Distributor's Reliable Price and Inventory Policy for Decaying Items under Permissible Delay in Payments and Freight Discount Cost in a Supply Chain

- 신용거래와 수송비의 할인을 허용하는 공급사슬에서 퇴화성제품에 대한 신뢰성있는 재고 및 가격정책 -

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Abstract

본 논문은 제품공급자, 중간분배자 그리고 고객으로 구성된 2 단계 공급사슬에서 공급자가 수요 증대를 목적으로 중간분배자에게 일정기간 동안 제품대금에 대한 지불연기를 허용하는 상황을 고려하여 중간분배자의 신뢰성있는 판매가격 및 재고보충정책을 결정하는 문제를 다루었다. 문제 분석을 위하여 고려하는 제품은 시간에 따라 일정률로 퇴화한다는 가정과 함께 수송량에 따라 할인되는 수송비를 고려하여 모형을 수립하였고, 중간분배자의 수익 증대를 위한 경제적 판매가격 및 재고보충정책 결정을 위한 해법을 개발하였다.

keywords: Supply Chain, Reliable Price and Inventory Policy, Delay in Payments, Freight Discount Cost, Decaying Items

1. Introduction

An effective supply chain network requires a cooperative relationship between the vendor/supplier and the buyer/distributor. The cooperation include the sharing of information, resources and profit or cost saving. One of the most common strategies is to set up a pricing policy to attract both the vendor and the buyer. Other strategies may include better credit terms and other strategic partnership advantages. In today's business transactions, it is more and more common to see that the buyers are allowed some grace period before they settle the account with

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the vendor. As implicitly stated by Mehta[10], a major reason for the vendor to offer a credit period to the buyers is to stimulate the demand for the product that he produces, and the vendor usually expects that the increased sales volume can compensate the capital losses incurred during the credit period. Also, the buyers who are allowed a period to pay back for the products bought without paying any interest, can earn interest on the sales of the inventory depending on the length of that payments period.

In this regard, a number of research papers appeared which deal with the inventory problem under a fixed credit period. Chung[4] and Goyal[7] analyzed the effects of trade credit on the optimal inventory policy. The common assumption held by the above authors is that the demand is a known constant and, thus, they disregarded the effects of credit period on the quantity demanded. Fewings[5] stated that the advantage of trade credit for the supplier is substantial in terms of influence on the distributor's purchasing and marketing decisions. The supplier usually expects that the profit increases due to rising sales volume can compensate the capital losses incurred during the credit period. The positive effects of credit period on the product demand can be integrated into the inventory model through the consideration of retailing situations where the demand rate is a function of the selling price the distributor sets for the product.. The availability of the credit period from the supplier enables the distributor to choose the selling price from a wider range of option. Since the distributor's replenishment cycle time is affected by the demand rate of the product, the problems of determining the distributor's selling price and the lot-size are interdependent and must be solved simultaneously(we will call the RPLS problem). Abad[1] dealt with the RPLS problem assuming that the supplier offers the quantity discounts and demand for the product is a decreasing function of price. Also, Kim *et al.*[8] introduced the RPLS problem under the condition of permissible delay in payments.

Note that all these RPLS problems assumed that the ordering cost contains a fixed cost alone. But, in many practical supply chain, the order may be delivered in unit loads, i.e., trucks, containers, pallets, boxes, etc. and a quantity discount may occur in terms of the number of unit loads due to the economies of scale. In this regard, Lee[9] studied the EOQ model with set-up cost including a fixed cost and freight cost where the freight cost has a quantity discount. Also, Shinn *et al.*[12] analyzed the RPLS problem when the supplier offers a certain credit period and the ordering cost of the retailer contains not only a fixed cost but also a freight cost which is a function of the lot-size.

All the research works mentioned above implicitly assumed that inventory is depleted by customer's demand alone. This assumption is quite valid for nonperishable or nondecaying inventory items. However, there are numerous types of inventory whose utility does not remain constant over time. In this case, inventory is depleted not only by demand but also by decay. Ghare and Schrader[6], assuming exponential deterioration of the inventory in the face of the constant demand, derived a revised form of the economic order quantity. Cohen[2] analyzed the RPLS problem for an exponentially decaying product. Recently, Chu *et al.*[3] examined the economic ordering policy of decaying product under the condition of permissible delay in payments.

This paper deals with the RPLS problem for an exponentially decaying product when the supplier permits delay in payments for an order of the product whose demand rate is represented by a constant price elasticity function of selling price. It is also assumed that the ordering cost of the distributor contains not only a fixed cost but also a freight cost, which is a function of the lot-size. In the next two sections, we formulate an mathematical model. A solution algorithm is developed in section 4 based on the properties of an optimal solution. A numerical example is provided in section 5, which is followed by concluding remarks.

2. Assumptions and Notations

The mathematical model is developed with the following assumptions and notations:

(1) Replenishments are instantaneous with a known and constant lead time.
(2) No shortages are allowed.
(3) The inventory system involves only one item.
(4) The supplier proposes a certain credit period and the purchasing cost of the products sold during the credit period is deposited in an interest bearing account with rate I. At the end of the period, the account is settled and the distributor starts paying the capital opportunity cost for the items in stock with rate $R(R \geq I)$.
(5) The distributor pays the freight cost for the transportation of the quantity purchased where the freight cost has a quantity discount.
(6) Inventory is depleted not only by demand but also by decay and decay follows an exponential distribution with parameter λ.

P : distributor's selling price.
C : unit purchasing cost.
K : scal;ing factor(>0)
β : index of price elasticity

D : annual demand rate as a function of the distributor's selling price(P); $D = KP^{-\beta}$.
Q : order size.
T : replenishment cycle time.
tc : credit period set by the supplier.
λ : a positive number representing the inventory decaying rate.
$q(t)$: inventory level at time t.
H : inventory carrying cost, excluding the capital opportunity cost.
R : capital opportunity cost(as a percentage).
I : earned interest rate(as a percentage).
S : fixed ordering cost.
N_j : jth freight cost break quantity, $j = 1, 2, \cdots, n$, where $N_0 < N_1 < \cdots < N_n < N_{n+1}$ with $N_0 = 0$ and $N_{n+1} = \infty$.
F_j : freight cost for Q, $N_{j-1} < Q \le N_j$, where $F_{j-1} < F_j$ and $F_{j-1}/N_{j-1} > F_j/N_j$, $j = 1, 2, \cdots, n$.

Note that the inequalities $F_{j-1} < F_j$ and $F_{j-1}/N_{j-1} > F_j/N_j$ are necessary to have some quantity discount in the freight cost for changing the order size from N_{j-1} to N_j. Thus the cost for setting an order becomes $S + F_j$ for $N_{j-1} < Q \le N_j$.

3. Development of the Mathematical Model

For the case of exponential deteriorate, the rate at which inventory deteriorates will be proportional to on hand inventory, $q(t)$. Thus, the depletion rate of inventory at any time t is

$$\frac{dq(t)}{dt} = -\lambda q(t) - D. \tag{1}$$

Observing that equation (1) is a first order linear differential equation, its solution is

$$q(t) = q(0)e^{-\lambda t} - \frac{D}{\lambda}(1 - e^{-\lambda t}). \tag{2}$$

Equation (2) gives the inventory level at time t representing the combined effects of demand usage and exponential decay. Now, we determine the inventory loss due to deterioration. Let $q^0(t)$ be the inventory level at time t where there were no deterioration. Then, the inventory loss due to deterioration becomes

$$q^0(t) - q(t) = (q(0) - Dt) - \left(q(0)e^{-\lambda t} - \frac{D}{\lambda}(1 - e^{-\lambda t})\right) \tag{3}$$

$$= q(t)(e^{\lambda t} - 1) - Dt + \frac{D}{\lambda}(e^{\lambda t} - 1). \tag{4}$$

Therefore, the quantity ordered per cycle becomes

$$Q = (q^0(T) - q(T)) + DT. \tag{5}$$

Note that due to the inventory carrying costs, it is clearly better off to have the inventory level reach zero just before reordering, i.e., $q(T) = 0$. With $q(T) = 0$, we have

$$Q = \frac{D}{\lambda}(e^{\lambda T} - 1). \tag{6}$$

Also, from the condition of $q(0) = Q$, the inventory level at time t is

$$q(t) = \frac{D}{\lambda}(e^{\lambda(T-t)} - 1), \quad 0 \le t \le T. \tag{7}$$

Now, we formulate the annual net profit $\Pi(P, T)$. For the formulation of $\Pi(P, T)$, let us consider the freight cost discount schedules, $N_{j-1} < Q \le N_j$ for $j = 1, 2, \cdots, n$. Using equation (6), the inequalities can be reduced to

$$L_{j-1} < T \le L_j, \quad j = 1, 2, \cdots, n \text{ where } L_j = \frac{1}{\lambda} \ln\left(\frac{\lambda}{D} N_j + 1\right). \tag{8}$$

Then, as stated by Shinn[11], the distributor's annual net profit consists of the following five elements.

(1) Annual sales revenue = DP.

(2) Annual purchasing cost = $\dfrac{CQ}{T} = \dfrac{CD(e^{\lambda T} - 1)}{\lambda T}$.

(3) Annual ordering cost = $\dfrac{S + F_j}{T}$ for $L_{j-1} < T \le L_j$, $j = 1, 2, \cdots, n$.

(4) Annual inventory carrying cost = $\frac{H}{T}\int_0^T q(t)dt = \frac{HD(e^{\lambda T}-\lambda T-1)}{\lambda^2 T}$.

(5) Annual capital opportunity cost:

(i) Case 1($tc \le T$): As product are sold, the purchasing cost of the products is used to earn interest with annual rate I during the credit period tc. And the average number of products in stock earning interest during time (0, tc) is $\frac{Dtc}{2}$ and the interest earned per order becomes $CI\left(\frac{Dtc}{2}\right)tc$. When the account is settled, the products still in stock have to be financed with annual rate R. Since the average number of products during time (tc, T) becomes $\frac{1}{(T-tc)}\int_{tc}^T q(t)dt$, the interest payable per order can be expressed as $CR\int_{tc}^T q(t)dt$. Therefore,

$$\text{the annual capital opportunity cost} = \frac{CR\int_{tc}^T q(t)dt - \frac{CIDtc^2}{2}}{T}$$

$$= \frac{1}{\lambda^2 T}CRD(e^{\lambda(T-tc)}-\lambda(T-tc)-1)-\frac{1}{2T}CIDtc^2.$$

(ii) Case 2($tc > T$): For the case of $tc > T$, all the purchasing cost of the products is used to earn interest with annual rate I during the credit period tc. The average number of products in stock earning interest during time (0, T) and (T, tc) become $\frac{DT}{2}$ and DT, respectively. Therefore,

$$\text{the annual capital opportunity cost} = -\frac{CI\left(\frac{DT}{2}T+DT(tc-T)\right)}{T}$$

$$= \frac{CIDT}{2} - CIDtc.$$

Therefore, the distributor's annual net profit can be expressed as

$\Pi(P, T)$ = Sales revenue - Purchasing cost - Ordering cost - Inventory carrying cost - Capital opportunity cost.

And depending on the relative size of tc to T, $\Pi(P, T)$ has two different expressions as follows:

1. Case 1($tc \le T$)

$$\Pi_{1,j}(P,T) = DP - \frac{DC(e^{\lambda T}-1)}{\lambda T} - \frac{S+F_j}{T} - \frac{HD(e^{\lambda T}-\lambda T-1)}{\lambda^2 T} - \left(\frac{CRD(e^{\lambda(T-tc)}-\lambda(T-tc)-1)}{\lambda^2 T} - \frac{CIDtc^2}{2T}\right),\ L_{j-1} < T \le L_j,\ j=1,2,\cdots,n. \quad (9)$$

2. Case 2($tc > T$)

$$\Pi_{2,j}(P,T) = DP - \frac{DC(e^{\lambda T}-1)}{\lambda T} - \frac{S+F_j}{T} - \frac{HD(e^{\lambda T}-\lambda T-1)}{\lambda^2 T} - \left(\frac{CIDT}{2} - CIDtc\right),\ L_{j-1} < T \le L_j,\ j=1,2,\cdots,n. \quad (10)$$

4. Determination of Optimal Policy

The problem is to find the distributor's economic selling price P^* and replenishment cycle time T^* which maximizes $\Pi(P,T)$. Once P^*, T^* are found, the distributor's economic lot size Q^* can be obtained by equation (6). Although the objective function can be differentiated, the resulting equation is mathematically intractable; that is, it is impossible to find the solution in explicit form. Thus the model will be solved approximately by using a truncated Taylor series expansion for the exponential term, i.e.,

$$e^{\lambda T} \approx 1 + \lambda T + \frac{1}{2}\lambda^2 T^2, \quad (11)$$

which is a valid approximation for smaller values of λT. With the above approximation, the total annual variable cost can be rewritten as

$$\Pi_{1,j}(P,T) = D(P - C(1-Rtc)) - \frac{1}{T}\left(S + F_j + \frac{DC(R-I)tc^2}{2}\right) - \frac{DT}{2}(H + C\lambda + CR), \quad (12)$$

$$\Pi_{2,j}(P,T) = D(P - C(1-Itc)) - \frac{S+F_j}{T} - \frac{DT}{2}(H + C\lambda + CI). \quad (13)$$

Note that equation (11) is exact when $\lambda = 0$ so that equation (9) and (10) reduce to the exact formulas equation (12) and (13) for nondeteriorating product as stated by Shinn *et al.*[12]. For the fixed P, $\Pi_{i,j}(P,T)$ is a convex function for every i and j. And so, there exists a unique value $T_{i,j}$, which maximizes $\Pi_{i,j}(P,T)$ and they are:

$$T_{1,j}=\sqrt{\frac{2(S+F_j)+C(R-I)Dtc^2}{D(H+C\lambda+CR)}}\ , \tag{14}$$

$$T_{2,j}=\sqrt{\frac{2(S+F_j)}{D(H+C\lambda+CI)}}\ . \tag{15}$$

And also, as proved by Shinn[11], for $P=P^o$ fixed, only the elements in set $B=\{L_j,\ T_{i,j}(P^o)\ \text{for}\ i=1,2\ \text{and}\ j=1,2,\cdots,n\}$ become candidates for an economic replenishment cycle time $T^*(P^o)$ where L_j, $T_{i,j}(P^o)$ are obtained by substituting P with P^o in equations (8), (14) and (15). Noting that some elements of B can be dropped from consideration in search of $T^*(P)$, we formulate the following conditions $T_{i,j}(P)$ and L_j must satisfy to become a candidate of $T^*(P)$.

(C-1): The conditions for $T_{i,j}(P)$ to be a candidate of $T^*(P)$.

$T_{1,j}(P)\ge tc$ and $L_{j-1}<T_{1,j}(P)\le L_j$ for Case 1 (16)

$T_{2,j}(P)<tc$ and $L_{j-1}<T_{2,j}(P)\le L_j$ for Case 2 (17)

(C-2): The conditions for L_j to be a candidate of $T^*(P)$.

$L_j\ge tc$ and $L_j<T_{1,j}(P)$ for Case 1 (18)

$L_j<tc$ and $L_j<T_{2,j}(P)$ for Case 2 (19)

For $T_{1,j}(P)$ to be a candidate of $T^*(P)$ in Case 1, $T_{1,j}(P)$ must lie on $(L_{j-1}, L_j]$ and also $T_{1,j}(P)\ge tc$ must hold. For L_j to be a candidate of $T^*(P)$ in Case 1, $\Pi_{1,j}(P,T)$ must be increasing at L_j. In other words, the conditions $L_j<T_{i,j}(P)$ and $L_j\ge tc$ must be satisfied. The conditions for Case 2, equation (17) and (19), are justified in similar way. Now, let us consider $T_{1,j}(P)\ge tc$ in equation (16). Since the demand rate D is also a function of P, the inequality can be rewritten as

$$T_{1,j}(P)=\sqrt{\frac{2(S+F_j)+DC(R-I)tc^2}{D(H+C\lambda+CR)}}\ge tc\ . \tag{20}$$

Rearranging equation (20),

$$P\ge\left[\frac{Ktc^2(H+C\lambda+CI)}{2(S+F_j)}\right]^{\frac{1}{\beta}}\ . \tag{21}$$

Let

$$P1_j = \left[\frac{Ktc^2 (H + C\lambda + CI)}{2(S + F_j)} \right]^{\frac{1}{\beta}} . \tag{22}$$

It is self evident that for any $P \geq P1_j$, the inequality $T_{1,j}(P) \geq tc$ holds. Similarly, $L_j \geq tc$ in equation (18) can be rewritten as

$$P \geq P2_j \quad \text{where } P2_j = \left[\frac{K}{\lambda N_j} (e^{\lambda tc} - 1) \right]^{\frac{1}{\beta}} . \tag{23}$$

With the similar procedure, we can find the price range which satisfy the above inequalities. Let $P3_j$ be the minimum price value which satisfies the inequality $L_{j-1} < T_{1,j}(P)$ and $P4_j$ be the maximum price value which satisfies the inequality $T_{1,j}(P) \leq L_j$. Also, Let $P5_j$ be the minimum price value which satisfies the inequality $L_{j-1} < T_{2,j}(P)$ and $P6_j$ be the maximum price value which satisfies the inequality $T_{2,j}(P) \leq L_j$. We conclude that $T_{1,j}(P)$ determined with P value which satisfies $P \geq P1_j$ and $P3_j < P \leq P4_j$, can be a candidate of $T^*(P)$. In other words, $T_{1,j}(P)$ can be a candidate of $T^*(P)$ only if P is an element of price interval $PIT_j = \{P | P3_j < P \leq P4_j \text{ and } P \geq P1_j\}$. Utilizing the above price ranges, we find the following price intervals which correspond to conditions (C-1) and (C-2).

(PI-1): Price interval on which $T_{i,j}(P)$ becomes a candidate for $T^*(P)$.

$$PIT_j = \{P | P3_j < P \leq P4_j \text{ and } P \geq P1_j\} \quad \text{for Case 1} \tag{24}$$

$$PIT_j = \{P | P5_j < P \leq P6_j \text{ and } P < P1_j\} \quad \text{for Case 2} \tag{25}$$

(PI-2): Price interval on which L_j becomes a candidate for $T^*(P)$.

$$PIL_j = \{P | P > P4_j \text{ and } P \geq P2_j\} \quad \text{for Case 1} \tag{26}$$

$$PIL_j = \{P | P > P6_j \text{ and } P < P2_j\} \quad \text{for Case 2} \tag{27}$$

The price intervals we present have a significant role in solving this model. We consider (PI-1) for example. If $P \in PIT_j$, $T_{i,j}(P)$ satisfies condition (C-1) and becomes a candidate for $T^*(P)$. Substituting T with $T_{i,j}(P)$ in $\Pi_{i,j}(P, T)$, we have a problem of maximizing $\Pi_{i,j}(P, T_{i,j}(P))$ which is a single variable function of P. Let $\Pi^1_{i,j}(P)$ = $\Pi_{i,j}(P, T_{i,j}(P))$, $i = 1, 2$ and $j = 1, 2, \cdots, n$. Note that $\Pi^1_{i,j}(P)$ is valid only on the interval $P \in PIT_j$. Similarly, if $P \in PIL_j$, then L_j satisfies condition (C-2). Substituting T with L_j in $\Pi_{i,j}(P, T)$, we also have a single variable function $\Pi^2_{i,j}(P)$ = $\Pi_{i,j}(P, L_j)$, $i = 1, 2$ and $j = 1, 2, \cdots, n$. Altogether we have at most 4n number of single variable functions in the form of $\Pi^1_{i,j}(P)$ and $\Pi^2_{i,j}(P)$. And the distributor's economic price and replenishment cycle time (P^*, T^*) which maximizes $\Pi(P, T)$ is found by searching over $\Pi^1_{i,j}(P)$ and $\Pi^2_{i,j}(P)$, and

$$\underset{P,T}{Max}\ \Pi(P, T) = \text{Max}\left\{ \underset{\substack{P\in PIT_j \\ i,j}}{Max}\ \Pi^1_{i,j}(P),\ \underset{\substack{P\in PIL_j \\ i,j}}{Max}\ \Pi^2_{i,j}(P),\ i = 1, 2 \text{ and } j = 1, 2\ \cdots,\ n \right\}. \quad (28)$$

Based on the above properties, we develop the following solution procedure to determine the economic selling price and replenishment cycle time for distributor.

Solution Algorithm

Step 1: This step identifies all the candidate values T_o of T satisfying $T_o \geq tc$(for Case 1). For each T_o, its optimal value $P_{1,j}$ is determined from the corresponding price interval.

1.1. Determine $P_{1,j}$ which maximizes $\Pi^1_{1,j}(P)$ among the following price intervals: $P \in PIT_j$ with $T_o = T_{1,j}(P)$, $j = 1, 2, \cdots, n$.

1.2. Determine $P_{1,j}$ which maximizes $\Pi^2_{1,j}(P)$ among the following price intervals: $P \in PIL_j$ with $T_o = L_j$, $j = 1, 2, \cdots, n$.

Step 2: This step identifies all the candidate values T_o of T satisfying $T_o < tc$(for Case 2). For each T_o, its optimal value $P_{2,j}$ is determined from the corresponding price interval.

2.1. Determine $P_{2,j}$ which maximizes $\Pi^1_{2,j}(P)$ among the following price intervals: $P \in PIT_j$ with $T_o = T_{2,j}(P)$, $j = 1, 2, \cdots, n$.

2.2. Determine $P_{2,j}$ which maximizes $\Pi^2_{2,j}(P)$ among the following price intervals: $P \in PIL_j$ with $T_o = L_j$, $j = 1, 2, \cdots, n$.

Step 3: Select the distributor's economic selling price(P^*) and replenishment cycle time(T^*) which gives the maximum annual net profit among those obtained in the previous steps.

5. Numerical Example and Sensitivity Analysis

To illustrate the solution algorithm, the following problem is considered.

S = \$ 50, K = 2.5 * 100,000, $\beta = 2.5$, C = \$ 3, h = \$ 0.1, R = 0.15(= 15%), I = 0.1(= 10%), tc = 0.3, $N_j = j\times500$, $j = 1, 2, \cdots, 10$ and $F_j = 10\times j\times(1-0.02\times(j-1))$, $j = 1, 2, \cdots, 10$.

5.1. Numerical Example

The solution procedure with $\lambda = 0.01$ generates the distributor's economic selling price(P^*) and replenishment cycle time(T^*), and these results are presented in Table 1.

<Table 1> Results of numerical example with $\lambda = 0.01$

Case	Distributor's Economic Policy	
Distributor's economic policy for Case 1	Selling price	4.98
	Replenishment cycle time	0.2999
	Distributor's annual net profit	$8797.3
Distributor's economic policy for Case 2	Selling price	4.94
	Replenishment cycle time	0.2652
	Distributor's annual net profit	$8831.4
Distributor's economic policy for both cases	Selling price(P^*)	4.94
	Replenishment cycle time(T^*)	0.2652
	Distributor's annual net profit $\Pi(P^*, T^*)$	$8831.4

5.2. Sensitivity Analysis

An interesting questions are how much effects the length of credit period and the inventory decaying rate have on the selling price, the replenishment cycle time and the distributor's annual net profit. Since the problem structure of equation (9) and (10) does not permit sensitivity analysis, the same example problems are solved to answer the above question. Six levels of λ are adopted, λ = 0.005, 0.01, 0.05, 0.1, 0.2. 0.3. For each level of λ, five levels of tc, ranging from 0.1 to 0.5 with an increment of 0.1, are tested. The results are shown in Table 2 and the following observations can be made which are consistent with our expectation.

(i) As tc increases, P^* decreases while $\Pi(P, T)$ increases.

(ii) As λ increases, P^* increases and T^* decreases while $\Pi(P, T)$ decreases.

6. Conclusion

This paper dealt with the distributor's economic selling price and replenishment cycle time determination problem for an exponentially decaying product when the supplier offers a certain fixed credit period. Recognizing that a major reason for the supplier to offer a credit period to the buyers is to stimulate the demand of the product, we expressed the customer's demand rate of the product with a

<Table 2> Sensitivity analysis with various values of λ and tc

	tc = 0.1	tc = 0.2	tc = 0.3	tc = 0.4	tc = 0.5
$\lambda=0.005$	$P^*=5.04$ $T^*=0.2260$ $\Pi(P^*,T^*)=8580.6$	$P^*=4.98$ $T^*=0.2401$ $\Pi(P^*,T^*)=8737.8$	$P^*=4.94$ $T^*=0.2700$ $\Pi(P^*,T^*)=8840.6$	$P^*=4.89$ $T^*=0.2665$ $\Pi(P^*,T^*)=8980.5$	$P^*=4.84$ $T^*=0.2629$ $\Pi(P^*,T^*)=9124.0$
$\lambda=0.01$	$P^*=5.04$ $T^*=0.2230$ $\Pi(P^*,T^*)=8573.2$	$P^*=4.99$ $T^*=0.2372$ $\Pi(P^*,T^*)=8729.8$	$P^*=4.94$ $T^*=0.2652$ $\Pi(P^*,T^*)=8831.4$	$P^*=4.89$ $T^*=0.2620$ $\Pi(P^*,T^*)=8971.1$	$P^*=4.84$ $T^*=0.2586$ $\Pi(P^*,T^*)=9114.5$
$\lambda=0.05$	$P^*=5.05$ $T^*=0.2035$ $\Pi(P^*,T^*)=8517.3$	$P^*=5.00$ $T^*=0.2165$ $\Pi(P^*,T^*)=8668.8$	$P^*=4.95$ $T^*=0.2181$ $\Pi(P^*,T^*)=8805.1$	$P^*=4.90$ $T^*=0.2153$ $\Pi(P^*,T^*)=8944.5$	$P^*=4.84$ $T^*=0.2122$ $\Pi(P^*,T^*)=9087.6$
$\lambda=0.10$	$P^*=5.06$ $T^*=0.1853$ $\Pi(P^*,T^*)=8454.1$	$P^*=5.01$ $T^*=0.1965$ $\Pi(P^*,T^*)=8599.8$	$P^*=4.96$ $T^*=0.1940$ $\Pi(P^*,T^*)=8734.7$	$P^*=4.91$ $T^*=0.1915$ $\Pi(P^*,T^*)=8873.2$	$P^*=4.86$ $T^*=0.1890$ $\Pi(P^*,T^*)=9015.3$
$\lambda=0.20$	$P^*=5.08$ $T^*=0.1601$ $\Pi(P^*,T^*)=8343.0$	$P^*=5.04$ $T^*=0.1653$ $\Pi(P^*,T^*)=8481.0$	$P^*=4.98$ $T^*=0.1631$ $\Pi(P^*,T^*)=8614.3$	$P^*=4.93$ $T^*=0.1611$ $\Pi(P^*,T^*)=8751.2$	$P^*=4.88$ $T^*=0.1590$ $\Pi(P^*,T^*)=8891.7$
$\lambda=0.30$	$P^*=5.11$ $T^*=0.1434$ $\Pi(P^*,T^*)=8246.3$	$P^*=5.06$ $T^*=0.1458$ $\Pi(P^*,T^*)=8379.5$	$P^*=5.01$ $T^*=0.1440$ $\Pi(P^*,T^*)=8511.5$	$P^*=4.95$ $T^*=0.1420$ $\Pi(P^*,T^*)=8647.1$	$P^*=4.90$ $T^*=0.1402$ $\Pi(P^*,T^*)=8786.2$

constant price elasticity function. The function value depends on the distributor's selling price which in turn is affected by the length of credit period. For the analysis, we also assumed that the ordering cost consists of a fixed set-up cost and a freight cost, where the freight cost has a quantity discount offered due to the economies of scale.

For a distributor who benefits from the supplier's offer of permissible delay in payments, it is not uncommon that he lowers the selling price to a certain degree expecting that he can make more profit by stimulating the customer's demand. The ordering cost sometimes depends upon the ordering quantity, owing to discounts allowed by a shipping company for large order. In this regard, we think that the model presented in this paper may be more realistic for some real world problems. Sensitivity analysis with an example problem generated the results which are consistent with our expectation.

7. References

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