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모폴로지 연산과 가우시안 혼합 모형에 기반한 컬러 영상 분할

(Color Image Segmentation Based on Morphological Operation and a Gaussian Mixture Model)

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요 약

본 논문에서는 수학적 모폴로지 연산과 가우시안 혼합 모형에 기초한 새로운 칼라 영상 분할 알고리즘을 제안한다. 우리는 혼합 모형에서 구성 성분의 수를 결정하고, 각 구성 성분의 중심값을 계산하는데 모폴로지의 연산과 라벨링 연산을 이용한다. 그리고 칼라 특징 벡터의 확률 모형으로 가우시안 혼합 모형을 사용하고, 이들의 모수 값들을 추정하는데 결정적 어닐링 EM 알고리즘을 사용한다. 최종적으로 혼합 모형으로부터 계산된 사후 확률을 이용하여 칼라 영상을 분할한다. 실험 결과를 통하여 모폴로지 연산이 혼합모형의 수를 자동으로 결정하고 각 성분의 모드를 계산하는데 아주 효율적인 방법임을 보였고, 또한 결정적 어닐링 EM 알고리즘에 의하여 추정된 가우시안 혼합 모형을 사용하여 계산된 사후 확률에 의한 영상 분할 방법이 기존의 분할 알고리즘보다 정확한 분할 방법임을 보였다.

Abstract

In this paper, we present a new segmentation algorithm for color images based on mathematical morphology and a Gaussian mixture model(GMM). We use the morphological operations to determine the number of components in a mixture model and to detect their modes of each mixture component. Next, we have adopted the GMM to represent the probability distribution of color feature vectors and used the deterministic annealing expectation maximization (DAEM) algorithm to estimate the parameters of the GMM that represents the multi-colored objects statistically. Finally, we segment the color image by using posterior probability of each pixel computed from the GMM. The experimental results show that the morphological operation is efficient to determine a number of components and initial modes of each component in the mixture model. And also it shows that the proposed DAEM provides a global optimal solution for the parameter estimation in the mixture model and the natural color images are segmented efficiently by using the GMM with parameters estimated by morphological operations and the DAEM algorithm.

Keywords : Color Image Segmentation, Morphological Operation, Gaussian Mixture Model, Deterministic Annealing EM.

Ⅰ. Introduction

The segmentation of a natural color image into an unknown number of distinct and homogeneous regions is a difficult problem and become a fundamental issue in low-level computer vision tasks.

Many of the existing segmentation techniques such as direct clustering methods generally use a feature vector space for solving image segmentation problems. Given an image, feature vectors are extracted from local neighborhoods and mapped into the space spanned by their coordinates. Significant features in the image then correspond to high-density regions in this space. The finite mixture of multivariate probability distributions has been used as the statistical modeling of a continuous feature

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space[1]. In this case, these highest density regions represent clusters centered on the modes of each component in the finite mixture model.

The widely often used assumption in modeling by using a finite mixture of distribution is that the number of components or clusters is small and known a priori and the individual components obey multivariate normal distributions. That is, the feature space can be modeled as a finite mixture of Gaussian distributions with a known number of components. However, we cannot recognize the number of colors composing an observed real image before analyzing its image. So, we need the method that can automatically estimate the number of mixture components. And also the GMM is commonly used to represent the probability distribution of the feature vector in feature space. The expectation maximization (EM) algorithm is naturally used for estimating the parameters of the GMM. But the estimates of parameters obtained by the EM algorithm are strongly dependent upon their initial values and they are sometimes achieved by the local maximum of total log likelihood.

In this paper, to overcome this problem, we are going to consider the morphological operations and the DAEM algorithm. We will show how to apply them for the estimation of components and parameters in a mixture model. We adopt the GMM to represent the probability distribution of the observed feature vector and perform the image segmentation using this model. And this paper demonstrates the performance of our segmentation algorithm for the natural color scenes.

Ⅱ. Color histogram and Morphological Operations

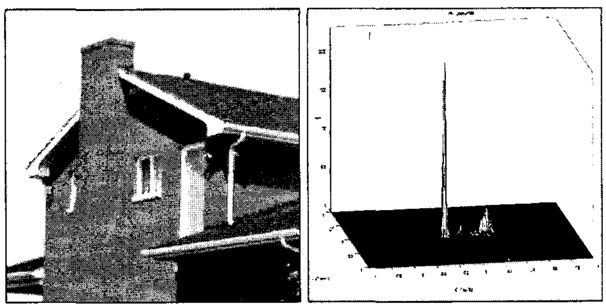
1. 2D color histogram and its smoothing using Morphological Operations

Normally, natural color images consist of several objects and they have native color stimuli. The RGB model is commonly used for representing their colors and it is a hardware-oriented color model used in optic device such as the TV monitor, the computer screen and the color printer. But, it is difficult for humans to recognize color of objects with this model. In this case, humans feel colors through the hue, saturation and brightness percepts. We will use HSI (Hue, Saturation, Intensity) color model in this paper.

We first translate the RGB color space into the HSI color space and only consider the chromatic color components of H and S. Then color distribution is obtained by projecting the pixel values in the selected each object into the color space.

To consider the statistical modeling for an observed color image, we first construct the 2D histogram with color components such as hue and saturation of pixels consisting of a color image and seek to explicit peaks in the histogram. This relies on the fact that the colors of objects in the image give rise to make peaks in the histogram. The main peaks of the 2D histogram are considered as the centroids of each cluster of pixels representing dominant colors and correspond to their regional maxima of the 2D histogram[2]. The number and position of main peaks are important features to segment the observed image statistically. However, we can't have any information about them without a priori knowledge. So we may need the method that can detect the number and position of each peak using a morphological operation of the image[3]. Fig. 1 shows the color histogram of HS components generated from the House image.

Here, since the original histogram of the image has a large amount of perturbation as being noticed, we



(a) House Image (b) Color Histogram

그림 1. 하우스 영상에 대한 HS 성분의 히스토그램

Fig. 1. The histogram of HS components for a House Image.

need to simplify the histogram to remove the unnecessary trivial peaks from the image histogram while preserving the most relevant ones. The smoothing of the histogram of the original image can be generally conducted by means of the morphological opening and closing by reconstruction. First, we have employed the opening by reconstruction to smooth the histogram as follows.

For a given 2-D histogram f generated from HS components of an observed image, the opening by reconstruction of f by a structuring element b is defined by the following iterative procedure[4].

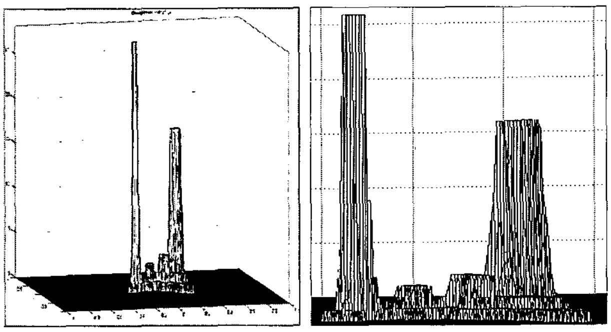
(1) Obtain the marker histogram $f \ominus b$ by eroding a histogram with a structuring element b.

(2) Initialize h_1 to be the marker histogram $f \ominus b$

(3) Repeat the dilation operation of a marker histogram h_k with respect to a mask histogram f such that $h_{k+1} = (h_k \oplus b) \cap f$ until $h_{k+1} = h_k$

The morphological opening by reconstruction is not only to maintain the basic structure of histogram but also to smooth the histogram by eliminating trivial peaks that cannot contain the structuring element.

Next, the result of the opening by reconstruction is applied to the closing by reconstruction operation for better smoothing. It can be implemented by complementing a histogram, computing its opening by reconstruction, and then complementing the result. The idea behind the morphological closing is to build an operator tending to recover the initial shape of the image structures that have been dilated. Fig. 2 shows the smoothed histogram of a House image using the opening and closing by reconstruction. We can note that the original noisy histogram has been smoothed successfully by yielding a piece-wise constant type histogram which can reflect the cluster components precisely.



(a) Smoothed Histogram. (b) Partly zoomed result of histogram (a)

그림 2. 하우스 영상의 평활화된 히스토그램

Fig. 2. The smoothed histogram of a House Image.

2. Detection of Modes in the Smoothed Histogram

Now, we will consider an algorithm to detect the modes from the smoothed histogram. First, the regional maxima of the smoothed histogram can be used to extract the binary components of each cluster. Regional maxima are connected components of pixels with the same intensity value whose external boundary pixels have a value less than this value. Pixels that are set to one identify regional maxima and all other pixels are set to zero[5]. Next, we have applied a labeling operation to the points in color domain with regional maxima to identify each cluster. Labeling process makes it possible to determine automatically the number of cluster components.

Finally, we compute the center of mass of each binary connected component to use it as the mode of each cluster. Fig. 3 shows the binary image of cluster components and positions of the center of mass for the color histogram of the house image. We

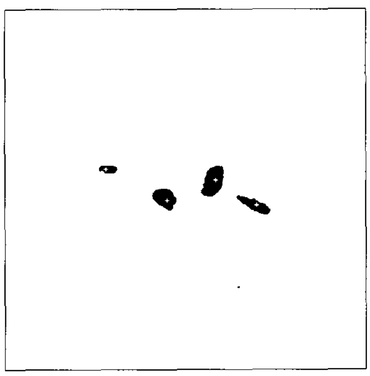


그림 3. 하우스 영상의 히스토그램에 대한 군집의 이진화 영상과 모드의 위치

Fig. 3. The binary image of clusters and positions modes for the House Image.

can note that the number of detected modes corresponds to the number of clusters in the original House image. The modes will be applied to the DAEM algorithm to estimate the parameters of the GMM unsupervisely.

III. Segmentation of a Color image Using a Gaussian Mixture Model

1. Modeling of Color images with a GMM

Suppose that a color image consists of a set of disjoint pixel labeled 1 to N, and that each pixel is assumed to belong to one of the K distinct color regions. Here, we will employ a GMM to characterize the distribution of color feature vectors observed from each object consisting of a color image[6]. We let the groups $G_1 \cdots G_K$ represent the K possible regions. Also we let $\mathbf{y}$ denotes the finite dimensional feature vector observed from i th pixel($i = 1 \cdots N$). Then, we adopt the GMM for a distribution of each feature vector $\mathbf{y}$ as defined as the following model

$$p(\mathbf{y}_i|\Theta) = \sum_{k=1}^{K} \pi_k \phi(\mathbf{y}_i; \boldsymbol{\mu}_k, \Sigma_k) \tag{1}$$

where π_k is the mixture proportion for each group and $\phi(\mathbf{y}_i; \boldsymbol{\mu}_k, \Sigma_k)$ denotes a bivariate or trivariate normal distribution with mean vector $\boldsymbol{\mu}_k$ and a covariance matrix Σ_k.

Furthermore, we let $Z_1 \cdots Z_N$ denote the unobservable group indicator vectors, where the k th element Z_{ik} of Z_i is taken to be one or zero according to the case in which the i th pixel does or does not belong to the k th group. Here, if the parameter vector, π is denoted as the prior probabilities in which each pixel belongs to a particular group, then the probability function of Z_i is given as follows:

$$p(\mathbf{z}_i; \boldsymbol{\pi}) = \prod_{k=1}^{K} \pi_k^{Z_{ik}} \tag{2}$$

Then the distribution of a color image is expressed by the joint distribution of a complete data vector, $\mathbf{x} = (\mathbf{y}, \mathbf{z})$ and the log likelihood function that can be formed on the basis of the complete data x if we adopt the GMM for an observed feature vector, is given by

$$\begin{aligned}\log L_C(\boldsymbol{\Theta} \mid \mathbf{x}) &= \log p(\mathbf{y} \mid \mathbf{z}; \Theta) + \log p(\mathbf{z}; \boldsymbol{\pi}) \\ &= \sum_{k=1}^{K}\sum_{i=1}^{N} Z_{ik} \log(\phi(\mathbf{y}_i; \boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)) + \sum_{k=1}^{K}\sum_{i=1}^{N} Z_{ik} \log(\pi_k)\end{aligned} \tag{3}$$

where $\boldsymbol{\Theta}$ is the vector containing the elements of $\boldsymbol{\Theta}$ and $\boldsymbol{\pi}$.

The problem of maximum likelihood estimation of $\boldsymbol{\Theta}$ given the observed vector y can be solved by applying the EM algorithm proposed by Dempster et al. for the incomplete data[7]. However the EM algorithm has two kinds of disadvantages. The first is hard to avoid unfavorable local maximum of the log likelihood and the second is overfitting problem. Thus we have to think about the new method that is able to improve the EM algorithm. It is known as the DAEM algorithm based on the principle of maximum entropy to estimate the parameter[8,9,10].

We consider the complete data log likelihood $\log L_c(\boldsymbol{\Theta} \mid \mathbf{x})$ as a function of the hidden variable z for a fixed parameter vector $\boldsymbol{\Theta}$, and define a cost function on the hidden variable space $\Omega_\mathbf{z}$ as follows:

$$H(\mathbf{z}; \mathbf{y}, \boldsymbol{\Theta}) = -\log L_c(\boldsymbol{\Theta} \mid \mathbf{x}) \tag{4}$$

Then we need to minimize $E(H(\mathbf{z}; \mathbf{y}, \boldsymbol{\Theta}))$ with respect to probability distribution over the distribution $p(\mathbf{z}; \boldsymbol{\pi})$ space subject to a constraint on the entropy. It yields a quantity, which is known as the generalized free energy in statistical physics. Introducing a Lagrange parameter β, we arrive at the following objective function:

$$\vartheta(P_{\mathbf{z}}^{(t)},\mathbf{\Theta}) = E_{P_{\mathbf{z}}^{(t)}}(H(\mathbf{z};\mathbf{y},\mathbf{\Theta})) + \beta \cdot E_{P_{\mathbf{z}}^{(t)}}(\log P_{\mathbf{z}}^{(t)}) \tag{5}$$

The solution of the minimization problem associated with the generalized free energy in $\vartheta(P_{\mathbf{z}}^{(t)},\mathbf{\Theta})$ with respect to probability distribution $p(\mathbf{z};\boldsymbol{\pi})$ with a fixed parameter $\mathbf{\Theta}$ is the following Gibbs distribution:

$$P_\beta(\mathbf{z}\,|\,\mathbf{y},\mathbf{\Theta}) = \frac{1}{\sum_{\mathbf{z}'\in\Omega_{\mathbf{z}}} \exp(-\beta H(\mathbf{z}'))} \cdot \exp(-\beta H(\mathbf{z})) \tag{6}$$

Hence we can obtain a new posterior distribution, $p_\beta(\mathbf{z}\,|\,\mathbf{y},\mathbf{\Theta})$ parameterized by β.

Next, we should find the minimum of with $\vartheta(P_{\mathbf{z}}^{(t)},\mathbf{\Theta})$ respect to with fixed posterior distribution $p_\beta(\mathbf{z}\,|\,\mathbf{y},\mathbf{\Theta})$. It means finding the estimates that minimizes. Since the second term on the right hand side of the generalized free energy in Equation (5) is independent of $\mathbf{\Theta}$, we should find the value of $\mathbf{\Theta}$ minimizing the first term

$$Q_\beta(\mathbf{\Theta}) = E_{p_{\mathbf{z}}^{(t)}}(H(\mathbf{z};\mathbf{y},\mathbf{\Theta})) \tag{7}$$

To achieve this purpose, we can add a new β -loop, which is called annealing loop, to the original EM-algorithm and replace the original posterior with the new posterior distribution, $p_\beta(\mathbf{z}\,|\,\mathbf{y},\mathbf{\Theta})$ parameterized by β. Thus we can obtain the following DAEM algorithm:

/* Initialize all kinds of parameters */

Parameter $\Theta^{(0)}$; prior distribution,

$p_{\mathbf{Z}}^{(0)}$; posterior distribution

/* annealing loop */

Label : If $\beta < \beta_{Final}$ then

/* EM loop */

$t = 0$

Do

E-step:

Compute a posterior density function and an expectation of a cost function:

$$p_z^{(t)} = p_\beta^{(t)}(\mathbf{z}\,|\,\mathbf{y}, \Theta^{(t-1)}) ;$$

$$Q_\beta(\Theta\,|\,\Theta^{(t-1)}) = E_{p_\beta^{(t)}}(H(\mathbf{z}))$$

M-step:

Find the parameter values that minimize Q-function:

$$\Theta^{(t)} \Leftarrow \min_\Theta Q_\beta(\Theta\,|\,\Theta^{(t-1)})$$

$t = t+1$

While(satisfying the convergence condition)

β←β + δ

Go to Label

Else exit

Finally, after finishing fully iteration, we can obtain the conditional expectation of the hidden variable, z_{ik} given the observed feature data from E-step as follows.

$$\tau_k^\beta(\mathbf{y}_i) = E(Z_{ik}) = \frac{(\hat{\pi}_k N(\mathbf{y}_i\,;\,\hat{\boldsymbol{\mu}}_k, \hat{\Sigma}_k))^\beta}{\sum_{j=1}^{K} (\hat{\pi}_j N(\mathbf{y}_i\,;\,\hat{\boldsymbol{\mu}}_j, \hat{\Sigma}_j))^\beta} \tag{8}$$

And also we can obtain the estimators of mixing proportions, the component mean vector and the covariance matrix from M-step. These are respectively given as

$$\hat{\pi}_k = \tfrac{1}{N}\sum_{i=1}^{N} \tau_k^\beta(\mathbf{y}_i),\ k = 1, \cdots, K$$

$$\hat{\boldsymbol{\mu}}_k = \frac{\sum_{i=1}^{N} \tau_k^\beta(\mathbf{y}_i)\mathbf{y}_i}{\sum_{i=1}^{N} \tau_k^\beta(\mathbf{y}_i)},\ k = 1, \cdots, K$$

and

$$\hat{\Sigma}_k = \frac{\sum_{i=1}^{N} \tau_k^{\beta}(\mathbf{y}_i)(\mathbf{y}_i - \hat{\boldsymbol{\mu}}_k)(\mathbf{y}_i - \hat{\boldsymbol{\mu}}_k)^t}{\sum_{i=1}^{N} \tau_k^{\beta}(\mathbf{y}_i)}, \; k = 1, \cdots, K \tag{9}$$

2. Segmentation of a Color Image

Suppose that a color image consists of a set of the K distinct objects or regions. We usually segment a color image to assign each pixel to some regions or objects. To do this, we use a posterior probability of the i th pixel belonging to kth region in Equation (8).

Next, we try to find what the component or region has the maximum value among the estimated posterior probabilities. This is define as

$$\hat{Z}_i = \mathrm{argmax}_{1 \le k \le K} \, \tau_k^{\beta}(\mathbf{y}_i)., \; i = 1, \cdots, N \tag{10}$$

Then, we can segment a color image by assigning each pixel to the region or object having the maximum a posterior probability.

Ⅳ. Experimental results

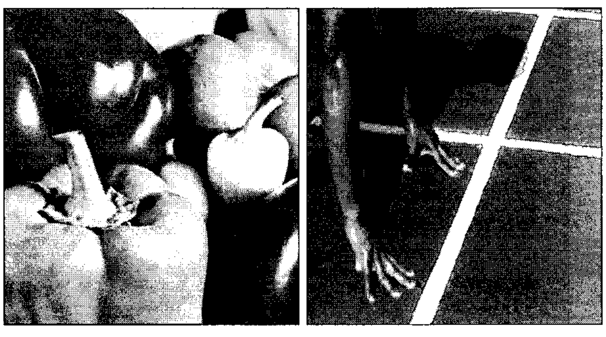
To demonstrate the performance of the proposed segmentation algorithm, we have applied the algorithm to the color images "peppers" and "runner" in Fig. 4(a) and (b), respectively.

The conversion of the RGB color image to HSI model is carried out and the hue and saturation components are only used as values of the feature vectors. To find the mode, we have first applied morphological reconstruction operations to the histogram consisting of the HS components. Here, we adopted the structuring element with radius 3.

Fig. 5 shows the smoothed histogram of "peppers" and "runner" images using the opening and closing by reconstruction.

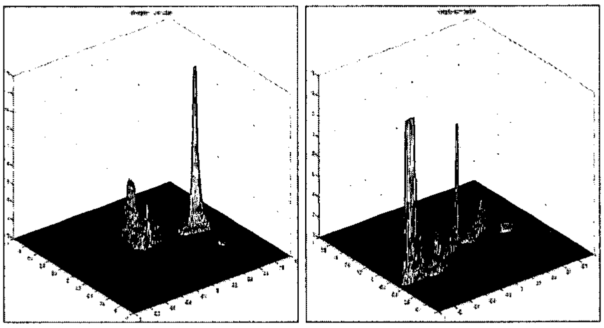
Next, a labeling operation has been applied to the binary components of each cluster being extracted by the regional maxima of the smoothed histogram to identify each cluster. Labeling process makes it possible to determine automatically the number of cluster components.

Finally, we have computed the center of mass of each binary connected component to use it as the mode of each cluster. Fig. 6 shows the binary image of cluster components and positions of the center of



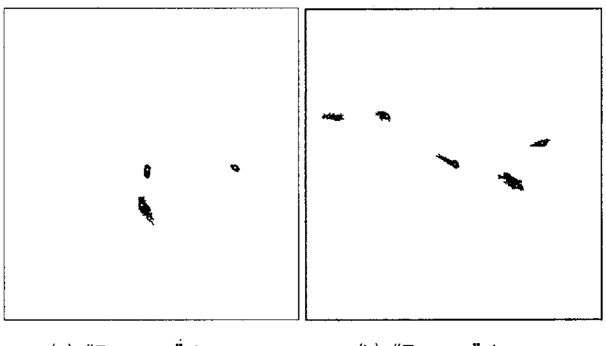
(a) "Peppers" Image (b) "Runner" Image

그림 4. "Peppers"와 "Runner" 원본 영상
Fig. 4. Original Image.



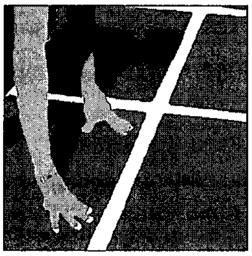
(a) "Peppers" image (b) "Runner" image

그림 5. "peppers"와 "runner"영상의 평활화된 히스토그램
Fig. 5. The smoothed histogram of "peppers" and "runner" images.



(a) "Peppers" image (b) "Runner" image

그림 6. "Peppers"와 "Runner" 영상의 모드 검출 결과
Fig. 6. Result of detected modes for the "Peppers" and "Runner" images.

(a) "Peppers" image (b) "Runner" image

그림 7. "Peppers"와 "Runner" 영상의 분할 결과
Fig. 7. Segmentation results.

mass for the "peppers" and the "runner" images.

As we can see, the modes have been precisely detected from the smoothed histogram. The mode information is fed to the DAEM algorithm to estimate the parameters of the GMM. The algorithm finally segments a color image by assigning each pixel to the region having the maximum a posterior probability. The segmentation results using the proposed algorithm are shown in Fig. 7.

We can see that the homogeneous objects are partitioned into the same region accurately and the fine structure is preserved.

V. Conclusions

In this paper, we have proposed algorithm combining the morphological operations and the deterministic annealing EM algorithm for an unsupervised segmentation of natural color image.

The morphological operations provide the unsupervised mode detection when the number of components is not known a priori. The DAEM algorithm is the estimation method of various parameters in the GMM derived from the principle of maximum entropy to overcome the local maximum problem associated with the conventional EM algorithm.

We conclude from the experiments for the real images that the morphological operations have been proven to perform well in detecting the number of components or clusters in complicated feature spaces, and the DAEM algorithm provides a global optimal solution for the ML estimates of the GMM parameters.

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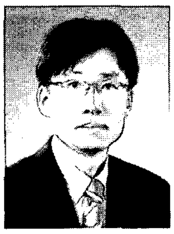

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