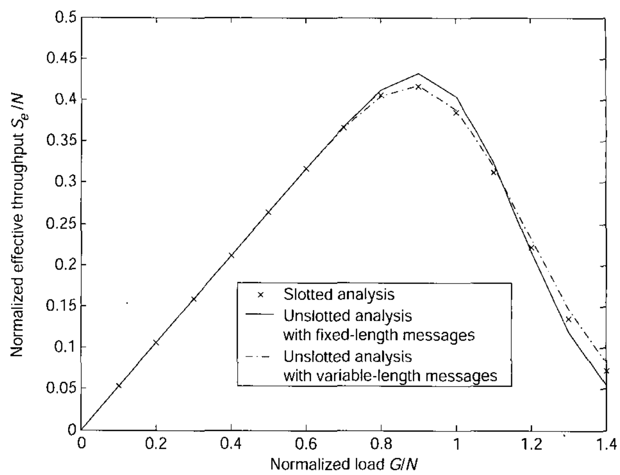

Fig. 8. Comparison of normalized effective throughputs for fixed and variable-length messages ($N = 32,\ t = 50$).

of maximum normalized effective throughput by about 5.2% in the fixed-length case and by about 3.5% in the variable-length case. Note that we took the geometric distribution parameter $p_g = 0.2$ for the latter case. Second, the presented analysis matches closely with the simulation results in both cases. Third, Sato's analysis underestimates the performance of the unslotted spread ALOHA system when compared with the presented analysis. Fourth, the maximum throughput of the unslotted system is given to be lower than that of the slotted system in the Sato's analysis. We may interpret such discrepancy to be resulted because Sato's analysis did not take account of the state dependency of the departure process. Based on this result, we may claim that the presented analysis yields a more rigorous system model than other existing ones.

Fig. 7 shows how the normalized effective throughput of the variable-length message system changes as the maximum block size, $B_{\max}$, changes. We observed that the throughput is independent of $B_{\max}$ as long as it is maintained larger than or equal to $10(= 2/p_g)$. This validates the geometric approximation of the message length given in Section IV.

Fig. 8 compares the normalized effective throughputs of the unslotted system with fixed and variable length messages. We observe that the maximum throughput is larger for the fixed-length message case than for the variable-length message case. This happens because the interference averaging effect weakens in the variable-length case due to the decreased fluctuation of packet level interference.

VI. CONCLUSION

So far we have analyzed the throughput of the DS/SSMA unslotted ALOHA system. The unique feature of the presented analysis is the adoption of the two-dimensional state transition model that reflects the state-dependent nature of the departure processes in the unslotted system. Based on this model, we first conducted performance analysis for the case of fixed-length messages and then extended it to encompass the case of variable-length messages. Numerical results revealed that the presented analysis is more accurate than other existing analyses, closely matching with the simulation results.

In examining the performance of the unslotted ALOHA system, we discovered that the maximum achievable throughput of the unslotted system becomes higher than that of the slotted system when a strong error-correction capability is supported. This is a new phenomenon that contradicts the existing misconception that the unslotted ALOHA system would perform poorer than the slotted ALOHA system. We were able to interpret this phenomenon by introducing the concept of interference averaging effect, which in fact is an inherent characteristic of the unslotted system. Due to this interference averaging effect, it is less probable for the unslotted system to suffer from uncorrectably severe interference than for the slotted system, which consequently leads to a higher throughput for the unslotted system. This effect has become visible in the presented analysis for the first time because it originates from the state-dependency of the departure rate.

APPENDIX

Proof of (24):

We prove the equation by the mathematical induction. Since message arrival is Poisson for the interval $(t_0 - (x+1)T_p, t_0 - xT_p]$ and each message lasts until t_0 with probability P_x, we obtain the probability of $k_x = m$ by

$$P(k_x = m) = \sum_{j=0}^{\infty}\{\frac{(\lambda T_p)^j}{j!}e^{-\lambda T_p}\binom{j}{m}P_x^m(1-P_x)^{j-m}\} = \frac{(P_x\lambda T_p)^m}{m!}e^{-P_x\lambda T_p} \quad (28)$$

which implies that (24) holds for $x' = 0$. We assume that (24) holds for $x' = 0, 1, \cdots, l$. Then, for $x' = l+1$, we get

$$P(k_0 + k_1 + \cdots + k_{l+1} = k) = \sum_{m=0}^{k} P(k_{l+1} = m) \times P(k_0 + k_1 + \cdots + k_l = k - m). \quad (29)$$

From (28) and the assumption for $x' = l$, (29) becomes

$$P(k_0 + k_1 + \cdots + k_{l+1} = k) = \sum_{m=0}^{k} \left[\frac{(P_{l+1}\lambda T_p)^m}{m!} e^{-P_{l+1}\lambda T_p} \times \frac{\{(P_0 + P_1 + \cdots + P_l)\lambda T_p\}^{k-m}}{(k-m)!} \times e^{-(P_0+P_1+\cdots+P_l)\lambda T_p} \right] \tag{30}$$

which reduces to $\frac{\{(P_0+P_1+\cdots+P_{l+1})\lambda T_p\}^k}{k!} e^{-(P_0+P_1+\cdots+P_{l+1})\lambda T_p}$. This proves that (24) holds for $x' = l + 1$. □

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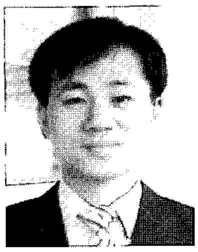

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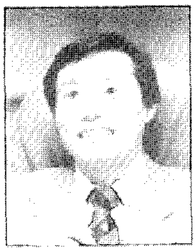

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