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# ON THE EXPONENTIAL FUZZY PROBABILITY

Yong Sik Yun, Jae Choong Song, and Sang Uk Ryu

Abstract. We study the exponential fuzzy probability for quadratic fuzzy number and trigonometric fuzzy number defined by quadratic function and trigonometric function, respectively. And we calculate the exponential fuzzy probabilities for fuzzy numbers driven by operations.

## 1. Introduction

Let $(\Omega, \mathfrak{F}, P)$ be a probability space, where $\Omega$ denotes the sample space, $\mathfrak{F}$ the $\sigma-$algebra on $\Omega$, and $P$ a probability measure. A fuzzy set $A$ on $\Omega$ is called a fuzzy event. Let $\mu_A(\cdot)$ be the membership function of the fuzzy event $A$. Then the probability of the fuzzy event $A$ is defined by Zadeh([6]) as

$$\widetilde{P}(A) = \int_\Omega \mu_A(\omega)\, dP(\omega), \qquad \mu_A(\omega) : \Omega \to [0,1].$$

The operations of two fuzzy numbers $(A, \mu_A)$ and $(B, \mu_B)$ are based on the Zadeh's extension principle([7], [8], [9]). We consider four operations addition $A(+)B$, subtraction $A(-)B$, multiplication $A(\cdot)B$ and division $A(/)B$.

We defined the normal fuzzy probability using the normal distribution and calculated the normal fuzzy probability for quadratic fuzzy number([3]). And then we had the explicit formula for the normal fuzzy probability for trigonometric fuzzy number. Furthermore we calculated the normal fuzzy probability for trigonometric fuzzy numbers driven by the above four operations([4]).

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In this paper, we define the exponential fuzzy probability using the exponential distribution and calculate the exponential fuzzy probabilities for quadratic fuzzy number and trigonometric fuzzy number. And then we calculate the exponential fuzzy probabilities for fuzzy numbers driven by the above four operations.

## 2. Preliminaries

Let $(\Omega, \mathfrak{F}, P)$ be a probability space, and $X$ be a random variable defined on it. Let $g$ be a real-valued Borel-measurable function on $\mathbb{R}$. Then $g(X)$ is also a random variable.

DEFINITION 2.1. We say that the mathematical expectation of $g(X)$ exists if

$$E[g(X)] = \int_{\Omega} g(X(\omega))\, dP(\omega) = \int_{\Omega} g(X)\, dP$$

is finite.

EXAMPLE 2.2. Let the random variable $X$ have the normal distribution given by the probability density function

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-m)^2}{2\sigma^2}}, \quad x \in \mathbb{R},$$

where $\sigma^2 > 0$ and $m \in \mathbb{R}$. Then $E[|X|^\gamma] < \infty$ for every $\gamma > 0$, and we have

$$E[X] = m \quad \text{and} \quad E[(X-m)^2] = \sigma^2.$$

The induced measure $P_X$ is called the normal distribution.

EXAMPLE 2.3. Let the random variable $X$ have the exponential distribution given by the probability density function

$$f(x) = \lambda e^{-\lambda x}$$

for $x \geq 0$ and $\lambda > 0$. Then we have

$$E[X] = \frac{1}{\lambda} \quad \text{and} \quad E[(X - \frac{1}{\lambda})^2] = \frac{1}{\lambda^2}.$$

The induced measure $P_X$ is called the exponential distribution.

A fuzzy set $A$ on $\Omega$ is called a *fuzzy event*. Let $\mu_A(\cdot)$ be the membership function of the fuzzy event $A$. Then the probability of the fuzzy event $A$ is defined by Zadeh([5], [6]) as

$$\widetilde{P}(A) = \int_{\Omega} \mu_A(\omega)\, dP(\omega), \quad \mu_A(\omega) : \Omega \to [0, 1].$$

DEFINITION 2.4. The normal fuzzy probability $\widetilde{P}(A)$ of a fuzzy set $A$ on $\mathbb{R}$ is defined by

$$\widetilde{P}(A) = \int_{\mathbb{R}} \mu_A(x)\, dP_X,$$

where $P_X$ is the normal distribution.

DEFINITION 2.5. The exponential fuzzy probability $\widetilde{P}(A)$ of a fuzzy set $A$ on $\mathbb{R}$ is defined by

$$\widetilde{P}(A) = \int_{\mathbb{R}} \mu_A(x)\, dP_X,$$

where $P_X$ is the exponential distribution.

DEFINITION 2.6. A triangular fuzzy number is a fuzzy number $A$ having membership function

$$\mu_A(x) = \begin{cases} 0, & x < a_1, \ a_3 \leq x, \\ \frac{x-a_1}{a_2-a_1}, & a_1 \leq x < a_2, \\ \frac{a_3-x}{a_3-a_2}, & a_2 \leq x < a_3. \end{cases}$$

The above triangular fuzzy number is denoted by $A = (a_1, a_2, a_3)$.

DEFINITION 2.7. The addition, subtraction, multiplication, and division of two fuzzy numbers are defined as

1. Addition $A(+)B$ :

$$\mu_{A(+)B}(z) = \sup_{z=x+y} \min\{\mu_A(x), \mu_B(y)\}, \ x \in A, y \in B.$$

2. Subtraction $A(-)B$ :

$$\mu_{A(-)B}(z) = \sup_{z=x-y} \min\{\mu_A(x), \mu_B(y)\}, \ x \in A, y \in B.$$

3. Multiplication $A(\cdot)B$ :

$$\mu_{A(\cdot)B}(z) = \sup_{z=x\cdot y} \min\{\mu_A(x), \mu_B(y)\}, \ x \in A, y \in B.$$

4. Division $A(/)B$ :

$$\mu_{A(/)B}(z) = \sup_{z=x/y} \min\{\mu_A(x), \mu_B(y)\}, \ x \in A, y \in B.$$

EXAMPLE 2.8. ([2]) For two triangular fuzzy numbers $A = (1, 2, 4)$ and $B = (2, 4, 5)$, we have

1. Addition : $A(+)B = (3, 6, 9)$.
2. Subtraction : $A(-)B = (-4, -2, 2)$.
3. Multiplication :

$$\mu_{A(\cdot)B}(x) = \begin{cases} 0, & x < 2, \ \ 20 \le x, \\ \frac{-2+\sqrt{2x}}{2}, & 2 \le x < 8, \\ \frac{7-\sqrt{9+2x}}{2}, & .8 \le x < 20. \end{cases}$$

Note that $A(\cdot)B$ is not a triangular fuzzy number.

4. Division :

$$\mu_{A(/)B}(x) = \begin{cases} 0, & x < \frac{1}{5}, \ \ 2 \le x, \\ \frac{5x-1}{x+1}, & \frac{1}{5} \le x < \frac{1}{2}, \\ \frac{-x+2}{x+1}, & \frac{1}{2} \le x < 2. \end{cases}$$

Note that $A(/)B$ is not a triangular fuzzy number.

Similar to triangular fuzzy number, the quadratic fuzzy number and the trigonometric fuzzy number are defined by quadratic curve and trigonometric curve, respectively.

DEFINITION 2.9. A quadratic fuzzy number is a fuzzy number $A$ having membership function

$$\mu_A(x) = \begin{cases} 0, & x < \alpha, \ \ \beta \le x, \\ -a(x-\alpha)(x-\beta) = -a(x-k)^2 + 1, & \alpha \le x < \beta, \end{cases}$$

where $a > 0$.

The above quadratic fuzzy number is denoted by $A = [\alpha, k, \beta]$.

THEOREM 2.10. ([3]) *For two quadratic fuzzy number $A = [x_1, k, x_2]$ and $B = [x_3, m, x_4]$, we have*

1. $A(+)B = [x_1 + x_3, k + m, x_2 + x_4]$.
2. $A(-)B = [x_1 - x_4, k - m, x_2 - x_3]$.
3. $\mu_{A(\cdot)B}(x) = 0$ *on the interval* $[x_1x_3, x_2x_4]^c$ *and* $\mu_{A(\cdot)B}(x) = 1$ *at* $x = km$. *Note that* $A(\cdot)B$ *need not to be a quadratic fuzzy number.*
4. $\mu_{A(/)B}(x) = 0$ *on the interval* $[\frac{x_1}{x_4}, \frac{k}{m}, \frac{x_2}{x_3}]^c$ *and* $\mu_{A(/)B}(x) = 1$ *at* $x = \frac{k}{m}$. *Note that* $A(/)B$ *need not to be a quadratic fuzzy number.*

EXAMPLE 2.11. ([3]) For two quadratic fuzzy numbers $A = [1, 2, 3]$ and $B = [2, 5, 8]$, we have

1. Addition : $\mu_{A(+)B}(x) = [3, 7, 11]$.
2. Subtraction : $\mu_{A(-)B}(x) = [-7, -3, 1]$.
3. Multiplication :

$$\mu_{A(\cdot)B}(x) = \begin{cases} 0, & x < 2, \ \ 24 \le x, \\ -\frac{1}{18}(6x + 43 - 11\sqrt{12x+1}), & 2 \le x < 24. \end{cases}$$

Note that $A(\cdot)B$ is not a quadratic fuzzy number.

4. Division :

$$\mu_{A(/)B}(x) = \begin{cases} 0, & x < \frac{1}{8}, \ \ \frac{3}{2} \le x, \\ \dfrac{-(8x-1)(2x-3)}{(3x+1)^2}, & \frac{1}{8} \le x < \frac{3}{2}. \end{cases}$$

Note that $A(/)B$ is not a quadratic fuzzy number.

DEFINITION 2.12. A trigonometric fuzzy number is a fuzzy number $A$ having membership function

$$\mu_A(x) = \begin{cases} 0, & x < \theta_1, \ \ \theta_3 \le x, \\ \sin(x - \theta_1), & \theta_1 \le x < \theta_3, \end{cases}$$

where $\theta_3 - \theta_1 = \pi$.

The above trigonometric fuzzy number is denoted by $A = < \theta_1, \theta_2, \theta_3 >$, where $\theta_2 = \theta_1 + \frac{\pi}{2}$. For all trigonometric fuzzy number $A = < \theta_1, \theta_2, \theta_3 >$, we define $\sin^{-1}(\cdot)$ as an inverse of $\sin(\cdot) : [0, \ \theta_2 - \theta_1] \to [0, 1]$.

THEOREM 2.13. ([4]) *For two trigonometric fuzzy numbers* $A =< c_1, c_2, c_3 >$ *and* $B =< d_1, d_2, d_3 >$, *we have*

1. $A(+)B =< c_1 + d_1, c_2 + d_2, c_3 + d_3 >$.
2. $A(-)B =< c_1 - d_3, c_2 - d_2, c_3 - d_1 >$.
3. *The membership functions of* $A(\cdot)B$ *and* $A(/)B$ *are expressed by trigonometric function.*

EXAMPLE 2.14. For two trigonometric fuzzy numbers $A = <\frac{\pi}{2}, \pi, \frac{3\pi}{2}>$ and $B = <\frac{\pi}{3}, \frac{5\pi}{6}, \frac{4\pi}{3}>$, we calculate exactly the above four operations using $\alpha$-cuts. Note that

$$\mu_A(x) = \begin{cases} 0, & x < \frac{\pi}{2},\ \frac{3}{2}\pi \le x, \\ \sin(x - \frac{\pi}{2}), & \frac{\pi}{2} \le x < \frac{3}{2}\pi, \end{cases}$$

and

$$\mu_B(x) = \begin{cases} 0, & x < \frac{\pi}{3},\ \frac{4}{3}\pi \le x, \\ \sin(x - \frac{\pi}{3}), & \frac{\pi}{3} \le x < \frac{4}{3}\pi. \end{cases}$$

Put $A_\alpha = [a_1^{(\alpha)}, a_2^{(\alpha)}]$ and $B_\alpha = [b_1^{(\alpha)}, b_2^{(\alpha)}]$. Since $\alpha = \sin(a_1^{(\alpha)} - \frac{\pi}{2})$ and $a_2^{(\alpha)} = 2\pi - a_1^{(\alpha)}$, we have $A_\alpha = [a_1^{(\alpha)}, a_2^{(\alpha)}] = [\frac{\pi}{2} + \sin^{-1}\alpha, \frac{3}{2}\pi - \sin^{-1}\alpha]$. Similarly, $B_\alpha = [b_1^{(\alpha)}, b_2^{(\alpha)}] = [\frac{\pi}{3} + \sin^{-1}\alpha, \frac{4}{3}\pi - \sin^{-1}\alpha]$.

1. Addition : By the above facts, $A_\alpha(+)B_\alpha = [a_1^{(\alpha)} + b_1^{(\alpha)}, a_2^{(\alpha)} + b_2^{(\alpha)}] = [\frac{5}{6}\pi + 2\sin^{-1}\alpha, \frac{17}{6}\pi - 2\sin^{-1}\alpha]$. Thus $\mu_{A(+)B}(x) = 0$ on the interval $[\frac{5}{6}\pi, \frac{17}{6}\pi]^c$ and $\mu_{A(+)B}(x) = 1$ at $x = \frac{11}{6}\pi$. Therefore

$$\mu_{A(+)B}(x) = \begin{cases} 0, & x < \frac{5}{6}\pi,\ \frac{17}{6}\pi \le x, \\ \sin\frac{1}{2}(x - \frac{5}{6}\pi), & \frac{5}{6}\pi \le x < \frac{17}{6}\pi, \end{cases}$$

i.e., $A(+)B = <\frac{5}{6}\pi, \frac{11}{6}\pi, \frac{17}{6}\pi>$.

2. Subtraction : Since $A_\alpha(-)B_\alpha = [a_1^{(\alpha)} - b_2^{(\alpha)},\ a_2^{(\alpha)} - b_1^{(\alpha)}] = [2\sin^{-1}\alpha - \frac{5}{6}\pi, \frac{7}{6}\pi - 2\sin^{-1}\alpha]$, we have $\mu_{A(-)B}(x) = 0$ on the interval $[-\frac{5}{6}\pi, \frac{7}{6}\pi]^c$ and $\mu_{A(-)B}(x) = 1$ at $x = \frac{\pi}{6}$. Therefore

$$\mu_{A(-)B}(x) = \begin{cases} 0, & x < -\frac{5}{6}\pi,\ \frac{7}{6}\pi \le x, \\ \sin\frac{1}{2}(x + \frac{5}{6}\pi), & -\frac{5}{6}\pi \le x < \frac{7}{6}\pi, \end{cases}$$

i.e., $A(-)B = <-\frac{5}{6}\pi, \frac{1}{6}\pi, \frac{7}{6}\pi>$.

3. Multiplication : Since

$$\begin{aligned} A_\alpha(\cdot)B_\alpha &= [a_1^{(\alpha)} \cdot b_1^{(\alpha)}, a_2^{(\alpha)} \cdot b_2^{(\alpha)}] \\ &= [\frac{\pi^2}{6} + \frac{5}{6}\pi\sin^{-1}\alpha + (\sin^{-1}\alpha)^2, \\ &\qquad 2\pi^2 - \frac{17}{6}\pi\sin^{-1}\alpha + (\sin^{-1}\alpha)^2], \end{aligned}$$

we have $\mu_{A(\cdot)B}(x) = 0$ on the interval $[\frac{\pi^2}{6}, 2\pi^2]^c$ and $\mu_{A(\cdot)B} = 1$ at $x = \frac{5}{6}\pi^2$. Therefore

$$\mu_{A(\cdot)B}(x) = \begin{cases} 0, & x < \frac{\pi^2}{6},\ \ 2\pi^2 \leq x, \\ \sin\Big(-\frac{5}{12}\pi + \sqrt{\frac{\pi^2}{144} + x}\Big), & \frac{\pi^2}{6} \leq x < 2\pi^2. \end{cases}$$

4. Division : Since

$$A_\alpha(/)B_\alpha = \left[\frac{a_1^{(\alpha)}}{b_2^{(\alpha)}}, \frac{a_2^{(\alpha)}}{b_1^{(\alpha)}}\right] = \left[\frac{2\sin^{-1}\alpha + 3\pi}{8\pi - 6\sin^{-1}\alpha}, \frac{9\pi - 6\sin^{-1}\alpha}{6\sin^{-1}\alpha + 2\pi}\right],$$

we have $\mu_{A(/)B}(x) = 0$ on the interval $[\frac{3}{8}, \frac{9}{2}]^c$ and $\mu_{A(/)B}(x) = 1$ at $x = \frac{4}{5}$. Therefore

$$\mu_{A(/)B}(x) = \begin{cases} 0, & x < \frac{3}{8},\ \ \frac{9}{2} \leq x, \\ \sin\frac{8\pi x - 3\pi}{2(1+3x)}, & \frac{3}{8} \leq x < \frac{9}{2}. \end{cases}$$

## 3. Main results

THEOREM 3.1. *The exponential fuzzy probability $\widetilde{P}(A)$ of a triangular fuzzy number $A = (a_1, a_2, a_3)$ is*

$$\widetilde{P}(A) = \frac{\lambda a_2}{(a_2 - a_1)}\Big[e^{-\lambda a_2}(-a_1 a_2 + \frac{a_2^2}{2}) - e^{-\lambda a_1}(-\frac{a_1^2}{2})\Big] + \frac{\lambda a_3}{a_3 - a_2}\Big[e^{-\lambda a_3}(\frac{a_3^2}{2}) - e^{-\lambda a_1}(a_3 a_2 - \frac{a_2^2}{2})\Big].$$

PROOF. Since

$$\mu_A(x) = \begin{cases} 0, & x < a_1, \quad a_3 \leq x, \\ \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x < a_2, \\ \frac{a_3 - x}{a_3 - a_2}, & a_2 \leq x < a_3, \end{cases}$$

we have

$$\widetilde{P}(A) = \int_{\mathbb{R}} \mu_A(x)\, dP_X = \int_{a_1}^{a_2} g_1(x) f(x)\, dx + \int_{a_2}^{a_3} g_2(x) f(x)\, dx,$$

where $g_1(x) = \frac{x-a_1}{a_2-a_1}$, $g_2(x) = \frac{a_3-x}{a_3-a_2}$ and $f(x) = \lambda e^{-\lambda x}(x \geq 0)$. Thus

$$\begin{aligned}\widetilde{P}(A) &= \frac{\lambda}{a_2 - a_1}\int_{a_1}^{a_2}(x-a_1)e^{-\lambda x}dx + \frac{\lambda}{a_3-a_2}\int_{a_2}^{a_3}(a_3-x)e^{-\lambda x}dx \\ &= \frac{\lambda a_2}{a_2-a_1}\Big[e^{-\lambda a_2}(-a_1a_2+\frac{a_2^2}{2}) - e^{-\lambda a_1}(-\frac{a_1^2}{2})\Big] \\ &\quad + \frac{\lambda a_3}{a_3-a_2}\Big[e^{-\lambda a_3}(\frac{a_3^2}{2}) - e^{-\lambda a_1}(a_3a_2 - \frac{a_2^2}{2})\Big].\end{aligned}$$

□

EXAMPLE 3.2. Let $A = (1,4,6)$ be a triangular fuzzy number. Then the exponential fuzzy probability with respect to $\lambda = 2$ is

$$\begin{aligned}\widetilde{P}(A) &= \int_1^4 \frac{x-1}{3}\cdot 2e^{-2x}dx + \int_4^6 \frac{6-x}{2}\cdot 2e^{-2x}dx \\ &= 0.0225.\end{aligned}$$

We derive the explicit formula of the exponential fuzzy probability for quadratic fuzzy number and trigonometric fuzzy number.

THEOREM 3.3. *The exponential fuzzy probability $\widetilde{P}(A)$ for quadratic fuzzy number $A = [\alpha, k, \beta]$ is*

$$\begin{aligned}\widetilde{P}(A) &= -e^{-\lambda\beta}\Big(\frac{2a+b\lambda+c\lambda^2}{\lambda^2} + \frac{2a+b\lambda}{\lambda}\beta + a\beta^2\Big) \\ &\quad + e^{-\lambda\alpha}\Big(\frac{2a+b\lambda+c\lambda^2}{\lambda^2} + \frac{2a+b\lambda}{\lambda}\alpha + a\alpha^2\Big),\end{aligned}$$

*where the membership function of $A$ is $-d(x-\alpha)(\alpha-\beta) = ax^2+bx+c$.*

PROOF. Since

$$\mu_A(x) = \begin{cases} 0, & x<\alpha, \quad \beta \leq x, \\ ax^2+bx+c = -d(x-k)^2+1, & \alpha \leq x < \beta, \end{cases}$$

we have

$$\begin{aligned}\widetilde{P}(A) &= \int_\alpha^\beta (ax^2+bx+c)\lambda e^{-\lambda x}dx \\ &= \Big[-e^{-\lambda x}(\frac{2a+b\lambda+c\lambda^2}{\lambda^2} + \frac{2a+b\lambda}{\lambda}x + ax^2)\Big]_\alpha^\beta \\ &= -e^{-\lambda\beta}\Big(\frac{2a+b\lambda+c\lambda^2}{\lambda^2} + \frac{2a+b\lambda}{\lambda}\beta + a\beta^2\Big) \\ &\quad + e^{-\lambda\alpha}\Big(\frac{2a+b\lambda+c\lambda^2}{\lambda^2} + \frac{2a+b\lambda}{\lambda}\alpha + a\alpha^2\Big).\end{aligned}$$

EXAMPLE 3.4. Let $A = [1, 2, 3]$ be a quadratic number. Then the exponential fuzzy probability of $A$ with respect to $\lambda = 2$ is

$$\begin{aligned}\widetilde{P}(A) &= \int_1^3 (-x^2 + 4x - 3)2 \cdot e^{-2x} dx \\ &= 0.0714.\end{aligned}$$

THEOREM 3.5. *The exponential fuzzy probability $\widetilde{P}(A)$ of a trigonometric fuzzy number $A =< \theta_1, \theta_2, \theta_3 >$ is*

$$\widetilde{P}(A) = -\frac{\lambda}{\lambda^2+1}(e^{-\lambda\theta_3}\cos(\theta_3 - \theta_1) + \lambda e^{-\lambda\theta_3}\sin(\theta_3 - \theta_1) - e^{-\lambda\theta_1}).$$

PROOF.

$$\begin{aligned}\widetilde{P}(A) &= \int_{\theta_1}^{\theta_3} \sin(x - \theta_1)\, \lambda\, e^{-\lambda x} dx \\ &= -\lambda\Big[\frac{e^{-\lambda x}\cos(x-\theta_1)}{\lambda^2+1} + \frac{\lambda e^{-\lambda x}\sin(x-\theta_1)}{\lambda^2+1}\Big]_{\theta_1}^{\theta_3} \\ &= -\frac{\lambda}{\lambda^2+1}(e^{-\lambda\theta_3}\cos(\theta_3 - \theta_1) + \lambda e^{-\lambda\theta_3}\sin(\theta_3 - \theta_1) - e^{-\lambda\theta_1}).\end{aligned}$$

□

EXAMPLE 3.6. Let $A =< 0, \frac{\pi}{2}, \pi >$ be a trigonometric fuzzy number. Then the exponential fuzzy probability of $A$ with respect to $\lambda = 2$ is

$$\begin{aligned}\widetilde{P}(A) &= \int_0^\pi \sin x\ 2\, e^{-2x} dx \\ &= -\frac{2}{5}(e^{-2\pi}\cos\pi + 2\, e^{-2\pi}\sin\pi - e^0) \\ &= 0.4007.\end{aligned}$$

It is well known that the results of addition and subtraction of two triangular fuzzy numbers are triangular fuzzy numbers. Thus we can calculate the exponential fuzzy probability using Theorem 3.1. The other cases can be calculated by definition.

EXAMPLE 3.7. The exponential fuzzy probabilities for the fuzzy numbers in Example 2.8 with respect to $\lambda = 2$ are

1. Multiplication

$$\begin{aligned}\widetilde{P} &= \int_2^8 \frac{-2+\sqrt{2x}}{2} \cdot 2e^{-2x}dx + \int_8^{20} \frac{7-\sqrt{9+2x}}{2} \cdot 2e^{-2x}dx \\ &= 0.0021.\end{aligned}$$

2. Division

$$\begin{aligned}\widetilde{P} &= \int_{\frac{1}{5}}^{\frac{1}{2}} \frac{5x-1}{x+1} \cdot 2\, e^{-2x}dx + \int_{\frac{1}{2}}^{2} \frac{-x+2}{x+1} \cdot 2\, e^{-2x}dx \\ &= 0.3604.\end{aligned}$$

Similarly, we can calculate the exponential fuzzy probabilities for the results of multiplication and division of two quadratic fuzzy numbers by definition.

EXAMPLE 3.8. The exponential fuzzy probabilities for the fuzzy numbers in Example 2.11 with respect to $\lambda = 2$ are

1. Multiplication

$$\begin{aligned}\widetilde{P} &= \int_2^{24} \frac{-(6x+43)+11\sqrt{12x+1}}{18}\, 2\, e^{-2x}dx \\ &= 0.0906.\end{aligned}$$

2. Division

$$\begin{aligned}\widetilde{P} &= \int_{\frac{1}{8}}^{\frac{3}{2}} \frac{-(8x-1)(2x-3)}{(3x+1)^2}\, 2\, e^{-2x}dx \\ &= 0.5062.\end{aligned}$$

Similarly, we can calculate the exponential fuzzy probabilities for the results of multiplication and division of two trigonometric fuzzy numbers by definition.

EXAMPLE 3.9. The exponential fuzzy probabilities of the fuzzy numbers in Example 2.14 with respect to $\lambda = 2$ are

1. Multiplication

$$\widetilde{P} = \int_{\frac{\pi^2}{6}}^{2\pi^2} \sin\Big(-\frac{5}{12}\pi + \sqrt{\frac{\pi^2}{144} + x}\Big)\ 2\, e^{-2x} dx = 9.5105 \times 10^{-10}.$$

2. Division

$$\widetilde{P} = \int_{\frac{3}{8}}^{\frac{9}{2}} \sin\Big(\frac{8\pi x - 3\pi}{2(1+3x)}\Big)\ 2\, e^{-2x} dx = 0.3184.$$

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Department of Mathematics and Information
Cheju National University
Jeju 690-756, Korea
*E-mail*: yunys@cheju.ac.kr
sjc0101@hanmail.net
ryusu81@cheju.ac.kr