Tip Position Control of a Flexible-Link Manipulator with Neural Networks

Yuan-Gang Tang, Fu-Chun Sun, Zeng-Qi Sun, and Ting-Liang Hu

Abstract: To control the tip position of a flexible-link manipulator, a neural network (NN) controller is proposed in this paper. The dynamics error used to construct NN controller is derived based on output redefinition approach. Without the filtered tracking error, the proposed NN controller can still guarantee the closed-loop system uniformly asymptotically stable as well as NN weights bounded. Furthermore, the tracking error of desired trajectory can converge to zero with the proposed controller. For comparison an NN controller with filtered tracking error is also designed for the flexible-link manipulator. Finally, simulation studies are carried out to verify the theoretic results.

Keywords: Filtered tracking error, flexible-link manipulator, Lyapunov function, neural network, output redefinition.

1. INTRODUCTION

Model and control of a flexible-link manipulator have attracted much attention due to the potential structure advantages. The structural flexibility is the key issue which can mainly account for most of the system characteristics: less mass, faster operation, lower energy consumption, higher load-carrying capability, wider operation range, as well as non-minimum phrase (unstable zero dynamics), distributed parameters, strong coupling, non-linearity, un-modeled dynamics and so on.

In an effort to control a flexible-link manipulator, many classical theories are considered, such as optimal control [1], robust control [2], inverse dynamics [3], feedback linearization [4], singular perturbation theory [5-7] and integral manifold theory [8], however, most of which fail to provide satisfied performances without exact knowledge of the dynamics and the nonlinearities of the plant. Therefore, more and more researchers turn to intelligent methods, especially NN techniques, in order to find preferable solutions to the control problems. Based on the feedback-error-learning approach Talebi [9] developed four different on-line NN controllers for tip position tracking, but the global asymptotic stability cannot be guaranteed only by this back-propagation method [10]. Although Cheng [11] employed two NNs to ensure the stability of the manipulator system, the control strategy cannot be generalized to time varying systems and nonlinear systems. With the idea of multilayer NN [12] and singular perturbation theory, Yesildirek [13] decomposed the flexible-link manipulator system into slow and fast subsystems. An LQR controller is used to stabilize the fast subsystem and an NN controller with PD and robust term is designed to regulate the slow subsystem. And yet, the flexible-link manipulator must be of sufficiently large stiffness and the robust control component relies on the exact knowledge of the fast dynamics. Similar deficiencies exist in [14].

In this paper we extend work in [12], which put stress on a general serial-link rigid manipulator, to the flexible-link case. Based on the fact that NNs are capable of uniformly approximating a given nonlinear function over a compact set to any degree of accuracy, a composite control strategy is adopted to deal with the tip tracking problem of flexible manipulators. The controller consists of an NN estimator to approximate a certain nonlinear function and a proportional component of tip tracking error to ensure the stability of the closed-loop system. Specially, the NN controller is constructed according to the structure information of manipulator dynamics. The uniformly asymptotic stability of the closed-loop system and bounded NN weights can be guaranteed by Lyapunov theory. The tracking error can go to zero

Manuscript received November 16, 2004; revised December 9, 2005; accepted March 3, 2006. Recommended by Editorial Board member Ju-Jang Lee under the direction of past Editor-in-Chief Myung Jin Chung. This work was jointly supported by the National Excellent Doctoral Dissertation Foundation (Grant No: 200041), the National Key Project for Basic Research of China (Grant No: G2002cb312205), the National Science Foundation of China (Grant No: 60474025, 60504003, 60321002 and 90405017), and specialized research fund for the doctoral program of higer education.

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asymptotically even with bounded unknown disturbances (including un-modeled dynamics, reconstruction error of neural network and high-order error of Taylor series expansion), whereas in [12] the filtered tracking error is only uniformly ultimately bounded (UUB) and cannot be eliminated completely. Moreover, vibration of tip position of the manipulator can also be effectively suppressed with the proposed NN controller.

The outline of this paper is as follows. Section 2 describes the basic idea of NN. Section 3 introduces the dynamic model and output redefinition of a flexible-link manipulator. Section 4 discusses the design of the NN controller. In Section 5, simulation results for a planner two-link flexible manipulator are presented. Finally, conclusions are drawn in Section 6.

2. BASIC STRUCTURE OF NN

Here a three-layer NN is considered, the input-output relation of which can be represented in the following form $g(x)=W^T\sigma(V^Tx)$, where $V\in R^{(m+1)\times h}$ is the bounded input-to-hidden layer weight matrix, $W\in R^{(h+1)\times k}$ is the bounded hidden-to-output layer weight matrix. m, h and k are the number of neurons in input, hidden and output layer respectively, x is the bounded input of NN and $g(x)$ is the output of NN. The active function $\sigma(V^Tx)=1/(1+\exp(-V^Tx))$.

In general, any continuous function $f(x)$ can be approximated as $f(x)=W^T\sigma(V^Tx)+\varepsilon$ on a compact set, where W and V are bounded optimal weight matrices, ε is a given positive constant. Since the size of neural network is difficult to be determined, we often use the weight estimates with a certain size to approximate the function, i.e. $\hat{f}(x)=\hat{W}^T\sigma(\hat{V}^Tx)$. Then the following estimation errors can be defined. Weights errors $\tilde{W}=W-\hat{W}$, $\tilde{V}=V-\hat{V}$. Functional approximation error $\tilde{f}=f-\hat{f}=f(x)-\hat{f}(x)$. Hidden layer output error $\tilde{\sigma}=\sigma-\hat{\sigma}=\sigma(V^Tx)-\sigma(\hat{V}^Tx)$ and its Taylor series expansion about $\hat{V}^Tx$ is $\tilde{\sigma}=\sigma'(\hat{V}^Tx)\tilde{V}^Tx+\mathrm{O}(\tilde{V}^Tx)^2=\hat{\sigma}'\tilde{V}^Tx+\mathrm{O}(\tilde{V}^Tx)^2$.

3. DYNAMIC MODEL AND OUTPUT REDEFINITION OF A FLEXIBLE-LINK MANIPULATOR

The closed-form dynamic equations of a flexible-link manipulator obtained from Lagrangian approach have the following form [15,16]

$$\begin{bmatrix} M_{\theta\theta} & M_{\theta\delta} \\ M_{\theta\delta}^T & M_{\delta\delta}\end{bmatrix}\begin{bmatrix}\ddot{\theta}\\ \ddot{\delta}\end{bmatrix}+\begin{bmatrix} C_{\theta\theta} & C_{\theta\delta} \\ C_{\delta\theta} & C_{\delta\delta}\end{bmatrix}\begin{bmatrix}\dot{\theta}\\ \dot{\delta}\end{bmatrix}+\begin{bmatrix}F_\theta\\ F_\delta\end{bmatrix}+\begin{bmatrix}G_\theta\\ G_\delta\end{bmatrix}+\begin{bmatrix}\tau_{d\theta}\\ \tau_{d\delta}\end{bmatrix}=\begin{bmatrix}\tau\\ \mathbf{0}\end{bmatrix}, \tag{1}$$

where $\theta\in R^n$ is the vector of joint position variables, $\delta\in R^p$ is the vector of flexible modes, $M_{\theta\theta}$, $M_{\theta\delta}$, $M_{\delta\delta}$ are the blocks of inertia matrix M, $C_{\theta\theta}$, $C_{\theta\delta}$, $C_{\delta\theta}$ and $C_{\delta\delta}$ are the blocks of Coriolis and centrifugal matrix C, F_θ and F_δ are the components of the friction vector F, G_θ and G_δ are the components of the gravity vector G, $\tau_{d\theta}$ and $\tau_{d\delta}$ are the bounded un-modeled dynamics, τ is the vector of control torques. Two main matrix properties [17] are used in this paper: 1) M is symmetric positive definite, 2) $\dot{M}-2C$ is skew symmetric.

The dynamic equations (1) can also be rewritten as

$$M_{\theta\theta}\ddot{\theta}+M_{\theta\delta}\ddot{\delta}+C_{\theta\theta}\dot{\theta}+C_{\theta\delta}\dot{\delta}+F_\theta+G_\theta+\tau_{d\theta}=\tau, \tag{2}$$

$$M_{\theta\delta}^T\ddot{\theta}+M_{\delta\delta}\ddot{\delta}+C_{\delta\theta}\dot{\theta}+C_{\delta\delta}\dot{\delta}+F_\delta+G_\delta+\tau_{d\delta}=\mathbf{0}. \tag{3}$$

From (3) we can obtain

$$\begin{aligned}\ddot{\delta}&=-M_{\delta\delta}^{-1}\left(M_{\theta\delta}^T\ddot{\theta}+C_{\delta\theta}\dot{\theta}+C_{\delta\delta}\dot{\delta}+F_\delta+G_\delta\right)\\ &\quad-M_{\delta\delta}^{-1}\tau_{d\delta}=S\left(\ddot{\theta},\dot{\theta},\theta,\dot{\delta},\delta\right)-M_{\delta\delta}^{-1}\tau_{d\delta}.\end{aligned} \tag{4}$$

Substituting (4) into (2) yields

$$\begin{aligned}&M_{\theta\theta}\ddot{\theta}+C_{\theta\theta}\dot{\theta}+M_{\theta\delta}S+C_{\theta\delta}\dot{\delta}+F_\theta+G_\theta\\ &-M_{\theta\delta}M_{\delta\delta}^{-1}\tau_{d\delta}+\tau_{d\theta}=\tau.\end{aligned} \tag{5}$$

To stabilize the zero dynamics of a flexible-link manipulator output redefinition is applied, thus tip position of link can be expressed as $y=\theta+\alpha d/l$, where α is chosen appropriately in [0 1] to ensure the minimum phase of the closed-loop system ($\alpha=0$ corresponds to the joint position, $\alpha=1$ corresponds to the tip position), l is the length of flexible link, d is the tip deflection. $d=[d_1,\cdots,d_n]^T$, $d_i=\sum_{j=1}^{N_i}\phi_{ij}\delta_{ij}$, ϕ_{ij} is the spatial mode shape function [15], δ_{ij} is the jth flexible mode of link i, N_i is the number of flexible modes of link i.

4. STABLE NN CONTROLLER DESIGN

In order to design the stable NN controller, the error dynamics of the flexible-link manipulator ought to be first considered. In adaptive control of a manipulator, an auxiliary filtered tracking error signal is often required to remove the acceleration components from the dynamic equations. With the same idea Lewis [12] introduced a filtered error to the NN control. but we find that the filtered error is not necessary to NN control. Here, a simple stable NN controller without the filtered error is proposed to control a flexible-link manipulator based on Lyapunov function, which can guarantee the uniformly asymptotic stability of the closed-loop system. And the convergence of the tracking error to zero is also guaranteed.

Given the desired tip-position trajectory $y_d \in R^n$ the tracking error is

$$e = y_d - y = y_d - \theta - \alpha d/l \tag{6}$$

then derivatives of tracking error can be obtained

$$\dot{e} = \dot{y}_d - \dot{y} = \dot{y}_d - \dot{\theta} - \alpha \dot{d}/l . \tag{7}$$

Considering equation (5) we have

$$\begin{aligned}M_{\theta\theta}\dot{e} &= M_{\theta\theta}\dot{y}_d - M_{\theta\theta}\dot{\theta} - \alpha M_{\theta\theta}\dot{d}/l \\ &= -C_{\theta\theta}e + C_{\theta\theta}e + M_{\theta\theta}\dot{y}_d - M_{\theta\theta}\dot{\theta} - \alpha M_{\theta\theta}\dot{d}/l \\ &= -C_{\theta\theta}e + C_{\theta\theta}e + M_{\theta\theta}\dot{y}_d - \alpha M_{\theta\theta}\dot{d}/l \\ &\quad -(\tau + M_{\theta\theta}\dot{\theta} - M_{\theta\theta}\ddot{\theta} - C_{\theta\theta}\dot{\theta} - M_{\theta\delta}S - C_{\theta\delta}\dot{\delta} \\ &\quad - F_\theta - G_\theta + M_{\theta\delta}M_{\delta\delta}^{-1}\tau_{d\delta} - \tau_{d\theta}) \\ &= -C_{\theta\theta}e - \tau + f + \bar{\tau}_d,\end{aligned}$$

where $f = C_{\theta\theta}e + M_{\theta\theta}\dot{y}_d - \alpha M_{\theta\theta}\dot{d}/l - M_{\theta\theta}\dot{\theta} + M_{\theta\theta}\ddot{\theta} + C_{\theta\theta}\dot{\theta} + M_{\theta\delta}S + C_{\theta\delta}\dot{\delta} + F_\theta + G_\theta$, $\bar{\tau}_d = M_{\theta\delta}M_{\delta\delta}^{-1}\tau_{d\delta} + \tau_{d\theta}$. If the control input torque is in the form of

$$\tau = \hat{f} + R , \tag{8}$$

where R is the robust term, the NN controller can be determined by the following Theorem.

Theorem 1: Consider the dynamic system of a flexible-link manipulator described by (1), for the bounded, continuous desired tip trajectory with bounded velocity and acceleration, NN controller (8) can guarantee the uniformly asymptotic stability of the close-loop system with the robust term $R = K_p \operatorname{sgn}(e)$, $K_p = diag\{k_{p1}, \cdots k_{pi}, \cdots k_{pn}\}$, $k_{pi} \ge \|\bar{\omega}\|$, $\bar{\omega} = \tilde{W}^T\hat{\sigma}'V^Tx + \tilde{W}^T O(\tilde{V}^Tx)^2 + \hat{W}^T O(\tilde{V}^Tx)^2 + \varepsilon + \bar{\tau}_d$. And the NN weight tuning algorithms are given by

$$\dot{\hat{W}} = E\hat{\sigma}e^T - E\hat{\sigma}'\hat{V}^Txe^T , \tag{9}$$

$$\dot{\hat{V}} = Qxe^T\hat{W}^T\hat{\sigma}' , \tag{10}$$

where E, Q are the constant positive matrices. Moreover, the weight estimates $\hat{W}$, $\hat{V}$ are bounded and the tracking error (6) goes to zero asymptotically.

Proof: Define the Lyapunov function

$$L = e^T M_{\theta\theta}e/2 + tr\left(\tilde{W}^T E^{-1}\tilde{W}\right)/2 + tr\left(\tilde{V}^T Q^{-1}\tilde{V}\right)/2 .$$

Differentiating yields

$$\begin{aligned}\dot{L} &= e^T M_{\theta\theta}\dot{e} + e^T \dot{M}_{\theta\theta}e/2 + tr\left(\tilde{W}^T E^{-1}\dot{\tilde{W}}\right) + tr\left(\tilde{V}^T Q^{-1}\dot{\tilde{V}}\right) \\ &= e^T\left(-C_{\theta\theta}e + f - \hat{f} + \bar{\tau}_d - R\right) + e^T\dot{M}_{\theta\theta}e/2 \\ &\quad + tr\left(\tilde{W}^T E^{-1}\dot{\tilde{W}}\right) + tr\left(\tilde{V}^T Q^{-1}\dot{\tilde{V}}\right) \\ &= e^T\left(\dot{M}_{\theta\theta} - 2C_{\theta\theta}\right)e/2 + e^T\left(f - \hat{f} + \bar{\tau}_d - R\right) \\ &\quad + tr\left(\tilde{W}^T E^{-1}\dot{\tilde{W}}\right) + tr\left(\tilde{V}^T Q^{-1}\dot{\tilde{V}}\right).\end{aligned}$$

Because $\dot{M} - 2C$ is skew-symmetric, $\dot{M}_{\theta\theta} - 2C_{\theta\theta}$ is skew-symmetric and $e^T\left(\dot{M}_{\theta\theta} - 2C_{\theta\theta}\right)e/2 = 0$. Now considering

$$\begin{aligned}& f - \hat{f} + \bar{\tau}_d - R \\ &= W^T\sigma + \varepsilon - \hat{W}^T\hat{\sigma} + \bar{\tau}_d - R \\ &= W^T\sigma + W^T\hat{\sigma} - W^T\hat{\sigma} - \hat{W}^T\hat{\sigma} + \varepsilon + \bar{\tau}_d - R \\ &= W^T\tilde{\sigma} + \tilde{W}^T\hat{\sigma} + \varepsilon + \bar{\tau}_d - R \\ &= W^T\tilde{\sigma} - \hat{W}^T\tilde{\sigma} + \hat{W}^T\tilde{\sigma} + \tilde{W}^T\hat{\sigma} + \varepsilon + \bar{\tau}_d - R \\ &= \tilde{W}^T\tilde{\sigma} + \hat{W}^T\tilde{\sigma} + \tilde{W}^T\hat{\sigma} + \varepsilon + \bar{\tau}_d - R \\ &= \tilde{W}^T\left(\hat{\sigma}'\tilde{V}^Tx + O\left(\tilde{V}^Tx\right)^2\right) + \hat{W}^T\left(\hat{\sigma}'\tilde{V}^Tx + O\left(\tilde{V}^Tx\right)^2\right) \\ &\quad + \tilde{W}^T\hat{\sigma} + \varepsilon + \bar{\tau}_d - R \\ &= \tilde{W}^T\hat{\sigma}'\tilde{V}^Tx + \hat{W}^T\hat{\sigma}'\tilde{V}^Tx + \tilde{W}^T\hat{\sigma} + \tilde{W}^T O\left(\tilde{V}^Tx\right)^2 \\ &\quad + \hat{W}^T O\left(\tilde{V}^Tx\right)^2 + \varepsilon + \bar{\tau}_d - R \\ &= -\tilde{W}^T\hat{\sigma}'\hat{V}^Tx + \hat{W}^T\hat{\sigma}'\tilde{V}^Tx + \tilde{W}^T\hat{\sigma} + \bar{\omega} - R,\end{aligned}$$

where

$$\bar{\omega} = \tilde{W}^T\hat{\sigma}'V^Tx + \tilde{W}^T O(\tilde{V}^Tx)^2 + \hat{W}^T O(\tilde{V}^Tx)^2 + \varepsilon + \bar{\tau}_d .$$

Since

$$\begin{aligned}& e^T\tilde{W}^T\hat{\sigma}'\hat{V}^Tx = tr\left(\tilde{W}^T\hat{\sigma}'\hat{V}^Txe^T\right), \\ & e^T\tilde{W}^T\hat{\sigma} = tr\left(\tilde{W}^T\hat{\sigma}e^T\right), \\ & e^T\hat{W}^T\hat{\sigma}'\tilde{V}^Tx = tr\left(\tilde{V}^Txe^T\hat{W}^T\hat{\sigma}'\right).\end{aligned}$$

We have

$$\begin{aligned}& e^T\left(f - \hat{f} + \bar{\tau}_d - R\right) + tr\left(\tilde{W}^T E^{-1}\dot{\tilde{W}}\right) + tr\left(\tilde{V}^T Q^{-1}\dot{\tilde{V}}\right) \\ &= -e^T\tilde{W}^T\hat{\sigma}'\hat{V}^Tx + e^T\tilde{W}^T\hat{\sigma} + e^T\hat{W}^T\hat{\sigma}'\tilde{V}^Tx + e^T\left(\bar{\omega} - R\right) \\ &\quad + tr\left(\tilde{W}^T E^{-1}\dot{\tilde{W}}\right) + tr\left(\tilde{V}^T Q^{-1}\dot{\tilde{V}}\right) \\ &= e^T\left(\bar{\omega} - R\right) + tr\left(\tilde{W}^T\left(E^{-1}\dot{\tilde{W}} - \hat{\sigma}'\hat{V}^Txe^T + \hat{\sigma}e^T\right)\right)\end{aligned}$$

$$+tr\left(\tilde{V}^T\left(Q^{-1}\dot{\hat{V}}+xe^T\hat{W}^T\hat{\sigma}'\right)\right).$$

Let the NN weight tuning be provided by

$$\dot{\hat{W}} = E\hat{\sigma}e^T - E\hat{\sigma}'\hat{V}^T xe^T, \quad \dot{\hat{V}} = Qxe^T\hat{W}^T\hat{\sigma}'.$$

i.e. $\dot{\tilde{W}} = -E(\hat{\sigma}e^T - \hat{\sigma}'\hat{V}^T xe^T)$, $\dot{\tilde{V}} = -Qxe^T\hat{W}^T\hat{\sigma}'$.

We obtain

$$tr\left(\tilde{W}^T\left(E^{-1}\dot{\tilde{W}} - \hat{\sigma}'\hat{V}^T xe^T + \hat{\sigma}e^T\right)\right)$$
$$+tr\left(\tilde{V}^T\left(Q^{-1}\dot{\tilde{V}} + xe^T\hat{W}^T\hat{\sigma}'\right)\right) = 0.$$

Lastly, choosing $R = K_p\,\mathrm{sgn}(e)$, $e = [e_1, e_2, \cdots e_n]^T$, $K_p = diag\{k_{p1}, \cdots k_{pn}\}$, we have

$$e^T(\bar{\omega} - R) = e^T\bar{\omega} - e^T K_p\,\mathrm{sgn}(e)$$
$$\leq \left\|e^T\right\|\left\|\bar{\omega}\right\| - \left(k_{p1}\left\|e_1\right\| + \cdots + k_{pn}\left\|e_n\right\|\right)$$
$$\leq \left(\left\|e_1\right\| + \cdots + \left\|e_n\right\|\right)\left\|\bar{\omega}\right\| - \left(k_{p1}\left\|e_1\right\| + \cdots + k_{pn}\left\|e_n\right\|\right).$$

If $k_{pi} \geq \|\bar{\omega}\|$, $i = 1, \cdots n$, $e^T(\bar{\omega} - R) \leq 0$. Thus, $\dot{L} \leq 0$.

In addition, $\|\tilde{W}\|$, $\|\tilde{V}\| \to 0$, then $\hat{W} \to W$ and $\hat{V} \to V$. W, V are bounded, so $\hat{W}$, $\hat{V}$ are bounded.

Remark 1: This Theorem indicates $\bar{\omega} \to \varepsilon + \bar{\tau}_d$, where given positive constant ε may be made arbitrarily small and $\bar{\tau}_d$ depends on the inertia matrix and external disturbance, so k_{pi} should be greater than $\|\bar{\tau}_d\|$ at least. In particular, gain k_{pi} can also be regarded as a variable relying on $\|\bar{\omega}\|$.

5. SIMULATION RESULTS

In this section we consider a planar two-link flexible manipulator. Partial physical parameters of arms are as follows: link uniform density $\rho_1 = \rho_2 = 0.2kg/m$, link length $l_1 = l_2 = 0.5m$, link center of mass $c_1 = c_2 = 0.25m$, link mass $m_1 = m_2 = 0.1kg$, hub mass $m_{h1} = m_{h2} = 1kg$, link inertia $J_{o1} = J_{o2} = 0.0083kgm^2$, hub inertia $J_{h1} = J_{h2} = 0.1kgm^2$, flexural link rigidity $(EI)_{1,2} = 1Nm^2$, flexible mode number $N_1 = N_2 = 2$. The other parameters can be found in [16].

In order to compare the proposed NN controller with the one based on a filtered error, some common parameters should be specified: the number of neurons in hidden layer $h = 5$, input vector of NN

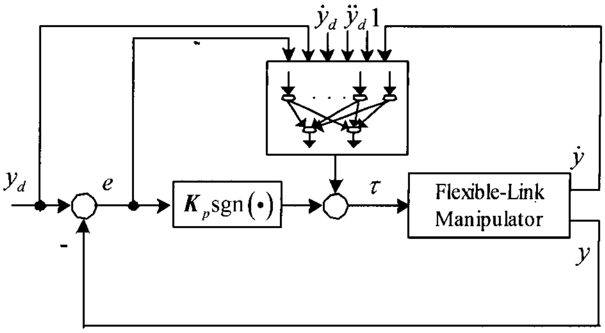

Fig. 1. The proposed NN control structure.

$x = [1, e, \dot{e}, y_d, \dot{y}_d, \ddot{y}_d]^T$; initial weight values of NN are zeros, initial values of the generalized coordinates are also zeros. Because the tip position control problem of flexible manipulators is considered in this paper, variable α is chosen as 1. Simulation results for other values of α are also carried out. As for the control goal, the final desired tip position of each flexible link

$$y_{1d} = \begin{cases}\sin(t) & 0 \leq t < 3\pi/2 \\ \sin(3\pi/2) & t \geq 3\pi/2,\end{cases} \quad y_{2d} = \begin{cases}\cos(t) & 0 \leq t < 3\pi/2 \\ \cos(3\pi/2) & t \geq 3\pi/2,\end{cases}$$

should be reached. The whole closed-loop system is shown schematically in Fig. 1.

Upon that we can compare the above two NN controllers for the flexible-link manipulator. With the filtered tracking error $r = \dot{e} + \Lambda e$, we design an NN controller

$$\tau = \hat{W}^T\sigma(\hat{V}^T x) + K_v r - v \tag{11}$$

for the flexible manipulator in the same control framework as [12] and let the improved weight tuning be given by

$$v(t) = -K_Z\left(\left\|\hat{Z}\right\|_F + Z_M\right)r,$$
$$\dot{\hat{W}} = F\hat{\sigma}r^T - F\hat{\sigma}'\hat{V}^T xr^T - kF\left\|r\right\|\hat{W},$$
$$\dot{\hat{V}} = Gx\left(\hat{\sigma}'^T\hat{W}r\right)^T - kG\left\|r\right\|\hat{V},$$

where $K_Z = 0.1$, $Z = diag\{W, V\}$, $Z_M = 1000$, $\Lambda = diag\{10, 10\}$, $F = G = diag\{1000, 1000\}$, $k = 1$ (scalar design parameter), $K_v = diag\{10, 10\}$ (gain matrix). The proposed NN controller (8) is of the following parameters

$$R = K_p\,\mathrm{sgn}(e),$$
$$\dot{\hat{W}} = E\hat{\sigma}e^T - E\hat{\sigma}'\hat{V}^T xe^T,$$
$$\dot{\hat{V}} = Qxe^T\hat{W}^T\hat{\sigma}',$$

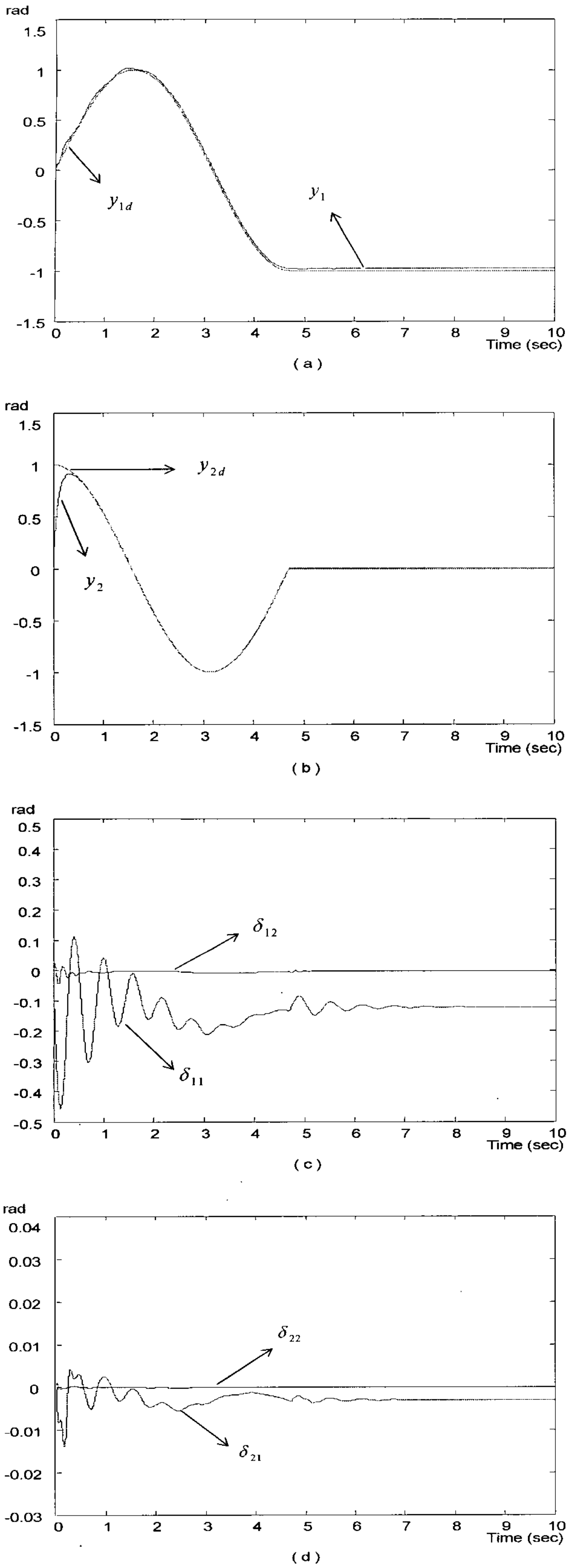

Fig. 2. Simulation results for case 1 with the controller (11). (a) Tip position of the first link. (b) Tip position of the second link. (c) Flexible modes of the first link. (d) Flexible modes of the second link.

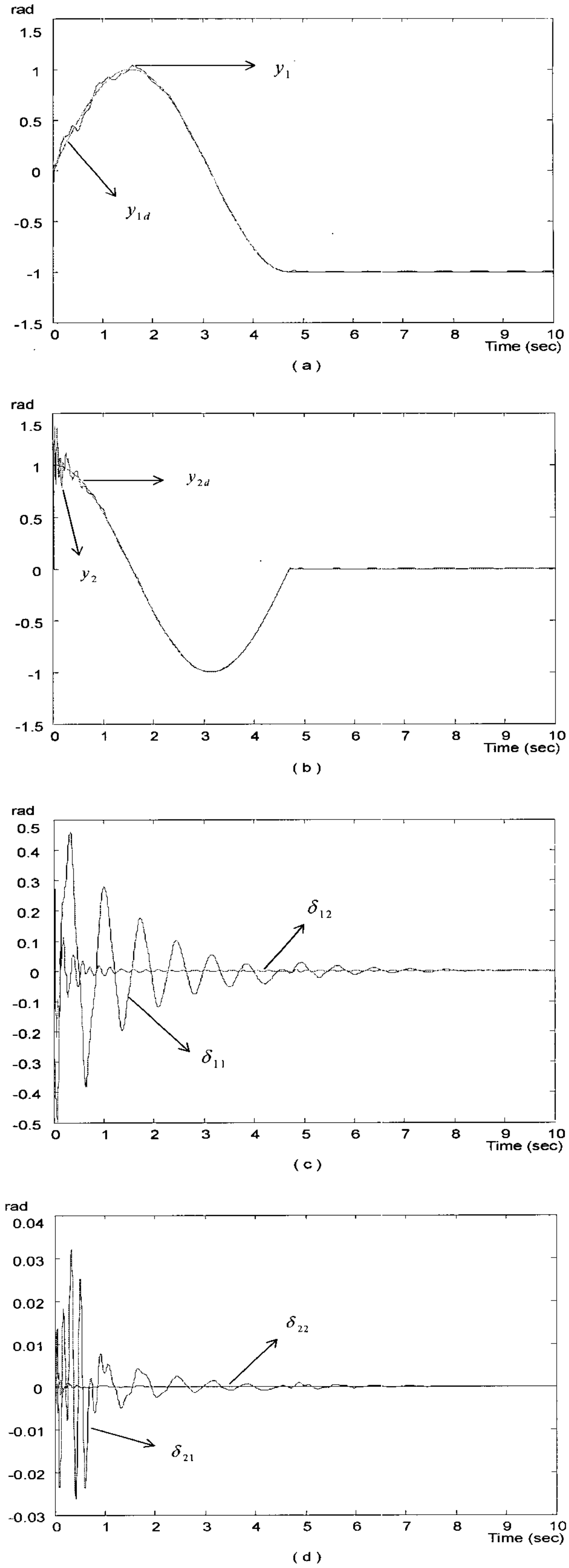

Fig. 3. Simulation results for case 1 with the proposed controller. (a) Tip position of the first link. (b) Tip position of the second link. (c) Flexible modes of the first link. (d) Flexible modes of the second link.

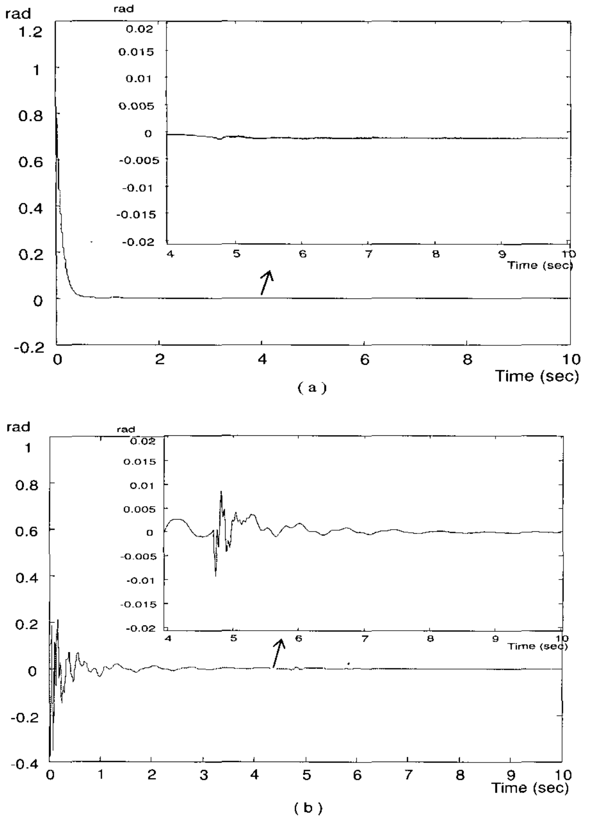

Fig. 4. Tip position tracking errors of the two-link flexible manipulator for case 1. (a) Tracking error with the controller (11). (b) Tracking error with the proposed controller.

where $K_p = diag\{160,160\}$, $E = Q = diag\{90,90\}$.

With the above parameters two different cases are considered: 1) payload mass m_p=0, payload inertia J_p=0, joint viscous friction and link structural damping D = 0.2* *sqrt*(*K*), *K* is stiffness matrix $\alpha = 1$, 2) $m_p = 0.01kg$, $J_p = 0.0005kgm^2$, $D = 0.6{*}sqrt(K)$ for $\alpha = 1$, 0.8, 0.5.

Simulation results for the first case can be seen from Figs. 2-4. Fig. 2 shows the responses of the closed-loop system to the desired tip-position trajectory of each link with extended Lewis's NN controller (11). The simulation results with the proposed NN controller are depicted in Fig. 3. For comparison purpose, Fig. 4 illustrates the tip position tracking errors of the second link with the above two kinds of controllers. For the second case Figs. 5-7 show the behavior of the system. Fig. 5 is the simulation results with extended Lewis's NN controller (11). Fig. 6 describes the behavior of the system with the proposed NN controller for $\alpha = 1$. Tip position tracking errors with two kinds of controllers are plotted in Fig. 7. With the proposed controller Fig. 8 and Fig. 9 depict the simulation results for $\alpha = 0.8$ and $\alpha = 0.5$.

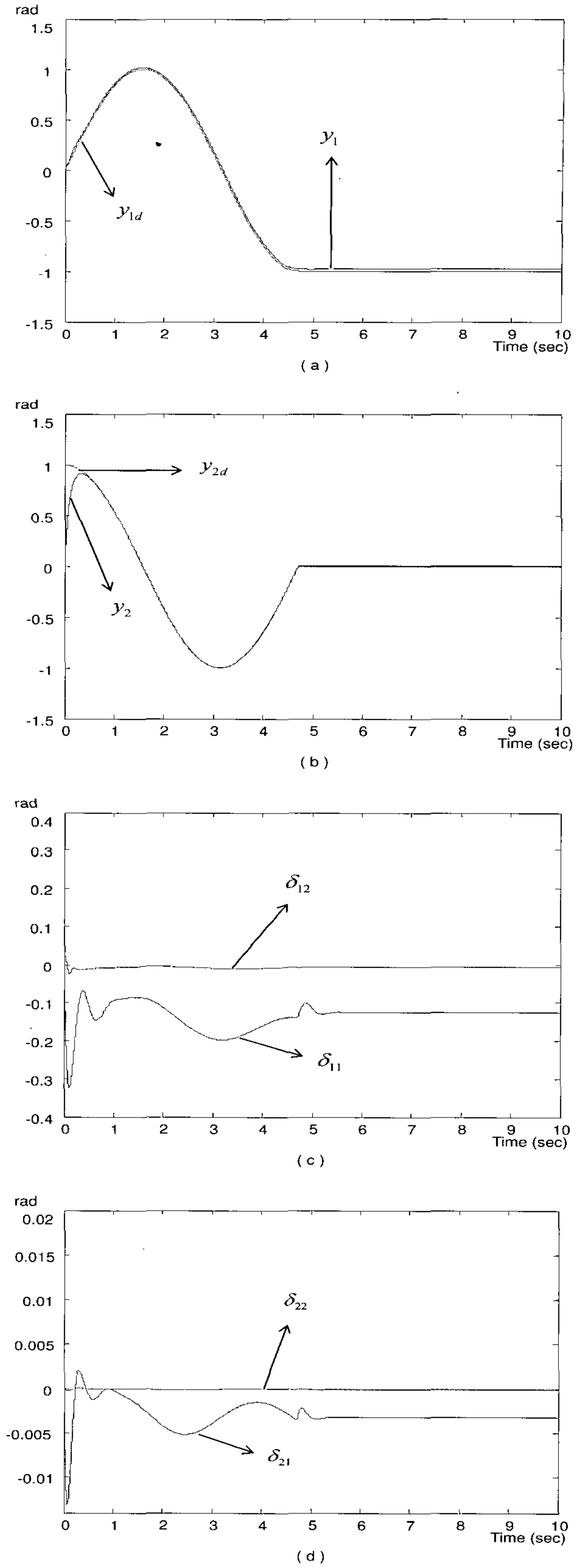

Fig. 5. Simulation results for case 2 with the controller (11). (a) Tip position of the first link. (b) Tip position of the second link. (c) Flexible modes of the first link. (d) Flexible modes of the second link.

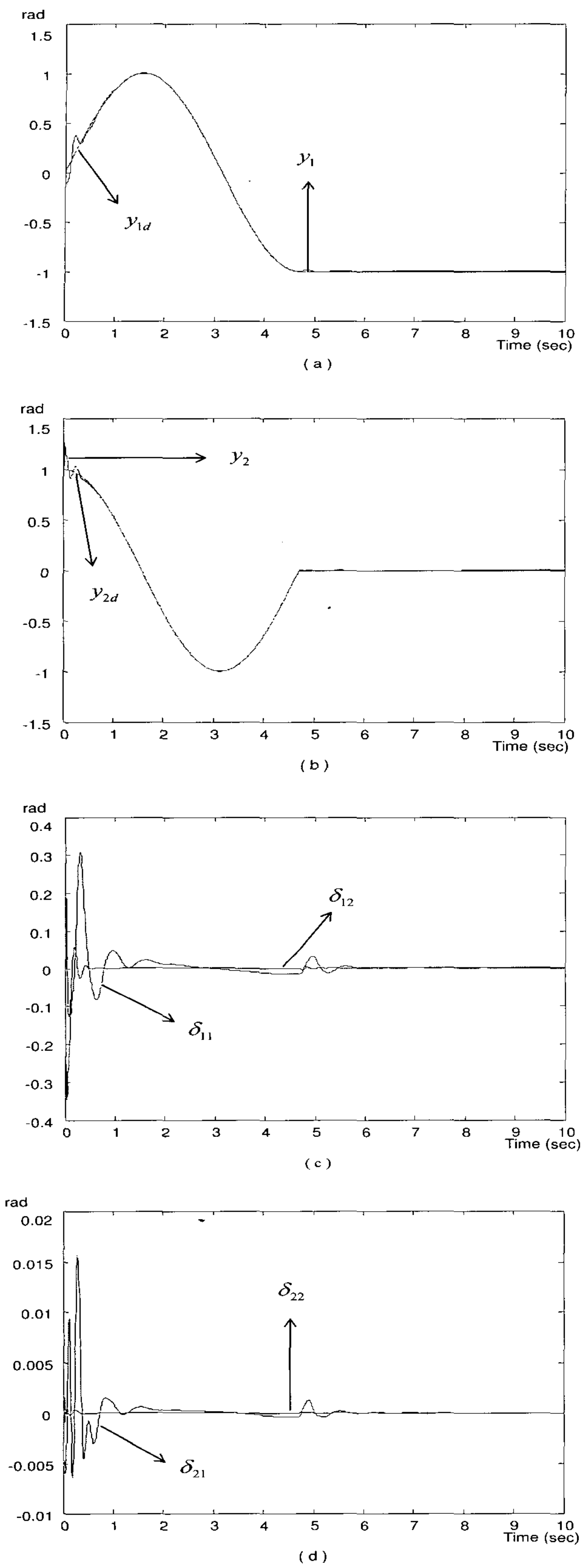

Fig. 6. Simulation results for case 2 with the proposed controller when $\alpha = 1$. (a) Tip position of the first link. (b) Tip position of the second link. (c) Flexible modes of the first link. (d) Flexible modes of the second link.

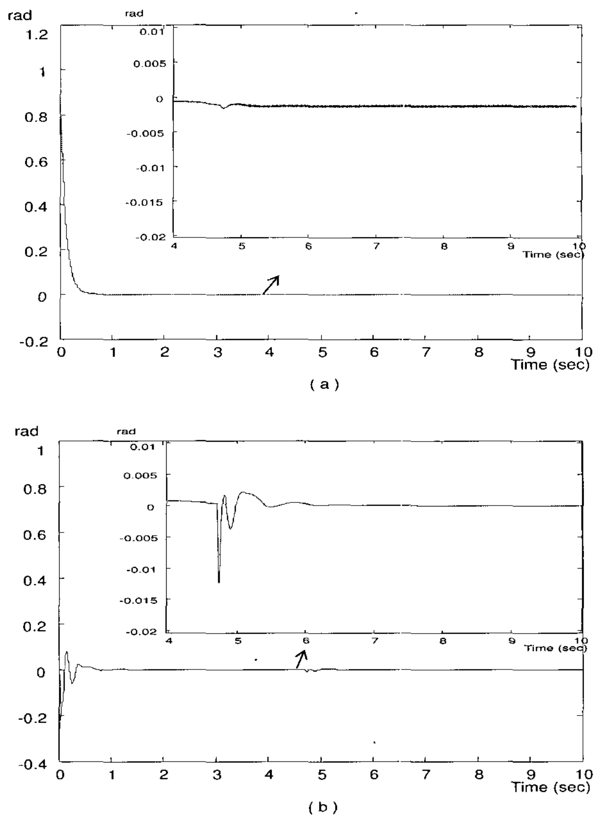

Fig. 7. Tip position tracking errors of the two-link flexible manipulator for case 2. (a) Tracking error with the controller (11). (b) Tracking error with the proposed controller.

From the simulations we can reveal the following facts: 1) Both NN controllers can stabilize the close-loop system of the flexible-link manipulator and give satisfied performances under the bounded unknown disturbances, 2) Fast tracking performance of tip position trajectories can be guaranteed with the on-line weight tuning, 3) End-point vibration of the manipulator can be suppressed effectively. On the contrary, two important differences between the two controllers should not be ignored: 1) Though, due to initialization, tip position of the second link vibrates at the beginning of tracking, the proposed controller with a simple structure can ensure it converging to the desired trajectory quickly. 2) More importantly, the proposed control strategy can make the tracking error go to zero asymptotically, while the tracking error cannot be eliminated by the NN controller (11) even thought it is small. When α is not equal to 1, nonzero tip tracking errors exist since the feedback signals are not the tip positions.

Remark 2: To add the filtered tracking error r is a reasonable way to remove the acceleration from (1), however, this may cause that the adaptive laws contain velocity signals [12]. In this paper, without using the filtered error an NN controller is redesigned for flexible manipulators and the deduced adaptive laws only

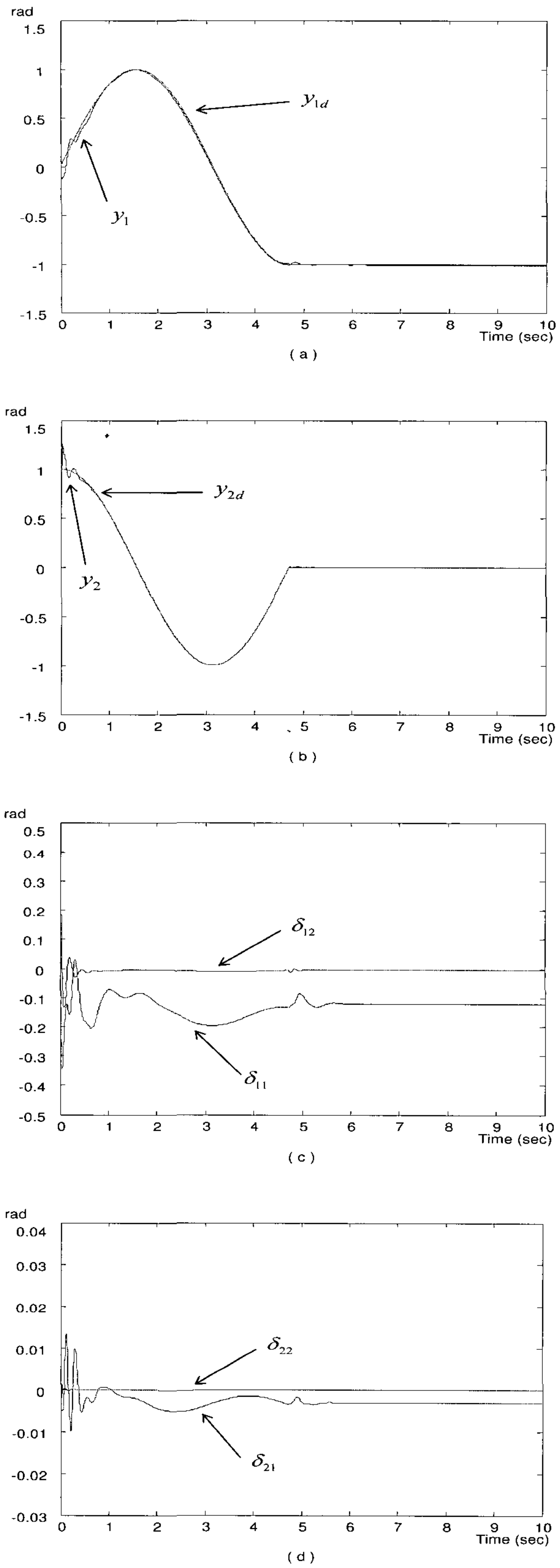

Fig. 8. Simulation results for case 2 with the proposed controller when $\alpha = 0.8$. (a) Tip position of the first link. (b) Tip position of the second link. (c) Flexible modes of the first link. (d) Flexible modes of the second link.

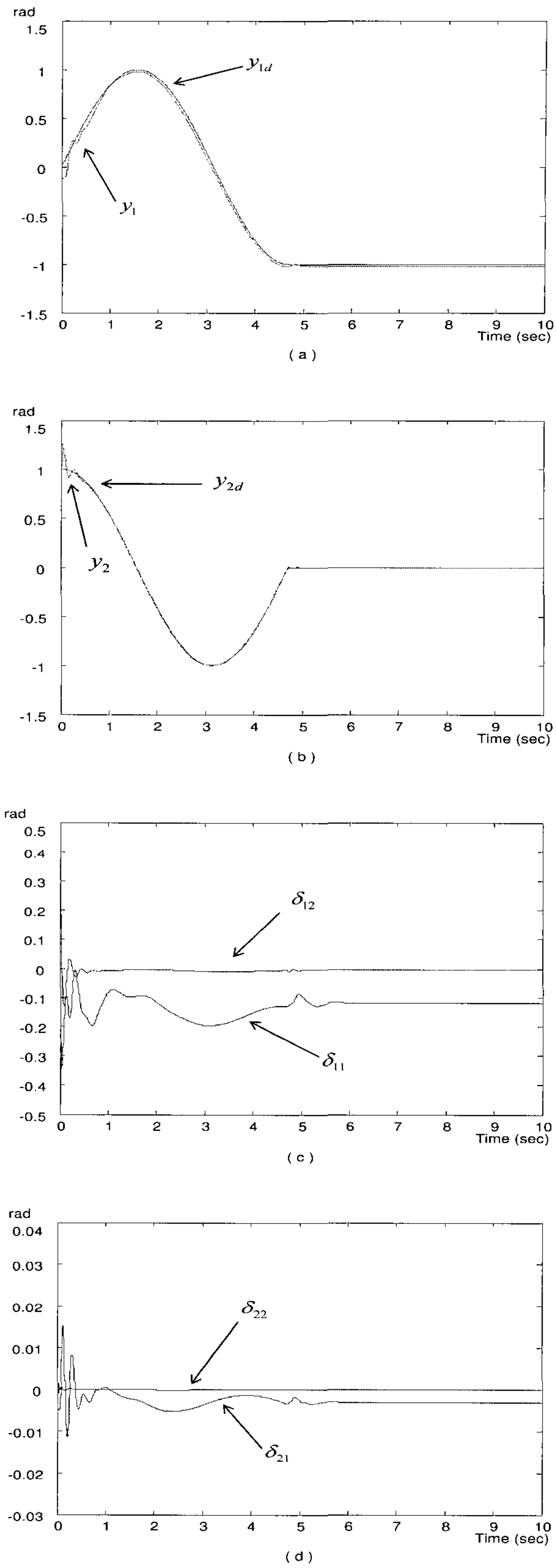

Fig. 9. Simulation results for case 2 with the proposed controller when $\alpha = 0.5$. (a) Tip position of the first link. (b) Tip position of the second link. (c) Flexible modes of the first link. (d) Flexible modes of the second link.

involve the position signals. Furthermore, in the proposed control scheme the error signals can directly adjust the adaptive laws, while in [12] an additional filter is needed before adjusting the adaptive laws. Therefore, our control method can guarantee the steady state error converge to zero asymptotically.

6. CONCLUSIONS

Tip position control problem of a flexible-link manipulator has been researched in this paper. Based on output redefinition a simple NN controller is designed without filtered tracking error, which has an on-line weight tuning algorithm. Besides, uniformly asymptotic stability of the close-loop system and bounded NN weights can be guaranteed by Lyapunov theory even in presence of bounded unknown disturbances. To compare with the proposed controller, Lewis's NN controller is also extended to the flexible-link manipulator. Simulation results of a planar two-link flexible manipulator show that the proposed NN controller can suppress the vibration of the manipulator effectively and also can make the tracking error converge to zero very quickly.

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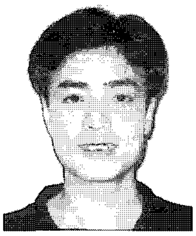

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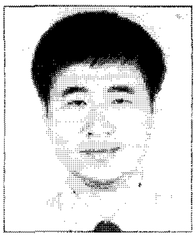

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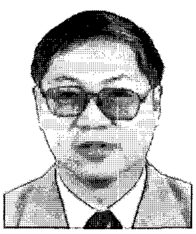

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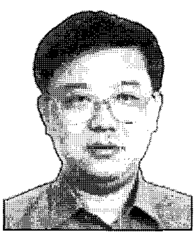

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