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이경근** · 문일경** · 송재복*** · †류시욱****

Coordinated Inventory Model for the Joint Replenishment Supply Chain*

Kyung-keun Lee** · Il-kyeong Moon**
Jae-bok Song*** · Si-wook Ryu****

■ Abstract ■

We consider an integrated supply chain model in which multiple suppliers replenish items for a single buyer's demand. Also the buyer specifies a basic replenishment cycle and the suppliers replenish the items only at those time intervals. Namely, we propose a model to study and analyze the benefit by coordinating supply chain inventories through the basic replenishment cycle time. The objective of this model is to minimize the total relevant annual cost of the integrated inventory model. After developing proposed coordinated models, we suggest heuristics for searching the solutions of our models. Finally, numerical and computational experiments for each policy are carried out to evaluate the benefits of those models and the compensation policy is addressed to share the benefits.

Keyword : Joint Replenishment, Coordinated Inventory Management, Supply Chain

1. Introduction

SCM (Supply Chain Management) has gained importance in the industrial field as one of the main marketing processes that have positive influences on shareholder value. The more it is

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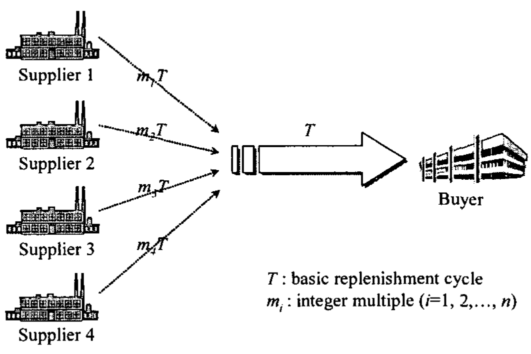
** 부산대학교 산업공학과
*** LG전자
**** 한중대학교 산학협력단 안전연구소
† 교신저자

growing importance of SCM, the more it is concentrated on not only the information sharing of the chain but also the interrelationship of the participants. The characteristic of buyers and suppliers relationship has been undergoing dramatic changes over the last decades. There have been some research works, showing that the buyers and suppliers relationship has evolved towards cooperation in order to respond to intensified competition in industry. Therefore, researchers extensively have studied the cooperative interactions between the buyers and suppliers.

In the context of inventory on cooperation between them, since the late 1970s, many researchers have studied the integrated inventory models. Goyal [5] suggests that if the supplier and the buyer, instead of determining their policies independently, decide to cooperate the economic joint inventory policy, then considerable savings can be achieved. And he also develops a joint economic lot size model where the objective is to minimize the total relevant costs for both the single supplier and the single buyer. Banerjee [1] analyzes the model by incorporating a finite production rate and following a lot-for-lot policy for a supplier. For all these models, it is assumed that the supplier previses buyers' annual demands, inventory holding cost and ordering cost. In this typical purchasing situation, single supplier and single buyer may negotiate both price and lot size, and the outcome depends on the negotiating power between the two parties or some contractual agreement. Moreover, these research works are devoted to single product models while most businesses are operating in a multi-item environment.

Even though there are many single supplier situations, it is quite common that there are a lot of big buyers who need a great variety of items replenished from many suppliers in practice. Such buyers are like motor companies, department stores, electronics companies and so on. In this circumstance, a big buyer orders the required products to many suppliers and he/she wants to be replenished at any particular time for the products. However, the buyer bearing the freight and inventory related costs wants to reduce the relevant costs without sacrificing product quality or service level. Thus, we must consider the coordinated inventory model with multiple suppliers and a single buyer in a supply chain. Furthermore we suggest the joint replenishment strategy as one of the many coordinated decision policies.

We consider the situation in which multiple suppliers implementing the joint replenishment strategy share the inventory information each other. Each supplier delivers the quantity produced during his period to the buyer, and this period is determined by the integer multiple of basic replenishment cycle. [Figure 1] presents the situation in which suppliers deliver their products at their own replenishment cycle time.



[Figure 1] Joint replenishment by basic replenishment cycle

Compared to the many researches for the integrated one-supplier one-buyer problem, a few research works address the integrated one-supplier multiple-buyer model. Lu [10] develops a model based on the assumption that the supplier has an advantage over the buyers in purchasing negotiation. The focus of this model is to minimize the supplier's total annual cost subject to the upper limit of the cost the buyer can bear. Viswanathan and Piplani [12] study the integrated supply chain similar to Lu's [10] framework. They propose a model to study and analyze the benefit of coordinating supply chain inventories for a single product by means of common replenishment time period.

Over the last two decades, many researchers have studied the problems that determine the economic order quantities for jointly replenished items. For the joint replenishment problem (JRP), an optimal solution approach is proposed by Goyal [4]. However, this approach is based on the enumeration and the running time for the optimal solution grows exponentially with the number of items. After Goyal [4], many more efficient heuristics have been developed to find solutions about the JRP. Silver [11] suggests an efficient heuristic method for solving the JRP. This approach has been further improved by Goyal and Belton [6] and Kaspi and Rosenblatt [9]. Kaspi and Rosenblatt [9] suggest a new heuristic method called RAND by adopting Silver's improved heuristics. The result of applying RAND shows that it outperforms all previous solution procedures which are not enumerative. Goyal and Deshmukh [7] provide a better lower bound for the basic replenishment cycle time, which further improves the performance of RAND. A tighter lower bound is helpful because it reduces the range of the basic replenishment cycle time from which m equally spaced values are obtained.

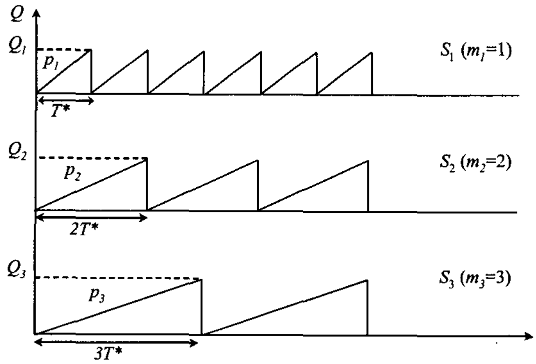
Most of the integrated inventory literatures deal with the environment of single supplier with single buyer or single supplier with multiple buyers. But, in this study, we basically consider another situation of the integrated inventory model with multiple suppliers and single buyer. In this circumstance, we adopt the joint replenishment strategy to coordinate the inventory management. The objective of our study is to minimize the total coordinated relevant cost for both the multiple suppliers and the single buyer. We suggest a new heuristics to determine the basic replenishment cycle time and integer multiples of the basic replenishment cycle time. Also, the performance of our new solution approach is compared with the one of RAND.

2. Model Development

We restrict our discussion and analysis to a relatively simple purchasing scenario like Banerjee [1]. That is, a buyer periodically orders some quantity, Q, of an inventory item. With the receipt of an order, the supplier produces the ordered quantity of the item (i.e, the suppliers follow a lot-for-lot policy) and, on completion of the batch, he delivers the entire lot to the buyer. In addition to deterministic conditions, we assume that there are many suppliers dealing with just single item and we ignore the time to deliver the completed lot to the buyer. In reality, demand rates and lead times are stochastic. Furthermore, real world scenarios are likely to be more complex than the one we outline. For example, in a just-in-time (JIT) environment, the buyer favors a relatively small lot size and seeks the most potential sources of supply. The supplier prefers to move closer to the buyer's position by lowering his/her setup cost through technological changes. In other cases the monopolistic supplier, knowing that no other supplier exists for the item in question, may be in a position to take undue advantage of the situation.

In this particular occasion, the supplier's production/delivery cycle equals to the buyer's replenishment cycle. Also, we assume that the buyer takes charge of the transportation cost for product deliveries and it is involved in buyer's ordering process. In this paper, we emphasize a basic replenishment cycle for the multiple suppliers to minimize the total coordinated system costs. The relevant costs considered for the coordinated decision policy are similar to those in the individual decision policy, except that the major ordering cost charged by the buyer. The cost of placing an order for a number of different items consists of two components : (1) a major ordering cost which is independent of the kind of supplied items per replenishment interval ; and (2) a minor ordering cost which depends on the kind of different items in the buyer's order. This problem is similar to the JRP. In this study, however, we consider the integrated supply chain with multiple suppliers and one buyer.

We compare two decision policies, i.e., the individual and coordinated decision policy. In the individual decision policy, a buyer individually makes the inventory decisions for the individual items. However in the coordinated decision policy, the joint replenishment strategy is used for all suppliers and the buyer. [Figure 2] depicts the coordinated inventory levels for three suppliers. It illustrates that suppliers deliver to the buyer with the multiple intervals of basic replenishment cycle.



[Figure 2] Multiple suppliers' inventory level with the basic replenishment cycle time

For our models, the following assumptions are used.

1) The demand rate and production rate, and relevant costs are known and constant.
2) No shortages are allowed and the buyer's storage space is not limited.
3) Replenishment lead time is constant and replenishment is instantaneous for all suppliers.
4) The buyer's ordering cost for each item i involves the transportation cost of the each item replenished from suppliers.
5) Each supplier deals with only one item.
6) The suppliers' production capacities are sufficient to meet demands.
7) Every participant shares all information.

The following notation is used in the model.

i = the supplier index, $i=1, 2, \cdots, n$

d_i = the annual demand rate (unit/period) of buyer for item i

p_i = the production rate (unit/period) of supplier for item i

A = the buyer's major ordering cost per order
a_i = the buyer's minor ordering cost per order for item i
s_i = the supplier's setup cost for item i per production
h_{bi} = the buyer's inventory holding cost for item i per unit time
h_{si} = the supplier's inventory holding cost for item i per unit time
T_{bi} = the buyer's order replenishment cycle time for item i under individual decision policy
T = the basic replenishment cycle time, common to all suppliers under coordinated decision policy
m_i = the positive integer multiples of T between two consecutive replenishment interval

We now present two cost models for two-level supply chain, one derived by the individual decision policy and the other by the coordinated decision policy. We assume that the buyer and the suppliers perfectly interchange information about inventories and purchasing schedules each other. In the individual decision policy, a buyer focuses on minimizing his own total inventory related cost without consideration for the suppliers. In the absence of any coordination between the buyer and the suppliers, a buyer places economic replenishment orders at time interval T_{bi}. The replenishment orders can be arrived at any time and thus deterministic demand can be met immediately. The suppliers follow a lot-for-lot policy, which means they produce as much as the order quantity placed by the buyer. The assumption of a lot-for-lot policy might be restrictive, but it is useful to focus the analysis on the benefits of implementing the coordinated strategy, and to make the analysis simpler. Before deriving the coordinated model, we first consider individual decision model for the two levels. Then, we propose the coordinated supply chain model to minimize the entire supply chain system cost.

2.1 Individual decision policy

In this policy, we assume that the ordering cost involves the fixed transportation cost. Under the individual decision policy, the buyer decides the replenishment cycle T_{bi} for each individual item independently. And the total annual cost for supply chain system is equal to the sum of every supplier's and buyer's total relevant cost.

2.1.1 Buyer's total cost

Such as a simple purchasing scenario in Banerjee [1], the buyer periodically orders some quantity, Q_i, of an inventory items to all the suppliers. Under the individual replenishment policy, the buyer decides the replenishment order cycle, T_{bi}, for each item on an EOQ basis. In EOQ policy, the buyer orders a quantity of $Q_i = d_i T_{bi}$ for each item i at every replenishment cycle time. Hence, the total relevant cost of n items per unit time for the buyer is given by

$$TC_b(T_{b1}, \dots, T_{bn}) = \Sigma_{i=1}^{n}((A+a_i)/T_{bi} + d_i T_{bi} h_{bi}/2) \quad (1)$$

It is easy to see that TC_b is convex function in T_{bi}. In addition, $T_{b1}, \dots, T_{bn}$ are independent each other. Therefore, the optimal replenishment cycle time for each item i can be obtained easily by differentiating equation (1) with respect to T_{bi} and by setting the n equations equal to zero. Accordingly, the optimal cycle between successive replenishments at the buyer for item i can be obtained as follows :

$$T_{bi}^* = (2(A+a_i)/d_i h_{bi})^{1/2} \quad (2)$$

Correspondingly the total cost per unit time for the buyer as follows :

$$TC_b(T_{b1}^*, \ldots, T_{bn}^*) = \Sigma_{i=1}^{n} (2d_i h_{bi}(A+a_i))^{1/2} \quad (3)$$

2.1.2 Suppliers' total cost

With the receipt of an order, each supplier following a lot-for-lot policy produces Q_i from the buyer respectively and, on completion of the batch, delivers the entire lot to the buyer. If the suppliers follow the buyer's replenishment policy, every supplier's total cost per unit time is given by

$$TC_s(T_{b1}, \ldots, T_{bn}) = \Sigma_{i=1}^{n} (s_i/T_{bi} + d_i^2 T_{bi} h_{si}/2p_i) \quad (4)$$

Therefore, the sum of TC_b and TC_s gives the total cost of two level inventories in a supply chain system.

$$TC_{ind} = TC_b + TC_s \quad (5)$$

2.2 Coordinated decision policy

In the coordinated decision policy, the basic replenishment cycle and multiple integer representing the replenishment time of each supplier are determined so that the total cost of supply chain system is minimized. The relevant costs considered in this policy are similar to those in the individual policy, except for the major and minor ordering costs to the buyer.

The buyer who is responsible for the transportation cost will specify that the ordering for multiple items is placed only at particular points in time. The suppliers adopting this policy must participate in the joint replenishment strategy. This means that the consolidated items among suppliers are periodically delivered to the buyer. If the practice already exists in the channel, then suppliers will accept it without any objection. However, if the suppliers are already used to deliveries of replenishment order at any point in time (more convenient to them), they may be reluctant to accept deliveries only at specific time periods. Therefore, suppliers may request adequate compensating incentives, such as a benefit sharing or price discounts.

Under the coordinated policy, the replenishment interval for each item i ordered by the buyer is

$$T_{bi} = m_i T \qquad \text{where } m_i \geq 1 \text{ and integer.}$$

By a joint replenishment among a family of items, the total cost for the buyer per unit time is as follows :

$$JTC_b(T, m_1, m_2, \ldots, m_n) = A/T + \Sigma_{i=1}^{n} a_i/m_i T + \Sigma_{i=1}^{n} m_i d_i h_{bi} T/2 \quad (6)$$

And the total cost for every supplier per unit time is as follows :

$$JTC_s(T, m_1, m_2, \ldots, m_n) = \Sigma_{i=1}^{n} s_i/m_i T + \Sigma_{i=1}^{n} m_i d_i^2 h_{si} T/2p_i \quad (7)$$

In the above models, $\{m_i\}$ represents a set of the integer multiples of the replenishment cycle for each item i. Similar to the individual policy, the total relevant cost of the coordinated decision model is obtained by summing of JTC_b and JTC_s.

$$\begin{aligned} JTC(T, m_1, m_2, \ldots, m_n) &= \Sigma_{i=1}^{n} s_i/m_i T + \Sigma_{i=1}^{n} m_i d_i^2 h_{si} T/2p_i \\ &+ A/T + \Sigma_{i=1}^{n} a_i/m_i T + \Sigma_{i=1}^{n} m_i d_i h_{bi} T/2 \end{aligned} \quad (8)$$

In equation (8), we can see that our integrated model is similar to the general JRP and produces $n+1$ decision variables corresponding to the basic replenishment cycle and n integer multiples.

3. Development of Heuristics

During the last two decades, many heuristic procedures for the JRP have appeared in the literature. Though Goyal [4] presents the efficient solution procedure based on an enumerative approach, the enumerative procedure requires substantial computational effort to obtain solutions for large problems. Thus, we use a much simpler and non-enumerative approach which is convenient and has produced excellent results on a number of problems. In our study, it is also very difficult to find the optimal solution except for the case when $\forall m_i = 1$. For any particular value of $T > 0$, the optimal values of m_i's can be determined easily, and likely, we can determine the optimal value of T when m_i's values are given. But m_i's cannot be determined without knowing T, and T in turn can not be determined without knowing m_i's.

The problem is to determine T and the integer m_i's to minimize equation (8). The integer requirements, as well as the interdependence of the effects of the values of the control variables (e.g., the effects of altering the value of a particular m_i depend upon the values of T and the other m_i's), make the determination of the optimal solution very difficult.

We can calculate the optimal replenishment cycle T^* for a given any particular integer multiple set $\{m_i\}$ as follows :

$$T^* = \left(\left(2(A + \Sigma_{i=1}^{n} a_i/m_i + \Sigma_{i=1}^{n} s_i/m_i\right)/\left(\Sigma_{i=1}^{n} m_i d_i^2 h_{si}/p_i + \Sigma_{i=1}^{n} m_i d_i h_{bi}\right)\right)^{1/2} \quad (10)$$

After ignoring A/T of equation (8) which is independent of m_i, we can select the m_i's to minimize the following equation (11)

$$\begin{aligned} Z(m_1, m_2, \cdots, m_n) &= \Sigma_{i=1}^{n} s_i/m_i T + \Sigma_{i=1}^{n} m_i d_i^2 h_{si} T/2p_i \\ &+ \Sigma_{i=1}^{n} a_i/m_i T + \Sigma_{i=1}^{n} m_i d_i h_{bi} T/2 \end{aligned} \quad (11)$$

The general idea suggested for solving this problem assumes that the m_i values are known so that the optimal T is obtained by the equation (10). Then, given the value of T, the optimal values of m_i^* is determined by the following sufficient condition :

$$Z(m_i^*) \le Z(m_i^* - 1) \text{ and } Z(m_i^*) \le Z(m_i^* + 1) \quad (12)$$

From above inequality (12), we obtain

$$\begin{aligned} m_i^*(m_i^* - 1) &\le \left(2p_i(a_i + s_i)/T^2 d_i(p_i h_{bi} + d_i h_{si})\right) \\ &\le m_i^*(m_i^* + 1) \end{aligned} \quad (13)$$

3.1 Iterative algorithm

In our model, we extend the iterative heuristics adopted by Brown [2] and Goyal [3]. This heuristic method uses equations (10) and equations (13) to get the efficient solutions. Our approach first requires an initial estimate of $\{m_i\}$, in our algorithm we assumed initially $m_i = 1 (i = 1, 2, \ldots, n)$ to determine T value from equation (10). This T value is then plugged into equation (13) and new estimates, m_i's are obtained. The iterations are stopped if values of m_i $(i = 1, 2, \ldots, n)$ in the q^{th} iteration are the same as those in the $(q-1)^{th}$ iteration. The procedure of our iterative algorithm to obtain solutions is as follows :

Step 1. (Initialization) Set $m_i = 1 (i = 1, 2, \ldots, n)$ and $q = 0$.

Step 2. Set $q = q + 1$. And compute
$$T = \left(\left(2(A + \Sigma_{i=1}^{n} a_i/m_i + \Sigma_{i=1}^{n} s_i/m_i\right)/\left(\Sigma_{i=1}^{n} m_i d_i^2 h_{si}/p_i + \Sigma_{i=1}^{n} m_i d_i h_{bi}\right)\right)^{1/2}.$$

Step 3. For T and each product i, compute

$m_{i,q}^2 = \left(2p_i(a_i+s_i)/T^2 d_i(p_i h_{bi}+d_i h_{si})\right)$.

Step 4. Find $m_{i,q}^2$ for each i where $m_{i,q}=L$ if $L(L-1) \le m_{i,q}^2 \le L(L+1)$.

Step 5. If $q=1$ or $m_{i,q} \ne m_{i,q-1}$ for any i, then go to step 2. If $m_{i,q}=m_{i,q-1}$, then go to next step.

Step 6. Compute JTC for this $(T^*, m_{1,q}^*, \ldots, m_{n,q}^*)$.

3.2 RAND algorithm

After an efficient heuristic algorithm has been developed for solving the JRP by Silver [11], the heuristics was further improved by Goyal and Belton [6], and Kaspi and Rosenblatt [9]. Silver's improved heuristics is a basis for a new heuristics proposed by Kaspi and Rosenblatt [9]. It is based on computing several equally spaced values of the basic replenishment cycle within its upper and lower bound $[T_{\min}, T_{\max}]$.

Then they apply Silver's improved heuristics at each value of T. This new heuristic algorithm called RAND, performs better than all previous heuristics except for the full enumeration.

Thus, we adopt the extended Kaspi and Rosenblatt [9]'s RAND algorithm to obtain the solution of our coordinated inventory model. The algorithm procedure is as follows :

Step 1. Compute $T_{\max} = \left(2\left(A+\sum_{i=1}^n (a_i+s_i)\right)/\left(\sum_{i=1}^n d_i^2 h_{si}/p_i + \sum_{i=1}^n d_i h_{bi}\right)\right)^{1/2}$ and

$T_{\min} = \min_i \left((2(a_i+s_i))/\left((d_i^2 h_{si}/p_i)+d_i h_{bi}\right)\right)^{1/2}$.

Step 2. Divide the range of $[T_{\min}, T_{\max}]$ into l equally spaced intervals of $T(T_1, \ldots, T_p, \ldots, T_l)$. And l is to be decided by the decision maker. Set $p=0$.

Step 3. Set $p=p+1$ and $q=0$.

Step 4. Set $q=q+1$.

For T_p and each product i, compute $m_{i,q}^2 = 2p_i(a_i+s_i)/(p_i h_{bi}+d_i h_{si})T_p^2 d_i$.

Step 5. Find $m_{i,q}^2$ for each i where $m_{i,q}=L$ if $L(L-1) \le m_{i,q}^2 \le L(L+1)$.

Step 6. Compute a new cycle time T_p according to

$T_p = \left(2(A+\sum_{i=1}^n a_i/m_{i,q} + \sum_{i=1}^n s_i/m_{i,q})\right.$
$\left./\left(\sum_{i=1}^n m_{i,q} d_i^2 h_{si}/p_i + \sum_{i=1}^n m_{i,q} d_i h_{bi}\right)\right)$.

Step 7. If $q=1$ or $m_{i,q} \ne m_{i,q-1}$ for any i, then go to step 4.

Compute JTC for this $(T_j, m_{1,q}, \ldots, m_{n,q})$

Step 8. If $p=l$ then select $(T_j^*, m_{i,q}^*, \ldots, m_{n,q}^*)$ with the minimum JTC.

Otherwise go to step 3.

It should be noted that, at the step 1, $T_{\min}$ is minimum value among those of all items when the major ordering cost, A, is zero.

3.3 Numerical Examples

Now we carry out numerical experiments to investigate the performance of joint replenishment for the coordinated inventory model. To set the solutions for our models, the iterative and RAND algorithms are coded by Excel Visual Basic for Applications. At first, optimal solutions for the individual model are obtained, and then we determine the value of basic replenishment cycle T and integer multiplier m_i by the iterative algorithm and RAND algorithm, which minimize the total inventory related cost of the coordinated inventory model.

For illustration, we use the set of parameters in <Table 1> which shows the buyer's demand rate and the costs associated major and minor ordering and inventory holding, and the suppliers' production rates and the costs associated with setup and inventory holding.

〈Table 1〉 Data for numerical example for 10 suppliers

Item	Buyer				Supplier		
	d_i (unit/yr)	A ($/cycle)	a_i ($/order)	h_{bi} ($/unit/yr)	p_i (unit/yr)	s_i ($/setup)	h_{si} ($/unit/yr)
1	5000	8	2.2	1.2	7000	7	1
2	10000	8	2	1	12000	5	0.75
3	1000	8	2	1.5	1200	8	1.2
4	2500	8	3	2	3600	6	1
5	15000	8	1	1	20000	2	0.5
6	4000	8	1	2	6000	5	1
7	2000	8	3	3	3300	8	1.5
8	530	8	5	2.3	650	11	2
9	1200	8	2	2	1800	7	1.75
10	1000	8	2	1.5	1500	7	0.5

We get the results for the independent decision policy and for the coordinated decision policy using the iterative algorithm and RAND algorithm. The computational results are summarized in <Table 2>, <Table 3> and <Table 4>.

As mentioned above, in the individual decision policy, the buyer determines the optimal replenishment cycles which minimize his inventory related cost of n items and the suppliers follow these replenishment cycles for their items. Therefore, we can get the entire supply chain system cost by summing up the costs of the buyer and every supplier. <Table 3> shows the result of joint replenishment strategy for the coordinated decision policy by iterative algorithm. We get cost saving of 22.33% compared with the independent decision policy. In this example, the basic replenishment cycle is 0.0385year. It implies that supply chain system can reduce its total annual cost by coordination of the replenishment of different suppliers. <Table 4> shows another solution by RAND algorithm. Remarkably, in joint replenishment strategy for the coordinated inventory model, RAND algorithm saves cost about 1.11% more than iterative algorithm. Also, we know that the basic replen-

〈Table 2〉 Result of the individual decision policy

Item	Buyer		Supplier	
	T_{bi}	TC_{bi}	T_{si}	TC_{si}
1	0.0583	349.86	0.0583	224.17
2	0.0447	447.21	0.0447	251.56
3	0.1155	173.21	0.1155	127.02
4	0.0663	331.66	0.0663	148.03
5	0.0346	519.62	0.0346	155.16
6	0.0474	379.47	0.0474	168.65
7	0.0606	363.32	0.0606	187.76
8	0.146	178.03	0.146	138.43
9	0.0913	219.09	0.0913	140.58
10	0.1155	173.21	0.1155	79.87
TC_{ind}	**$ 4755.32**			

〈Table 3〉 Result of the coordinated inventory model by iterative algorithm

Iteration	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9	m_{10}	T
1	1	1	1	1	1	1	1	1	1	1	0.0487
2	1	1	2	1	1	1	1	3	1	2	0.041
3	1	1	2	1	1	1	1	3	2	2	0.0391
4	1	1	2	1	1	1	1	3	2	3	0.0385
5	**1**	**1**	**2**	**1**	**1**	**1**	**1**	**3**	**2**	**3**	**0.0385**
T^*	**0.0385yr**										
JTC	**$ 3693.35**										
Saving	**22.33%**										

〈Table 4〉 Result of the coordinated inventory model by RAND algorithm(l=10, 20, 30)

l	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9	m_{10}	T
10	2	1	3	2	1	1	2	5	3	4	0.0275
20	2	1	3	2	1	1	2	4	2	3	0.0292
30	**2**	**1**	**3**	**2**	**1**	**1**	**2**	**4**	**2**	**3**	**0.0292**
T^*	**0.0292yr**										
JTC	**$ 3650.94**										
Saving	**23.22%**										

ishment cycle decreases by 0.0292year. That is, RAND algorithm performs slightly better than iterative algorithm in this numerical example.

Now we take one more numerical example for 20 suppliers. <Table 5>, <Table 6>, <Table 7 > and <Table 8> show the data and results.

〈Table 5〉 Data for numerical example for 20 suppliers

Item	Buyer				Supplier		
	d_i (unit/yr)	A ($/cycle)	a_i ($/order)	h_{bi} ($/unit/yr)	p_i (unit/yr)	s_i ($/setup)	h_{si} ($/unit/yr)
1	5000	8	4	2.5	6000	6	1
2	10000	8	2	1.5	12000	3	0.75
3	1500	8	5	3	3000	7	1.2
4	2500	8	4	1.5	3600	6	1
5	2000	8	5	1.25	3200	6.5	1.6
6	4000	8	2.5	1	5800	6	1.5
7	2000	8	3	0.5	3200	5	2
8	350	8	6	1	650	7	0.5
9	600	8	2	1	1000	5	0.6
10	1000	8	3	1.5	2500	6	1.3
11	5000	8	3	1.2	6200	5	1
12	10000	8	2	1	13000	3	0.75
13	1500	8	3	1.5	2700	3	1.2
14	2500	8	3	2	3600	5	1
15	2000	8	2	2	3600	7	1.6
16	4000	8	2	2	6000	10	1
17	2000	8	2	3	3300	5.5	1.5
18	350	8	1	2.5	550	6	2
19	600	8	1	1.25	1300	8	0.5
20	1000	8	2	1.5	1800	7	0.5

〈Table 6〉 Result of independent decision policy for 20 suppliers

Item	Buyer		Supplier	
	T_b	TC_b	Ts	TCs
1	0.0438	547.72	0.0438	228.22
2	0.0365	547.72	0.0365	196.27
3	0.076	342.05	0.076	126.3
4	0.08	300	0.08	144.44
5	0.102	254.95	0.102	165.72
6	0.0725	289.83	0.0725	232.72
7	0.0148	148.32	0.0148	219.11
8	0.2828	98.99	0.2828	23.38.07
9	0.1826	109.54	0.1826	47.1
10	0.1211	181.66	0.1211	81.03
11	0.0606	363.32	0.0606	20.465
12	0.0447	222.49	0.0447	196.09
13	0.0989	331.66	0.0989	79.78
14	0.0663	282.84	0.0663	132.96
15	0.0707	400	0.0707	161.85
16	0.05	346.41	0.05	266.67
17	0.0577	125.5	0.0577	147.75
18	0.1434	116.19	0.1434	73.78
19	0.1549	173.21	0.1549	62.36
20	0.1155	447.21	0.1155	76.66
TC_{ind}	**$ 8511.16**			

〈Table 7〉 Result of the coordinated inventory model by iterative algorithm

Iteration	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9	m_{10}	m_{11}	m_{12}	m_{13}	m_{14}	m_{15}	m_{16}	m_{17}	m_{18}	m_{19}	m_{20}	T
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.0523
2	1	1	1	1	1	1	1	5	3	2	1	1	1	1	1	1	1	2	3	2	0.0459
3	1	1	2	1	2	1	2	5	3	2	1	1	1	1	1	1	1	2	3	2	0.0418
4	1	1	2	2	2	1	2	6	3	2	1	1	2	1	1	1	1	3	3	2	0.0383
5	1	1	2	2	2	1	2	6	3	3	1	1	2	1	2	1	1	3	4	3	0.0361
6	1	1	2	2	2	1	2	7	4	3	1	1	2	1	2	1	1	3	4	3	0.0359
7	**1**	**1**	**2**	**2**	**2**	**1**	**2**	**7**	**4**	**3**	**1**	**1**	**2**	**1**	**2**	**1**	**1**	**3**	**4**	**3**	**0.0359**

T^*	**0.0359yr**
JTC	**$ 6386.22**
Saving	**24.97%**

〈Table 8〉 Result of the coordinated inventory model by RAND algorithm

l	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8	m_9	m_{10}	m_{11}	m_{12}	m_{13}	m_{14}	m_{15}	m_{16}	m_{17}	m_{18}	m_{19}	m_{20}	T
10	1	1	3	2	3	2	3	9	5	4	2	1	2	2	2	2	2	4	5	4	0.0263
20	1	1	3	2	3	2	3	9	5	4	2	1	2	2	2	2	2	4	5	4	0.0263
30	**1**	**1**	**3**	**2**	**3**	**2**	**3**	**9**	**5**	**4**	**2**	**1**	**2**	**2**	**2**	**2**	**2**	**4**	**5**	**4**	**0.0263**
T^*	**0.0263yr**																				
JTC	**$ 6381.27**																				
Saving	**25.02%**																				

Interestingly, for 20 suppliers case, performance of iterative algorithm is slightly better than that of RAND.

4. Compensation Policy

As mentioned in previous section, even the coordinated inventory decision policy cannot guarantee that all participants of supply chain are necessarily better off. If some of the suppliers joining the coordinated decision policy have less benefit than before, they are reluctant to join the coordinated supply chain. Accordingly, if they do not want to join the supply chain, the whole benefit from the coordinated decision policy is decreased. Namely, the more suppliers join the coordinated supply chain, the more cost savings can be achieved. Therefore, it is very important how cost savings are allocated among the participants. We suggest a method to compensate the suppliers experiencing a loss due to coordinated decision policy and to share the remained benefits.

At this point, we use the cost allocation method used in Goyal [5]. Goyal [5] suggested that the total annual cost should be allocated to the suppliers and buyer as follows :

$$Cost\ of\ suppliers = Z \cdot V(C^*, S^*)$$
$$Cost\ of\ buyer = (1-Z) \cdot V(C^*, S^*)$$

where $Z = V(S_0)/(V(S_0)+V(C_0))$.

$V(S_0)$ and $V(C_0)$ are the minimal objective values of the total cost per unit time for suppliers and the buyer in individual decision policy respectively. And $V(C^*, S^*)$ is the minimal objective value of the coordinated decision policy. We adopt this allocation method to our coordinated model and derive the following equations.

$$Cost\ of\ buyer = Z_b \cdot JTC(T^*, m_1^*, m_2^*, \ldots, m_n^*)$$
$$Z_b = TC_b(T_{b1}^*, T_{b2}^*, \ldots, T_{bn}^*) / \big(TC_b(T_{b1}^*, T_{b2}^*, \ldots, T_{bn}^*) + TC_s(T_{b1}^*, T_{b2}^*, \ldots, T_{bn}^*)\big) \quad (14)$$

and

$$Cost\ of\ supplier\ i = Z_{si} \cdot JTC(T^*, m_1^*, m_2^*, \ldots, m_n^*).$$
$$Z_{si} = TC_{si}(T_{bi}^*) / \big(TC_b(T_{b1}^*, T_{b2}^*, \ldots, T_{bn}^*) + TC_s(T_{b1}^*, T_{b2}^*, \ldots, T_{bn}^*)\big) \quad (15)$$

With the data in <Table 1>, we carry out the numerical example for the compensation policy. We get the 23.22% of cost savings by the coordinated decision policy. However, the buyer's total cost is decreased from $3134.67 to $1924.02 through the coordinated decision policy, whereas most of suppliers' cost are increased. Therefore, we need to share fairly the benefit of cost savings to all the participants. Adopting compensation policy to allocate cost savings by equation (13) and (14), we can obtain the following results :

Cost of buyer

$$= Z_b \cdot JTC(T^*, m_1^*, m_2^*, \ldots, m_n^*) = \$2406.7$$

$$Z_b = TC_b(T_{b1}^*, T_{b2}^*, \ldots, T_{bn}^*) / \big(TC_b(T_{b1}^*, T_{b2}^*, \ldots, T_{bn}^*) + TC_s(T_{b1}^*, T_{b2}^*, \ldots, T_{bn}^*)\big) = 0.6592$$

Cost of supplier 1

$$= Z_{s1} \cdot JTC(T^*, m_1^*, m_2^*, \ldots, m_n^*) = \$172.11$$

$$Z_{s1} = TC_{s1}(T_{b1}^*) / \big(TC_b(T_{b1}^*, T_{b2}^*, \ldots, T_{bn}^*) + TC_s(T_{b1}^*, T_{b2}^*, \ldots, T_{bn}^*)\big) = 0.0471$$

All other Z_{si} $(i = 2, 3, \ldots, n)$ can be obtained by the same way. <Table 9> shows the result of compensation policy for 10 suppliers.

〈Table 9〉 Result of compensation policy for the coordinated decision policy

<table>
<tr><th rowspan="2">Item</th><th colspan="2">Individual policy($)</th><th colspan="2">Coordinated policy($)</th><th colspan="2">Compensation policy($)</th></tr>
<tr><th>Supplier</th><th>Buyer</th><th>Supplier</th><th>Buyer</th><th>Supplier</th><th>Buyer</th></tr>
<tr><td>1</td><td>224.17</td><td rowspan="10">3134.67</td><td>224.17</td><td rowspan="10">1924.02</td><td>172.11</td><td rowspan="10">2406.7</td></tr>
<tr><td>2</td><td>251.56</td><td>262.61</td><td>193.14</td></tr>
<tr><td>3</td><td>127.02</td><td>135.2</td><td>97.52</td></tr>
<tr><td>4</td><td>148.03</td><td>153.52</td><td>113.65</td></tr>
<tr><td>5</td><td>155.16</td><td>150.6</td><td>119.13</td></tr>
<tr><td>6</td><td>168.65</td><td>210.38</td><td>129.48</td></tr>
<tr><td>7</td><td>187.76</td><td>190.21</td><td>144.15</td></tr>
<tr><td>8</td><td>138.43</td><td>144.73</td><td>106.28</td></tr>
<tr><td>9</td><td>140.58</td><td>160.87</td><td>107.93</td></tr>
<tr><td>10</td><td>79.87</td><td>94.62</td><td>61.32</td></tr>
<tr><td>TC_s</td><td colspan="2">1620.65</td><td colspan="2">1726.92</td><td colspan="2">1244.24</td></tr>
<tr><td>Total</td><td colspan="2">4755.32</td><td colspan="2">3650.94</td><td colspan="2">3650.94</td></tr>
</table>

According to this compensation policy, the buyer bears the suppliers' increased costs and shares the benefit of coordinated decision policy. Since the buyer compensates the cost savings of $1210.65 with the suppliers, it is reasonable for suppliers to accept and maintain the coordinated decision policy.

5. Computational Experiments

In this section, we perform the computational experiments comparing the coordinated decision policy with the individual decision policy using RAND algorithm. Fifty different examples are solved and compared for various set of data. For the buyer's data, the demand for each item is generated from a uniform distribution between 100 and 20000 ; the minor ordering cost from a uniform distribution between 0.5 and 5; the holding cost from a uniform distribution between 0.2 and 3. And for the supplier's data, the production rate for each item is generated from a uniform distribution between 1.5 and 2 which is a multiple of each item's demand; the setup cost from a uniform distribution between 0.5 and 5 ; the holding cost from a uniform distribution between 0.1 and 2. It should be noted that the generated supplier's holding costs exceeding buyer's holding cost for each item are excluded. In addition, four values of n are considered (n= 5, 10, 15, 20) and five different values of A, the major ordering cost, are also considered (A= 0, 5, 10, 15, 20). Thus, 20 different combinations are considered.

〈Table 10〉 Data for computational experiments

parameter	*Buyer*	*Supplier*
d_i	$U(100, 20000)$	·
a_i	$U(0.5, 5)$	·
h_i	$U(0.2, 3)$	$U(0.1, 2)$
p_i	·	$d_i \cdot U(1.5, 2)$
s_i	·	$U(0.5, 5)$

A summary of results for these combinations is presented in <Table 11>. Also we randomly generate 50 problems for each combination and total of 1000 different examples are solved by RAND algorithm and by iterative algorithm. The results of these computational experiments are summarized in <Table 12>.

〈Table 11〉 Results for the individual and the coordinated (RAND) policy

n	A	①	②	TCS (%)
5	0	2128.36	2065.44	3.21
	5	2984.73	2441.99	18.19
	10	3353.26	2463.44	26.52
	15	4144.72	2835.78	32.01
	20	4387.94	2845.37	35.09
10	0	4299.57	4165.33	3.12
	5	5643.25	4402.89	22.32
	10	7200.73	4898.15	32.18
	15	8077.77	4991.20	37.67
	20	8979.82	5211.02	41.78
15	0	**6505.37**	**6332.46**	**2.84**
	5	8894.89	6812.80	23.39
	10	10515.13	6947.08	34.05
	15	11994.16	7087.18	40.73
	20	13624.84	7455.21	45.18
20	0	8713.00	8462.66	3.09
	5	11591.42	8797.69	24.21
	10	13732.68	8843.00	35.69
	15	16439.45	9513.56	42.02
	20	**17604.75**	**9329.01**	**47.07**
Maximum Saving		17604.75	9329.01	47.07%
Minimum Saving		6505.37	6332.46	2.84%

Note) ① : TC_{ind},
② : JTC_{RAND}, TCS : Total Cost Savings (%).

Our computational experiments complement the weakness of deterministic situations against more real-world complexities, i.e., stochastic demands and productions. Analyzing the result in <Table 11>, it is obviously seen that the coordinated decision policy dominates the individual decision policy. For all examples, the coordinated decision policy is superior to the individual decision policy. Even though the major ordering cost is zero, we can obtain some savings. Also, we can find that the higher the major ordering cost becomes and the more the joining suppliers are, the more the cost savings are.

〈Table 12〉 Comparison of efficiency between Iterative and RAND algorithm

n	A	①	②
5	0	22	28
	5	40	10
	10	39	11
	15	43	7
	20	**45**	**5**
10	0	21	29
	5	31	19
	10	38	12
	15	41	9
	20	43	7
15	0	**12**	**38**
	5	24	26
	10	31	19
	15	38	12
	20	40	10
20	0	17	33
	5	25	25
	10	35	15
	15	39	11
	20	34	16

Note) ① : the number of $JTC_{ITERATION}$ = JTC_{RAND},
② : the number of $JTC_{ITERATION} \neq JTC_{RAND}$.

The results by RAND algorithm listed in <Table 11>, <Table 12> are based on dividing the ranges between T_{min} and T_{max} into 20 equally spaced values. We also experiment the case of different values of l (l=10, 20, 30). However, in our computational experiments, any differences in solutions between RAND20 (l=20) and RAND30 (l=30) are not occurred. Thus, from our perspective it is sufficient to consider only 20 values of T between T_{min} and T_{max}. From the results in <Table 12>, we can see that the coordinated total cost by iterative algorithm has a bigger probability to be equal to the total cost by RAND algorithm as the major ordering cost grows.

6. Concluding Remarks

In this study, we introduce the supply chain circumstance with multiple suppliers and single buyer. We first develop the coordinated inventory models and focus on minimizing the coordinated joint total cost in the context of information sharing. Through the numerical examples, we find that our coordinated model achieve cost savings due to the major ordering cost. Especially, since the basic replenishment cycle is a critical factor in the coordinated inventory model, we adopt the joint replenishment strategy. Also, we suggest the compensation policy to induce suppliers to remain in the coordinated supply chain. Finally, we conduct the computational experiments to complement the weakness of deterministic situations against more real-world complexities. And we compare RAND algorithm with the iterative algorithm.

The numerical examples present some implications to manage the supply chain. First, the coordinated inventory decision policy is always superior to the individual decision policy, especially when the major ordering cost is high. And in our integrated model, the RAND algorithm and iterative algorithm have a great chance to get the same joint total costs when the major ordering cost is high.

We can get some insights into the strategies to manage supply chain and the application of the suggested coordinated decision policy makes it possible to increase the efficiency of supply chain.

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