Performance Improvement of Wald Test for Resolving GPS Integer Ambiguity Using a Baseline-Length Constraint

Eunsung Lee, Sebum Chun, Young Jae Lee*, Teasam Kang, Gyu-In Jee, and Mamoun F. Abdel-Hafez

Abstract: In this paper, the baseline-length information is directly modeled as a measurement for the Wald test, which speeds up the resolution convergence of the integer ambiguity of GPS carrier phase measurements. The convergent speed improvement is demonstrated using numerical simulation and real experiments. It is also shown that the integer ambiguities can be resolved using only four actual satellite measurements with very reasonable convergence speed, if the baseline-length information is used just like one additional observable satellite measurement. Finally, it is shown that the improvement of convergence speed of the Wald test is due to the increase of the probability ratio with the use of the baseline-length constraint.

Keywords: Baseline-length constraint, integer ambiguity, multiple hypothesis Wald sequential probability test, real-time kinematics.

1. INTRODUCTION

We can estimate the attitude of the vehicle as well as the precise 3-dimenional position of a moving vehicle by multiple GPS antennas attached on the surface of the vehicle using GPS carrier phase measurements. To obtain the final position and attitude, the integer ambiguities should be resolved first. It is well known that the Real-Time Kinematics (RTK) has the major bottle neck in the processing of the integer ambiguity resolution.

During the last decade, a number of integer ambiguity resolution methods have been suggested. Firstly, various resolution techniques have been proposed to provide static or post-processed solutions for surveying applications because of the huge computational time for searching integer ambiguities. After this stage, RTK ambiguity resolution methods for determining precise positioning have been developed to deal with dynamic applications. These are the LAMBDA (Least squares AMBiguity Decorrelation Adjustment), the FARA (Fast Ambiguity Resolution Approach), the Euler and Landau method, the NIP (Nonlinear Integer Programming), LSAST (Least Squares Ambiguity Search Technique), AFM (Ambiguity Function Method), and ARCE (Ambiguity Resolution with Constraint Equation) were developed.

In most cases, the GPS integer ambiguity fixing problem is divided into two distinct parts, *i.e.*, the ambiguity *estimation* problem and the ambiguity *validation* problem [1]. For unknown variables, the estimation part searches for optimal estimations using observation equation models. If the optimal estimation methods, for example, are based on the principle of least-squares, the task is to find the least-squares solution for the unknown integer ambiguities. The second part of GPS ambiguity fixing problem is the validation of the estimated ambiguities. This part is usually independent of the estimation part, and very important in its own right. It determines whether the estimated integer ambiguity solution is to be accepted or not.

We can always compute integer ambiguity solutions, whether they are true or not. Therefore, the basic question addressed by the validation process is whether we accept the estimated integer ambiguity as the solution or not.

On the other hand, the multiple hypotheses Wald sequential probability test (MHWSPT) is a method for validating the integer ambiguity during the initial ambiguity resolution [2]. For convenience, hereafter Wald test means the MHWSPT. This is essentially a nonlinear sequential filter which, after conditioning

---

Manuscript received October 10, 2005; revised February 22, 2006; accepted February 27, 2006. Recommended by Editorial Board member Jae Weon Choi under the direction of Editor Keum-Shik Hong. This work was supported by the Research Program of Konkuk University, Seoul, Korea.

Eunsung Lee, Sebum Chun, Young Jae Lee, and Teasam Kang are with Department of Aerospace Engineering, Konkuk University, Hwayang-dong, Gwangjin-gu, Seoul 143-701, Korea (e-mails: koreagnss@koreagnss.com, tucano@tucanokorea.com, {younglee, tskang}@konkuk.ac.kr)

Gyu-In Jee is with the Department of Electronic Engineering, Konkuk University, Hwayang-dong, Gwangjin-gu, Seoul 143-701, Korea (e-mail: gijee@konkuk.ac.kr)

Mamoun F. Abdel-Hafez is with American University of Sharjah, Sharjah, United Arab Emirates (e-mail: mabdelhafez@aus.edu).

* Corresponding author.

the measurements, is optimal. References [2] and [3] have shown that the Wald test is a well-proven and efficient integer ambiguity resolution algorithm. The biggest advantage of the Wald test is that this method has one simple unified step for estimation and validation in fixing the integer ambiguities of GPS carrier phase measurements.

The attitude estimation is basically to determine the relative position, more specifically the baseline vectors between multiple GPS antennas. If the baseline-length is not long enough (for example, less than several hundred meters), the code measurements are not good enough because of the poor angular resolution resulting from the poor resolution of the relative position. In most cases, to determine the attitude of a moving vehicle, the baseline vector estimation is used based on the GPS carrier phase measurements. Therefore, the real-time attitude determination using the GPS carrier phase measurements has the same fundamental problem that is the integer ambiguity resolution in real time. On the other hand, since, in most cases, the GPS antennas are fixed on the body surface of the moving vehicle, the relative distances between antennas remain almost constant in spite of vehicle motion, or stay within a reasonable region of error. Furthermore, the relative distances of the antennas measured beforehand. In this case, the pre-measured distances between antennas are considered to be independent and are very valuable information for the attitude determination of the vehicle. This baseline-length information has been mostly used for setting up or reducing the searching space of the integer ambiguities, or validating the estimated integers, some details are summarized in the following section.

In this paper, the baseline-length information is directly used in producing residuals of measurements for the Wald test, which reduces very much the convergence time for $2x^T\Delta x = -\Delta x^T\Delta x$ resolving integer ambiguity.

In Section 2, a brief review of using the baseline-length information is discussed. In Section 3, mathematical model of GPS is summarized and a measurement model with baseline-length constraint is explained. In Section 4, the Wald test is explained. In Section 5, the probability increment ratio is introduced and the proof of the improvement of the convergence performance is shown. In Section 6, the results of experimental tests are explained. The final conclusions are summarized in Section 7.

2. APPLICATION OF THE BASELINE-LENGTH CONSTRAINT USING MULTIPLE ANTENNAS

The methods using the baseline-length constraint between GPS antennas can be divided into three types. The first is using the baseline-length constraint to reduce search volume of GPS carrier phase integer ambiguities [5]. The second method uses it for simplification of relative equations with baseline geometry [6]. The last one is using the baseline-length constraint to reduce the number of possible ambiguity solution candidates with the relationship of each ambiguity [7]. The first and second methods are usually used for attitude determination and the third one is used for positioning. If the known baseline lengths are used, the minimum number of satellites can be three when conventional GPS receivers are used for attitude determination [4]. The first attempt to use the baseline length is the development of a GPS multi-antenna system for the attitude determination, the three dimensional search space of GPS carrier ambiguities is reduced to the two dimensional search space [5]. There can be many carrier phase measurements, but the numbers of independent measurement basis exist. Suppose that four primary satellites have been chosen and the length of the baseline vector from the master antenna to the remote antenna is known, there are three related double difference carrier phase observation equations. After the first search volume is determined using the length of the baseline vector, the second search volume is determined smaller than the other cases. After the second volume is determined, the last search volume dose not need be solved when the length of the baseline vector is known. The second method is applied to the attitude determination problem when the vehicle has flexile surface. When the six channel receiver was practically developed and attitude was determined, the baseline-length constraint was used [6]. Suppose that the baseline vector, x, rotates along circle in space. Here the baseline vector is moved by the vector Δx. By simultaneously considering each Δx vector, the center of the sphere can be located along with the initial position of the baseline. After the geometry constraint of baseline vector is applied, $2x^T\Delta x = -\Delta x^T\Delta x$ remains. The third method is used when the carrier measurement equations are solved with known baseline lengths and relations between the ambiguities in the measurement equations can be applied. This application can reduce the ambiguity candidates [7]. If there is a network which is connected by 3 reference stations, there are three ambiguity sets *i.e.* N_1, N_2, N_3. As ambiguity sets can make the equation, $N_1 + N_2 + N_3 = 0$, we can calculate the other ambiguity set from ones. According to the knowledge of reference station position, the baseline lengths can be calculated. Multiple fixed receivers can be used to estimate and reduce noise for carrier phase measurements at each receiver, using a network adjustment methodology [8]

The new method, proposed in this paper, uses the baseline length as a new additive measurement when the Wald test is applied for eliminating integer ambiguities. So the measurement matrix H is changed, and the new measurement matrix includes direction cosines with the baseline vector. As the result, this new measurement reduces the estimation errors and provides more precise position.

3. MATHMETICAL MODELING

3.1. Measurement equations

In general, high-grade GPS receivers provide several kinds of measurement information which contains many types of errors (ionospheric, tropospheric delay *etc*). In RTK application, GPS carrier measurements are used throughout the whole process, but GPS code measurements are mostly used for guessing an initial position. To mitigate the errors of carrier phase measurements, double differencing is used commonly.

$$y = H \cdot dx + \lambda N + v_{\phi}, \quad (1)$$

where $dx = [\delta x \, \delta y \, \delta z]^T$

dx: relative position from nominal position on cartesian coordinates
y: double differenced GPS carrier measurements
H: measurement matrix
λ: carrier phase wave length
N: double differenced integer ambiguity
v_{ϕ}: double differenced GPS carrier measurements noise.

To apply the hypothesis test in resolving integer ambiguity, we plan to hypothesize the measurement residuals of the ambiguity candidates. The residual and its covariance of i-th hypothesis is defined respectively as

$$r_i = y - \hat{y}_i = y - (H \cdot d\hat{x} + \lambda \hat{N}_i), \quad (2)$$

where
y: double differenced GPS carrier measurements
$\hat{y}$: estimated double differenced GPS carrier measurements
H: measurement matrix
$d\hat{x}$: estimated relative position
y: double differenced GPS carrier measurements
λ: carrier phase wave length
$\hat{N}$: estimated double differenced integer ambiguity,

$$Q_r = Q_y - Q_{\hat{y}} = P_H^{\perp} \cdot Q_y, \quad (3)$$

where
Q_r: covariance of residual
Q_y: covariance of measurements
$Q_{\hat{y}}$: covariance of estimated measurements

$$P_H^{\perp} = I - H \cdot (H^T \cdot Q_y^{-1} \cdot H)^{-1} \cdot H^T \cdot Q_y^{-1}.$$

3.2. Measurement model with baseline-length constant

In this sub-section, the use of the baseline-length constraint to the double differenced carrier phase measurements is briefly described. Let us assume that two GPS antennas are fixed on the surface of a vehicle to determine the attitude as Fig. 1, and that the relative distance between the two antennas is measurable very accurately in advance and remains constant if structure of the vehicle is rigid. The initial position of antenna B is estimated using code measurements at the beginning of the process.

It is going to be claimed that the measured baseline length can be used as an additional information to the double differenced carrier phase measurements. (4) represents the nonlinear baseline measurement equation of the two antenna configuration represented in Fig. 1.

$$\ell_{BL} = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2 + (z_A - z_B)^2}, \quad (4)$$

where

$[x_A, y_A, z_A]$: position of antenna A
$[x_B, y_B, z_B]$: position of antenna B
ℓ_{BL}: baseline length.

The Taylor series expansion of (4) with respect to the initial position of antenna B, *i.e.* B_0, is given as

$$\ell_{BL} - \ell_0 = \quad (5)$$
$$\begin{bmatrix} -\frac{x_A - x_{B0}}{\ell_0} & -\frac{y_A - y_{B0}}{\ell_0} & -\frac{z_A - z_{B0}}{\ell_0} \end{bmatrix} \cdot dx + H.O.T,$$

where

dx: position difference between B and Bo
x_{B0}, y_{B0}, z_{B0}: nominal position of antenna Bo
ℓ_0: nominal baseline length between antenna A and Bo
H.O.T : high order terms.

(6) is the linearized form of (5) with respect to the guessed initial position using code measurements, which is to be used as the baseline-length constraint

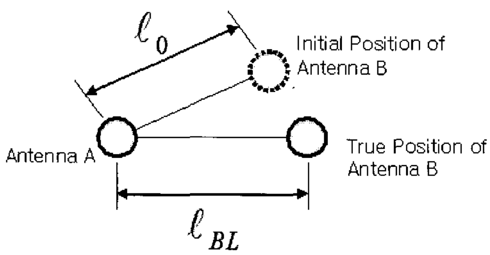


Fig. 1. Basic configuration of two antennas.

equation.

$$y_{BL} = H_{BL} \cdot dx + v_{BL}, \tag{6}$$

where

$$H_{BL} = \begin{bmatrix} -\frac{x_A - x_{B0}}{\ell_0} & -\frac{y_A - y_{B0}}{\ell_0} & -\frac{z_A - z_{B0}}{\ell_0} \end{bmatrix}$$

y_{BL} : baseline length error measurement, $y_{BL} = \ell_{BL} - \ell_0$

v_{BL} : measurement error of base line length.

Since the state vectors, dx, of (1) (*i.e.* the double differenced carrier phase measurements) and (6) (*i.e.* the baseline-length constraint equation) are the same, these two equations can be combined as in (7), which is called as *the all measurement equation.*

$$\begin{bmatrix} y_G \\ y_{BL} \end{bmatrix} = \begin{bmatrix} H_G \\ H_{BL} \end{bmatrix} \cdot dx + \lambda \begin{bmatrix} N \\ 0 \end{bmatrix} + \begin{bmatrix} v_G \\ v_{BL} \end{bmatrix}, \tag{7}$$

$$y_A = H_A \cdot dx + \lambda N_A + v_A,$$

where

$$y_A = \begin{bmatrix} y_G \\ y_{BL} \end{bmatrix}, H_A = \begin{bmatrix} H_G \\ H_{BL} \end{bmatrix}, N_A = \begin{bmatrix} N \\ 0 \end{bmatrix}, v_A = \begin{bmatrix} v_G \\ v_{BL} \end{bmatrix}.$$

As the result, the baseline-length constraint defined in (6) is considered as an additional measurement to improve the characteristics of the convergence of probability functions of the Wald test. Now let's consider two different cases of measurement information. The first one is using the GPS carrier phase measurements only, and the other one is using GPS carrier phase measurements plus the baseline-length constraint.

The least square solution and its covariance of each case can be summarized as (8) and (9).

Case 1: the GPS carrier phase measurements only

$$d\hat{x}_G = \left(H_G^T Q_{y_G}^{-1} H_G\right)^{-1} H_G^T Q_{y_G}^{-1} y_G,$$
$$Q_{dx_G} = \left(H_G^T Q_{y_G}^{-1} H_G\right)^{-1}. \tag{8}$$

Q_{y_G} is covariance when only GPS carrier phase measurements are used.

Case 2: Case of GPS carrier phase measurements with the baseline-length constraint.

$$d\hat{x}_A = \left(Q_{dx_G}^{-1} + H_{BL}^T Q_{y_{BL}}^{-1} H_{BL}\right)^{-1} \left(Q_{dx_G}^{-1} dx_G + H_{BL}^T Q_{y_{BL}}^{-1} H_{BL}\right),$$
$$Q_{dx_A} = \left(H_G^T Q_{y_G}^{-1} H_G + H_{BL}^T Q_{y_{BL}}^{-1} H_{BL}\right)^{-1}. \tag{9}$$

Q_{yG} is covariance when GPS carrier phase measurements and baseline-length constraint are used.

As can be seen in (9), addition of the baseline-length constraint to measurement information helps to reduce the final covariance of the estimator.

4. WALD TEST AND INTEGER AMBIGUITY RESOLUTION

The Wald test is a special case of the multiple hypothesis Shiryayev sequential probability ratio test, which determines the most likely event from a set of hypotheses, assuming that the event is true for all time [3].

In 1947, Wald was developed as a general sequential probability test [9], and it has been generalized and applied to fault detection and isolation by Malladi et al. in 1999 [10]. In 2001, the first application of the method to GPS carrier phase integer ambiguity resolution has been conducted by Wolfe et al. [2]. In 2002, to increase the efficiency of computation (*i.e.* to reduce the hypothesis sets), it has been combined with LAMBDA method by Abdel-Hafez et al. [3].

The Wald test can be used to find the conditional probability that each integer ambiguity under consideration is true. Hence it can be said that the Wald test is a statistical method used for validating the integer ambiguity. Eventually residual is used to obtain the probability of a certain integer hypothesis as a candidate of the correct integer ambiguity. For more details, refer References [2] and [3].

The Wald test recursively calculates the probability of each integer hypothesis under consideration that the hypothesis is the correct integer ambiguity set using the measurements up to the current time k. That is $F_i(k)$, i=0, ... m-1. This is explicitly expressed as (10).

$$F_i(k) = \frac{F_i(k-1) \cdot f_i(k)}{\sum_{j=1}^{m} F_j(k-1) \cdot f_j(k)}, \tag{10}$$

where

$F_i(k)$ = probability of the hypothesis being true
$f_i(k)$ = probability density function of residual of the i-th hypothesis.

Theoretically, the probability density function of residual can be any shape. However, if the residual is assumed as a Gaussian distribution, then the density function is modeled as follows.

$$f_i(k) = C \cdot \exp\left\{-\frac{1}{2}\left\{r_i(k)^T Q_r^{-1} r_i(k)\right\}\right\}, \tag{11}$$

where

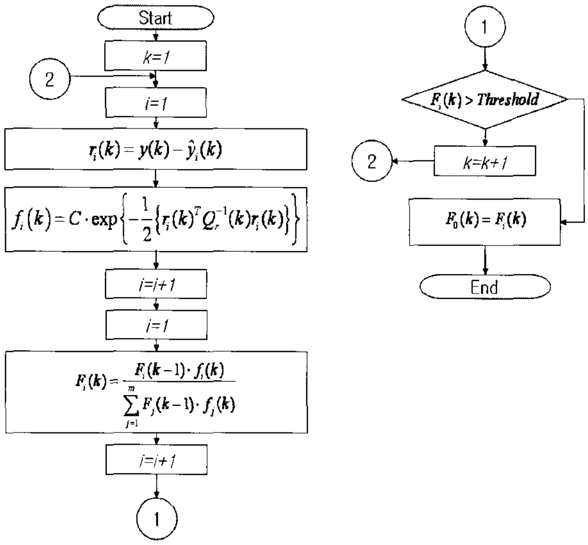


Fig. 2. Flow chart of the wald test.

$F_i(k)$ = probability of the hypothesis being true
$f_i(k)$ = probability density function of residual of the i-th hypothesis.

C is a constant for Gaussian density function and $r_i(k)$ is the residual with its covariance Q_r. It is necessary to calculate (11) for each hypothesis being the true integer ambiguity. Using (11), all the probability functions of all hypotheses are then updated at each epoch whenever new measurements are received. The algorithm terminates as soon as the probability of one hypothesis reaches one (or practically a desired threshold, *say 0.999*), while the others approach zero (or practically very close to zero, *say 0.001*). These phenomena occur at the same time, since the sum of all probabilities of hypotheses is always equal to one due to the definition of the probability function. Finally we select the integer ambiguity set of the hypothesis as the true one. Fig. 2 shows the flow chart of the Wald test.

5. CHARACTERISTICS OF PROBABILITY INCREMENT RATIO

5.1. Definition of probability increment ratio

The new variable is defined to prove the improvement of convergence speeds, which means that the true hypothesis makes the probability converge faster than the others.

Let us define the new variable L_i be a *probability increment ratio* of each hypothesis.

$$L_i = \frac{F_i(k)}{F_i(k-1)}. \tag{12}$$

(12) is defined for all i-th hypothesis, where $i = 0$, … , $m-1$, and m is the number of total hypothesis. Let set the 0-th hypothesis be true.

In the case of the true hypothesis, the probability ratio of (12) is as follows

$$L_0 = \frac{F_0(k)}{F_0(k-1)} = \frac{f_0(k)}{\sum_{j=0}^{m-1} F_j(k) f_j(k)} = \frac{C\cdot\exp\left\{-\frac{1}{2}\{r_0(k)^T Q_r^{-1} r_0(k)\}\right\}}{A}, \tag{13}$$

where $A = \sum_{j=0}^{m-1} F_j(k) f_j(k)$.

For every hypothesis except for the true one, the probability increment ratio is defined as follows

$$L_i = \frac{F_i(k)}{F_i(k-1)} = \frac{f_i(k)}{\sum_{j=0}^{m-1} F_j(k) f_j(k)} = \frac{C\cdot\exp\left\{-\frac{1}{2}\{r_i(k)^T Q_r^{-1} r_i(k)\}\right\}}{A}, \tag{14}$$

where $i = 1, \cdots, m-1$.

It is important to point out that A in the denominator of (14) is the same as that of (13). To prove that the true hypothesis converges faster than the other ones, the values of the probability ratio of every hypothesis need to be compared with that of the true hypothesis.

$$\frac{L_0}{L_i} = \frac{\dfrac{C\cdot\exp\left\{-\frac{1}{2}\{r_0(k)^T Q_r^{-1} r_0(k)\}\right\}}{A}}{\dfrac{C\cdot\exp\left\{-\frac{1}{2}\{r_i(k)^T Q_r^{-1} r_i(k)\}\right\}}{A}} = \exp\left\{-\frac{1}{2}\left[\{r_0(k)^T Q_r^{-1} r_0(k)\} - \{r_i(k)^T Q_r^{-1} r_i(k)\}\right]\right\}, \tag{15}$$

where again $i = 1, \cdots, m-1$.

$\{r_0(k)^T Q_r^{-1} r_0(k)\} - \{r_i(k)^T Q_r^{-1} r_i(k)\}$ in (15) determines whether the probability ration is greater than 1 or not. To determine the value, the statistical characters of r_0 and r_i must be considered beforehand. When $r_0(k)$ has zero mean, the covariance Q_r, and the degree of freedom n, $r_0(k)^T Q_r^{-1} r_0(k)$ has the centralized chi-square distribution. In this case, the mean of $r_0(k)^T Q_r^{-1} r_0(k)$ is n and its variance is $2n$. If r_i has the mean μ_i, the covariance $r_i(k)^T Q_r^{-1} r_i(k)$ has a non-centralized chi-square distribution, which has non-centralized parameter δ_i^2 ,where δ_i^2 is $\mu_i{}^T Q_r^{-1} \mu_i$ and μ_i is the bias of the i-th hypothesis from the true hypothesis. Then the mean is $n+\delta_i^2$ and the variance is $2n+4\delta_i^2$ [11].

$E\left[\left\{r_0(k)^T Q_r^{-1} r_0(k)\right\}-\left\{r_i(k)^T Q_r^{-1} r_i(k)\right\}\right]$ is $-\delta_i^2$. For every i-th hypothesis, $-\delta_i^2<0$ is always satisfied. Since $\left[\left\{r_0(k)^T Q_r^{-1} r_0(k)\right\}-\left\{r_i(k)^T Q_r^{-1} r_i(k)\right\}\right]$ is less than zero, $\exp\left\{-\frac{1}{2}\left[\left\{r_0(k)^T Q_r^{-1} r_0(k)\right\}-\left\{r_i(k)^T Q_r^{-1} r_i(k)\right\}\right]\right\}$ is greater than 1, which means that $\frac{L_0}{L_i}>1$ for every i. Thus, the probability increment ratio of the true hypothesis converges faster than the others in average.

5.2. Characteristics of probability increment ratio using baseline-length constraint

In this sub-section, it is shown that the addition of the baseline-length constraint to measurement information contributes to speed up resolution convergence.

We want to prove that when the hypothesis is true, the convergence speed of the probability function of *the all information case* that uses carrier measurements and baseline-length information is faster than that of *the GPS carrier phase measurements only case.*

Let us define the probability ratio of all information case for the correct hypothesis as follows.

$$L_{A,0}=\frac{F_{A,0}(k)}{F_{A,0}(k-1)}=\frac{f_{A,0}(k)}{\sum_{j=0}^{m-1} F_{A,j}(k) f_{A,j}(k)}=\frac{C_A\cdot\exp\left\{-\frac{1}{2}\left\{r_{A,0}(k)^T Q_{A,r}^{-1} r_{A,0}(k)\right\}\right\}}{B},$$

where $B=\sum_{j=0}^{m-1} F_{A,j}(k) f_{A,j}(k)$.

In the case of every hypothesis except for the true hypothesis, the probability ratio is defined as follows, where $i=1,\cdots,m-1$.

$$L_{A,i}=\frac{F_{A,i}(k)}{F_{A,i}(k-1)}=\frac{f_{A,i}(k)}{\sum_{j=0}^{m-1} F_{A,j}(k) f_{A,j}(k)}=\frac{C_A\cdot\exp\left\{-\frac{1}{2}\left\{r_{A,i}(k)^T Q_{A,r}^{-1} r_{A,i}(k)\right\}\right\}}{B}$$

When all information measurements are used, the ratio of the probability ratio can be defined as

$$\frac{L_{A,0}}{L_{A,i}}=\exp\left\{-\frac{1}{2}\left[\left\{r_{A,0}(k)^T Q_{A,r}^{-1} r_{A,0}(k)\right\}-\left\{r_{A,i}(k)^T Q_{A,r}^{-1} r_{A,i}(k)\right\}\right]\right\}$$

In the same way, when GPS measurements are only used, the ratio of the probability ratio is defined as

$$\frac{L_{G,0}}{L_{G,i}}=\exp\left\{-\frac{1}{2}\left[\left\{r_{G,0}(k)^T Q_{G,r}^{-1} r_{G,0}(k)\right\}-\left\{r_{G,i}(k)^T Q_{G,r}^{-1} r_{G,i}(k)\right\}\right]\right\}$$

If $\frac{L_{A,0}}{L_{A,i}}>\frac{L_{G,0}}{L_{G,i}}$ is correct, $\frac{L_{A,0}/L_{A,i}}{L_{G,0}/L_{G,i}}>1$ will be satisfied. Therefore, this can be expressed using residuals and their covariances as

$$\frac{\frac{L_{A,0}}{L_{A,i}}}{\frac{L_{G,0}}{L_{G,i}}}=\frac{\exp\left\{-\frac{1}{2}\left[\left\{r_{A,0}(k)^T Q_{A,r}^{-1} r_{A,0}(k)\right\}-\left\{r_{A,i}(k)^T Q_{A,r}^{-1} r_{A,i}(k)\right\}\right]\right\}}{\exp\left\{-\frac{1}{2}\left[\left\{r_{G,0}(k)^T Q_{G,r}^{-1} r_{G,0}(k)\right\}-\left\{r_{G,i}(k)^T Q_{G,r}^{-1} r_{G,i}(k)\right\}\right]\right\}}$$
$$=\exp\left\{-\frac{1}{2}\left(\begin{array}{l}\left[\left\{r_{A,0}(k)^T Q_{A,r}^{-1} r_{A,0}(k)\right\}-\left\{r_{A,i}(k)^T Q_{A,r}^{-1} r_{A,i}(k)\right\}\right]\\-\left[\left\{r_{G,0}(k)^T Q_{G,r}^{-1} r_{G,0}(k)\right\}-\left\{r_{G,i}(k)^T Q_{G,r}^{-1} r_{G,i}(k)\right\}\right]\end{array}\right)\right\}, \quad (16)$$

where

$$E\left[\left\{r_{A,0}(k)^T Q_{A,r}^{-1} r_{A,0}(k)\right\}-\left\{r_{A,i}(k)^T Q_{A,r}^{-1} r_{A,i}(k)\right\}\right]=n-\left(n+\delta_{A,i}^2\right)=-\delta_{A,i}^2,$$
$$E\left[\left\{r_{G,0}(k)^T Q_{G,r}^{-1} r_{G,0}(k)\right\}-\left\{r_{G,i}(k)^T Q_{G,r}^{-1} r_{G,i}(k)\right\}\right]=n-\left(n+\delta_{G,i}^2\right)=-\delta_{G,i}^2,$$
$$\delta_{A,i}^2=\mu_{A,i}{}^T Q_{A,r}^{-1} \mu_{A,i},$$
$$\delta_{G,i}^2=\mu_{G,i}{}^T Q_{G,r}^{-1} \mu_{G,i}.$$

Here $\mu_{A,i}$ is a bias of i-th hypothesis from the true hypothesis $H_{A,0}$ using all information, and $\mu_{G,i}$ is a bias of i-th hypothesis from the true hypothesis $H_{G,0}$ using the GPS measurements only.

By the way, since GPS carrier measurements and baseline length information do not have any correlation, all the non-centered parameters, $\delta_{A,i}^2$, can be expressed as

$$\delta_{A,i}^2=\mu_{A,i}{}^T Q_{A,r}^{-1} \mu_{A,i}=\begin{bmatrix}\mu_{G,i}\\ \mu_{BL,i}\end{bmatrix}^T\begin{bmatrix}Q_{G,r} & 0\\ 0 & Q_{BL,r}\end{bmatrix}^{-1}\begin{bmatrix}\mu_{G,i}\\ \mu_{BL,i}\end{bmatrix}=\mu_{G,i}{}^T Q_{G,r}^{-1}\mu_{G,i}+\mu_{BL,i}{}^T Q_{BL,r}^{-1}\mu_{BL,i}. \quad (17)$$

The difference of two non central parameters is $\mu_{BL,\ i}^{T} Q_{BL,\ r}^{-1} \mu_{BL,i}$, and it is greater than zero. It means that $\delta_{A,i}^2 - \delta_{G,i}^2 = \mu_{BL,\ i}^{T} Q_{BL,\ r}^{-1} \mu_{BL,i} > 0$, and also $-\left(\delta_{A,i}^2 - \delta_{G,i}^2\right) = -\mu_{BL,\ i}^{T} Q_{BL,\ r}^{-1} \mu_{BL,i} < 0$ is true. Therefore, in average

$$\exp\left\{-\frac{1}{2}\left\langle \begin{array}{l} \left[\left\{r_{A,0}(k)^T Q_{A,r}^{-1} r_{A,0}(k)\right\} - \left\{r_{A,i}(k)^T Q_{A,r}^{-1} r_{A,i}(k)\right\}\right] \\ -\left[\left\{r_{G,0}(k)^T Q_{G,r}^{-1} r_{G,0}(k)\right\} - \left\{r_{G,i}(k)^T Q_{G,r}^{-1} r_{G,i}(k)\right\}\right] \end{array}\right\rangle\right\} > 1.$$

This means

$$\frac{\dfrac{L_{A,0}}{L_{A,i}}}{\dfrac{L_{G,0}}{L_{G,i}}} > 1. \tag{18}$$

Finally, we obtain that

$$\frac{L_{A,0}}{L_{A,i}} > \frac{L_{G,0}}{L_{G,i}}. \tag{19}$$

As a conclusion, if the hypothesis is true, the probability function of the GPS carrier phase measurements with the baseline-length constraint information converges faster in average than GPS carrier phase measurements only case.

5. EXPERIMENTAL TEST OF INTEGER AMBIGUITY RESOLVING PERFORMANCE

5.1. Result of numerical simulation test

A numerically simulated data have been used to test the performance of the Wald test with and without the dynamic information constraint. In this experiment, a total of nine satellites are observable and the relative distance between two GPS antennas, *i.e.* the baseline length, is set to 9m. Fig. 3 shows an example of the configuration of antennas. Fig. 4 shows the circumstance of the experiment. Figs. 5 and 6 show the number of visible satellites and PDOP.

To obtain precise position using carrier phase measurements, the double difference method between measurements is used in most case. In this case, the number of observable satellites must be more than four or five. In this case the visible satellite number is 8. The PDOP is about 1.72.

Fig. 7 shows the difference of convergence speeds with and without the baseline-length constraint. In this result, six visible satellites are used.

As we can see at Fig. 7, the baseline-length constraint information improves the convergent speed

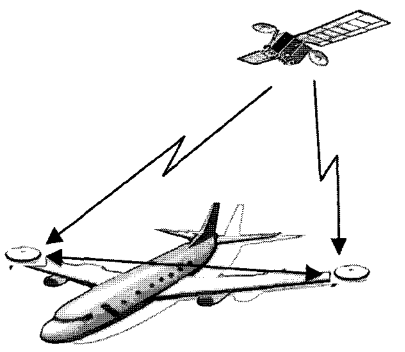


Fig. 3. Configuration of antennas.

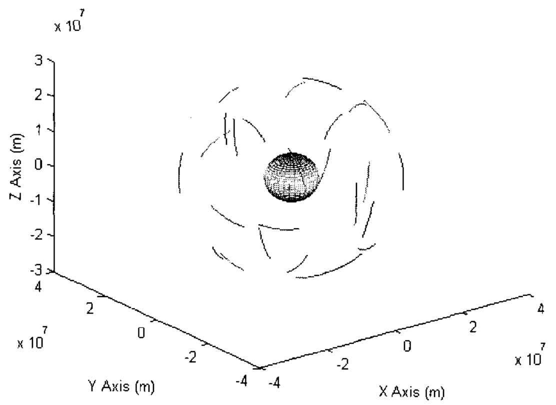


Fig. 4. The circumstance of the experiment.

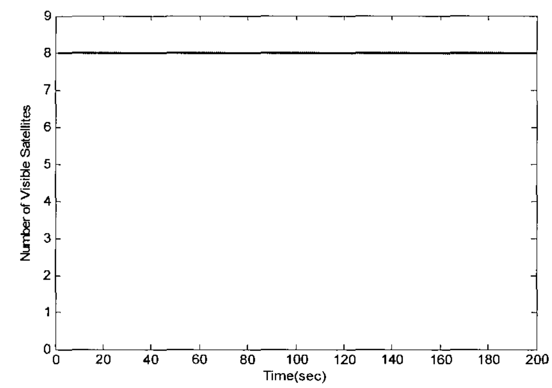


Fig. 5. The number of visible satellites.

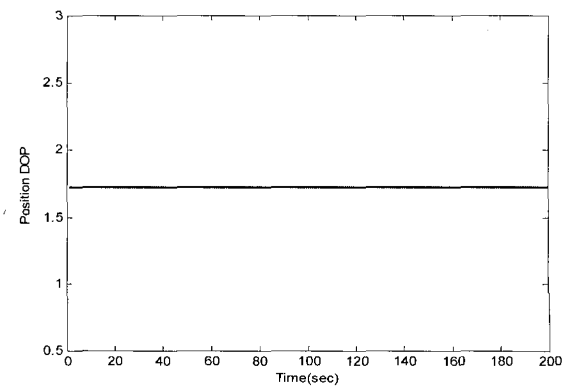
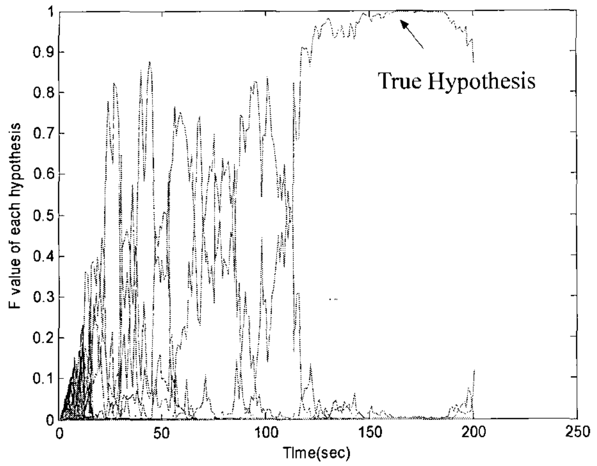
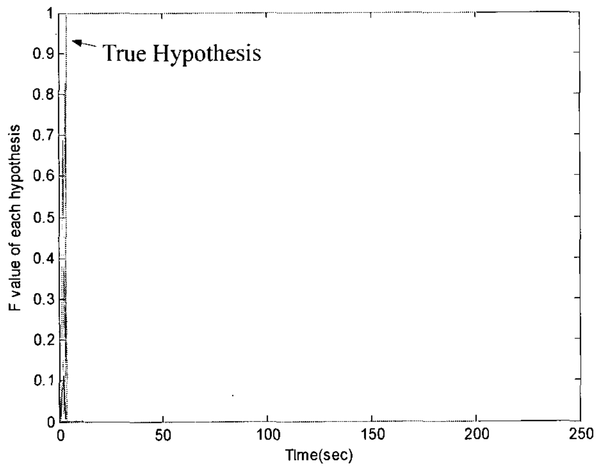


Fig. 6. The position DOP.

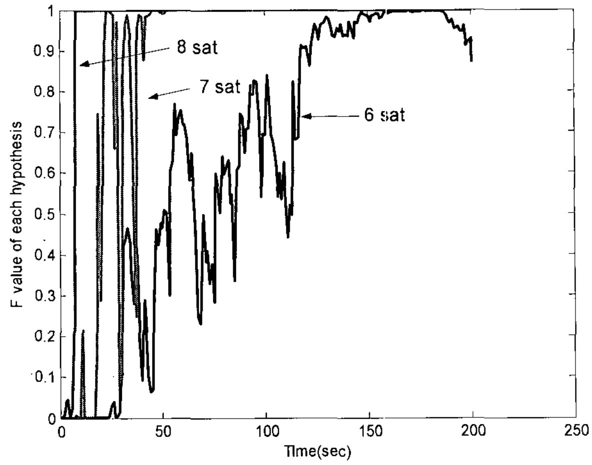


(a) Without baseline-length constraint.

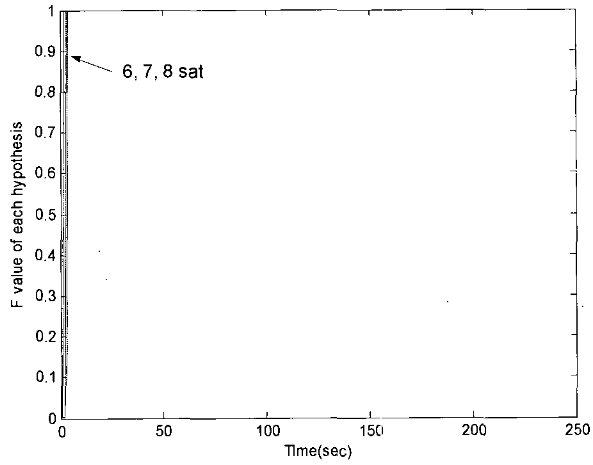


(b) With baseline-length constraint.

Fig. 7. Comparison of convergence speeds with and without baseline-length constraint (Numerically simulated data).



(a) Without baseline-length constraint.



(b) With baseline-length constraint.

Fig. 8. Comparison of convergence speeds with and without baseline-length constraint with varying the number of the observable satellites (Numerically simulated data).

considerably, and it stays within the probability very constantly once it enters into the threshold region. Fig. 8 shows the difference of convergence speeds with and without baseline-length constraint with varying the number of the observable GPS satellites.

The baseline-length constraint information improves the convergent speeds much so that it converges within 10 seconds with six to eight observable GPS satellites, while all the cases are very sluggish for the not-using-it cases.

To obtain precise position using carrier phase measurements, the double difference method between measurements is used in most case. In this case, the number of observable satellites must be more than four or five, depending on the method used. In general, it can be said that it is extremely difficult to fix the integer ambiguity with four observable satellites only. However, as we proposed, the baseline-length constrint information can be considered as an additional measurement of one observable satellite.

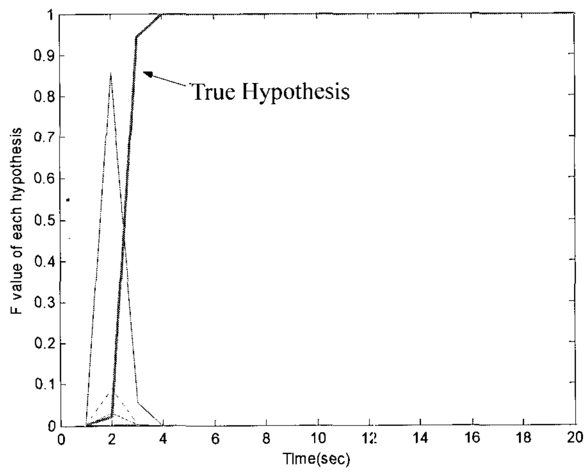


Fig. 9. Performance of integer ambiguity resolution with four observable satellites using the baseline-length constraint (Numerically simulated data).

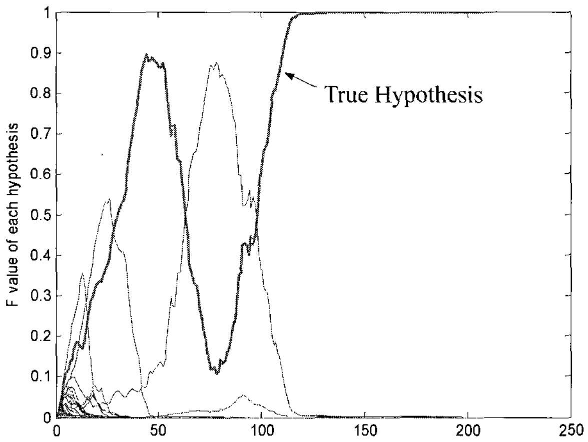
Therefore, we can resolve the integer ambiguities using four actual satellite measurements only.

If we use the baseline-length constraint, we can resolve the integer ambiguity with four observable satellites only as can be seen at Fig. 9. Furthermore, the convergence speed is compared to the case of seven, eight, or more number of observable satellites.

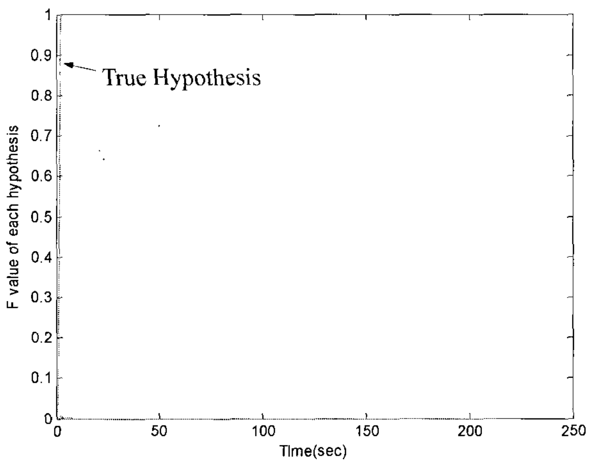
5.2. Result of real experiments

The real GPS carrier phase data have been taken using two dual frequency receivers at the rooftop of the engineering building of Konkuk University, Seoul, Korea, on March 26, 2003. The data have been sampled every second. All the cases using the real data are almost the same as the numerical simulation cases. In other words, when we use the baseline-length constraint, the convergence speed is much better than the case without it.

Fig. 10 shows the difference of convergence speeds with and without baseline-length constraint with varying the number of the observable GPS satellites.

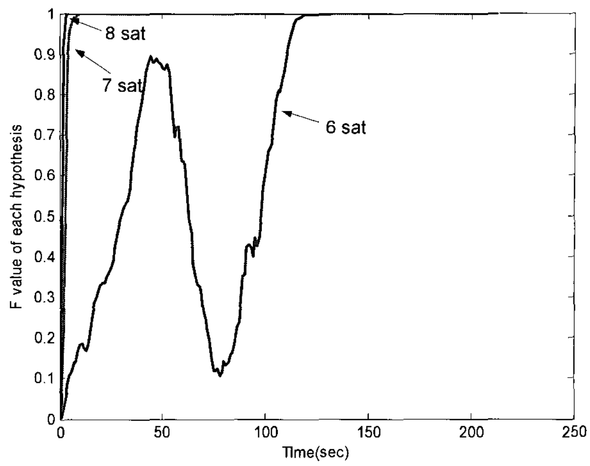


(a) Without baseline-length constraint.

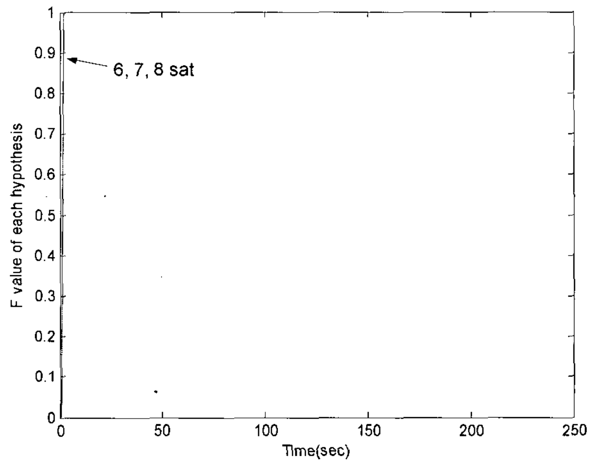


(b) With baseline-length constraint.

Fig. 10. Comparison of convergence speeds with and without baseline-length constraint (Real data).



(a) Without baseline-length constraint.



(b) With baseline-length constraint.

Fig. 11. Comparison of convergence speeds with and without baseline-length constraint with varying the number of the observable satellites (Real data).

Fig. 11 also shows that the baseline-length constraint makes a much bigger contribution for a fewer observable satellite case.

Once again we can see that we can obtain a high quality convergence performance in spite of the short number of observable satellites. It is impossible to fix the integer ambiguity with four observable satellites without the constraint.

When we consider the baseline-length information, it can be variable more or less because of flexibility of the vehicle body or other reasons. The length variation can be either random or biased, or both. This kind of variable length will become the error of the baseline-length information. Fig. 12 shows the convergence characteristics of the true hypothesis with various baseline length biases. It shows the sensitivity of the developed Wald test algorithm with respect to biased errors respectively. According to the test, the algorithm works successfully for the bias case of

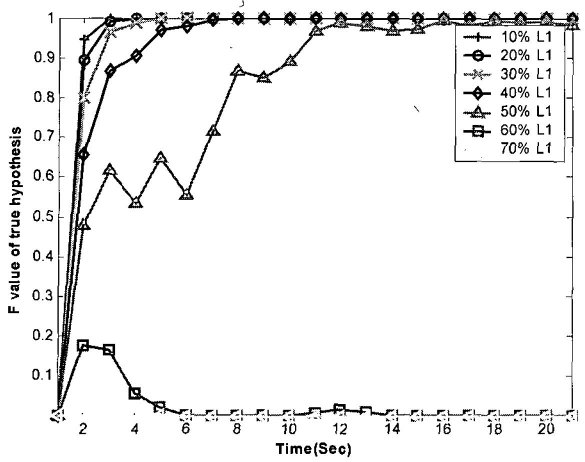


Fig. 12. Sensitivity of the developed Wald test algorithm with respect to biased error of baseline-length.

smaller than 50% of L1 carrier wave length, which is about 9.5cm. Practically, this kind of biased baseline constraints can be considered as the deformed body where surface multiple-GPS antennas are fixed.

7. CONCLUSIONS

In this paper, using numerically simulated and experimental data, it has been shown that the performance of integer ambiguity resolution of the Wald test can be improved considerably if the baseline-length constraint information is used as additional measurement information. A better and more stable convergence trend has been obtained. Furthermore, since the proposed baseline-length constraint information can be considered as an additional measurement of one observable satellite, it has been shown that the integer ambiguities can be resolved using four actual satellite measurements only with very reasonable convergence speed. It has been proven that the major contribution of applying the baseline-length constraint has resulted from the increment of the probability ratio, and the algorithm has been shown that it works successfully for the bias error which is smaller than 50% of L1 carrier wave length. This proposed algorithm can be used for the real time precise positioning as well as the attitude determination using GPS carrier phase measurements.

REFERENCES

[1] P. J. G. Teunissen, "GPS carrier phase ambiguity fixing concepts," *GPS for Geodesy*, Chapter 8, pp. 319-388, 2nd Edition, Springer, 1998.

[2] J. D. Wolfe, W. R. Williamson, and J. L. Speyer, "Hypothesis testing for resolving integer ambiguity in GPS," *Proc. of ION GPS-2001,* pp. 1522-1531, September 2001.

[3] M. F. Abdel-Hafez, Y. J. Lee, W. R. Williamson, J. D. Wolfe, and J. L. Speyer, "A methodology for reducing the admissible hypotheses for GPS integer ambiguity resolution," *Proc. of ION GPS-2002*, pp. 2788-2798, September 2002.

[4] F. van Graas and M. Braasch, "GPS interferometric attitude and heading determination: Initial flight test results," *Navigation*, vol. 38, no. 4, pp. 297-316, 1991.

[5] G. Lu, *Development of a GPS Multi-Antenna System for Attitude Determination*, Ph.D. Dissertation, Dept. of Geomatics Engineering, University of Calgary, Canada, December 1994.

[6] C. E. Cohen, *Attitude Determination using GPS*, Ph.D. Dissertation, Stanford University, 1992.

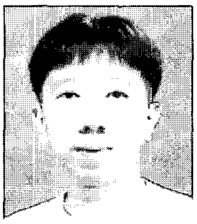
[7] G. Lachapelle, H. Sun, M. E. Cannon, and G. Lu, "Precise aircraft-to-aircraft positioning using a multiple receiver configuration," *Canadian Aeronautics and Space Journal*, vol. 40, no. 2, pp. 74-78, 1994.

[8] J. F. Raquet, "Multiple reference GPS receiver multipath mitigation technique," *Proc. of ION Annual Meeting*, pp. 681-690, 1996.

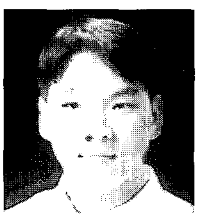
[9] A. Wald, *Sequential Analysis*, Wiley Mathematical Statistics Series, John Wiley and Sons, Inc., New York, 1947.

[10] D. P. Malladi and J. L. Speyer, "A generalized Shiryayev sequential probability ration test for change detection and isolation," *IEEE Trans. on Automatic Control*, vol. 44, pp. 1522-1534, Aug. 1999.

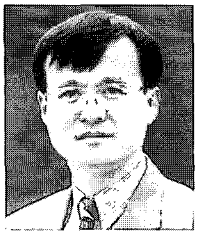
[11] S. M. Kay, *Fundamentals of Statistical Signal Processing, Volume II, Detection Theory*, Prentice Hall PTR, New Jersey, 1998.



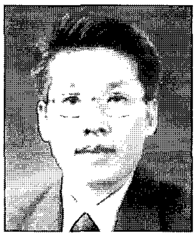
Eunsung Lee is a Research Staff in Mechanical and Aerospace Engineering at University of California, Los Angeles. He received the Ph.D. in Aerospace Engineering from Konkuk University in 2005. He received the M.S. degree in Aerospace Engineering from the Konkuk University, Korea in 1998. His research areas include GPS RTK, orbit determination of LEO satellites, and fault detection of GPS systems.



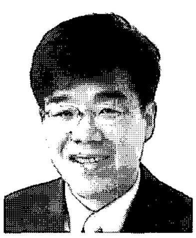
Sebum Chun is a Ph.D. candidate in the Department of Aerospace Engineering at Konkuk University, Korea. He received the M.S. degree in Aerospace Engineering from Konkuk University in 2002. His research areas include GPS RTK, system integration of multi-sensors for navigation, nonlinear estimation, and UAV.



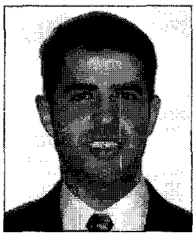
Young Jae Lee is a Professor in the Department of Aerospace Engineering at Konkuk University, Korea. He received the Ph.D. in Aerospace Engineering from the University of Texas at Austin in 1990. His research interests include application of GPS to aircraft attitude determination and orbit determination, precise positioning, development of aircraft landing system, and other GPS related engineering problems.



Teasam Kang is an Associate Professor of the Department of Aerospace Engineering at Konkuk University, Korea. He received the B.S., M.S., and Ph.D. degree from the Seoul National University in 1986, 1988 and 1992, respectively. His current research areas are robust control theories and its applications that include flight control, development of micro aerial vehicle, and MEMS inertial sensor development.



Gyu-In Jee is a Professor in the Department of Electronics Engineering at Konkuk University, Seoul, Korea. He received the Ph.D. in Systems Engineering from Case Western Reserve University. His research has been focused on GPS and its applications. He is a committee of the GNSS Technology Council which organizes and hosts GNSS Workshop every year in Korea.



Mamoun F. Abdel-Hafez is a Professor in the Department of Aerospace Engineering at American University of Sharjah, Sharjah, United Arab Emirates. He received the M.S. in Mechanical Engineering from the University of Wisconsin, Milwaukee in 1999, and Ph.D. degree in Mechanical Engineering from the University of California, Los Angeles (UCLA) in 2003. His research interests are in stochastic estimation, control systems, and algorithms and applications of GPS/INS systems.