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Elasto-plastic Analysis of Circular Cylindrical Shell under Horizontal Load by Rigid-bodies Spring Model

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Abstract

This paper is a study on the experiment and elasto-plastic discrete limit analysis of reinforced concrete circular cylindrical shell by the rigid-bodies spring model. In the rigid bodies-spring model, each collapsed part or piece of structures at the limiting state of loading is assumed to behave like rigid bodies. The present author propose new discrete elements for elasto-plastic analysis of cylindrical shell structures, that is, a rectangular-shaped cylindrical element and a rhombus-shaped cylindrical element for the improvement and expansion of this rigid-bodies spring model. In this study, it is proposed how this rigid element-bodies spring model can be applied to the elasto-plastic discrete limit analysis of cylindrical shell structures. Some numerical results of elasto-plastic discrete limit analysis and experimental results such as the curve of load-displacement and the yielding and fracturing pattern of circular cylindrical shell under horizontal load are shown.

keywords : circular cylindrical shell, elasto-plastic discrete limit analysis, rigid bodies spring model, load displacement curve

1. Introduction

In order to overcome some difficulties encountered in the present finite non-linear analysis, it is proposed a family of new discrete elements which are called a rigid bodies-spring model. In the rigid bodies-spring model, each collapsed part or piece of structures at the limiting state of loading before the collapse is assumed to behave like rigid bodies in the mechanics depended by crack pattern. When structures or solids reach their ultimate state of loading they may be yielded, collapsed and crushed into some pieces, and can see the result of experimental test of structures. At the limiting state each part or piece of the structures may move like a rigid body according to the cracks. And the relative slip movement of adjacent pieces on the divided element occurs along the contact surfaces. In the initiation and growth of **cracks, the discontinuous displacement occurs on the divided surfaces of structures. In rigid-bodies spring model, when the continuum or structures is analyzed by the numerical discrete technique** in order to overcome some difficulties encountered in the present finite non-linear analysis, **the discrete elements are assumed to rigid bodies, and these elements are interconnected with the spring system which transmits the internal forces. The behavior of overall structures can be estimated by the behavior of rigid elements and the spring system. In this paper, it is proposed how this rigid-bodies spring model can be applied to the elasto-plastic discrete limit analysis of circular cylindrical shell. Some numerical results of analysis and experimental results such as the curve of load-displacement and the yielding and fracturing pattern of a circular cylindrical shell under horizontal load are shown.**

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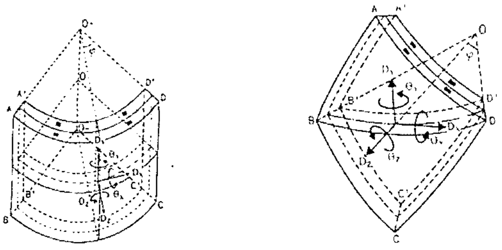
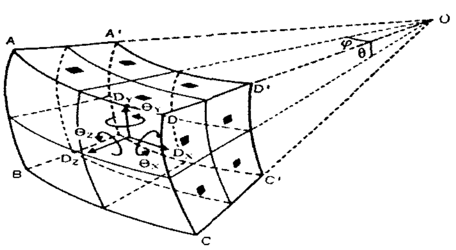
2. New Cylindrical Element

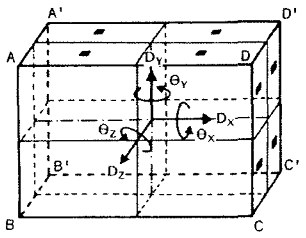
Rigid-bodies spring model of cylindrical shell structures assumed to be composed of the division of finite element cylindrical element and elasto-plastic spring system in the contact divided surface. The analytical model of cylindrical shell consist of the assemblage of rectangular-shaped cylindrical elements or rhombus-shaped cylindrical elements as shown in <Fig. 1>. Considering a circular cylindrical shell to be divided into finite cylindrical elements with circumferential angle ϕ, two typical element shapes can be defined to rectangular-shaped cylindrical element and rhombus-shaped cylindrical element. The cylindrical bending element has 3 movable rigid displacements (D_X, D_Y, D_Z) and 3 rotational rigid displacements ($\Theta_X, \Theta_Y, \Theta_Z$) at the center of element. Supposing that the rotational displacements are very small, the equation of the displacements u, v, w in a point around the element affected by 6 center rigid displacements in the center of the cylindrical element can be obtained. And the spherical shell element by the same metod can obtained as <Fig. 2.>


(a)Rectangular element (b)Rhombus element

<Fig. 1> Circular cylindrical shell elements



<Fig. 2> Spherical shell element



<Fig. 3> Plate element

3. Anlytical Procedure by Rigidbodies Spring Model

In the analytical method by the new discrete rigid-bodies spring model of cylindrical shell structures, at first the equation of the relations between the shell element and spring can be determined by considering the behavior of spring and 6 centroid rigid displacements. This is called adaptation equation of cylindrical shell element. The adaptation equation represents displacements by the rigid behavior in a point around the element affected by 6 centroid rigid displacements in the center of the element. And the total stiffness equation is obtained by the principle of virtual work. Substituting boundary condition into the total stiffness equation, the relative displacement and internal forces of springs are obtained. The axial forces, normal forces, shear forces and bending moments on contact surfaces of cylindrical shell element are obtained by the internal forces of springs.

The spring-α is connected with a point 'i' of cylindrical shell element-M and 'j' of cylindrical shell element-N, and the adaptation equation of spring-α of shell element can be expressed as

$$U_i = A_i D_M \qquad (1)$$

$$U_j = A_j D_N \qquad (2)$$

The equation that transforms the displacements u, v and w into the displacements d_A, d_S, d_N corresponding to each side of the cylindrical shell element is derived as

$$\begin{Bmatrix} d_A \\ d_S \\ d_N \end{Bmatrix} = \begin{bmatrix} \cos\eta & \sin\eta & 0 \\ -\sin\eta & \cos\eta & 0 \\ 0 & 0 & \xi \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} \tag{3}$$

$$d = T^1 U \tag{4}$$

In Eq.(3), η and ξ are changed according to the direction of each cylindrical shell element side. From Eqs.(1), (2) and (4), the following formulae are derived.

$$d_i = T_i^1 \ A_i \ D_M \tag{5}$$

$$d_j = T_j^1 \ A_j \ D_N \tag{6}$$

By consideration of the transformation matrix T^2 between the global coordinate system for overall structure and the local coordinate system for each cylindrical shell element, Eqs.(5) and (6) can be rewritten as

$$d_i = T_i^1 \ A_i \ T^2 \ I_N \ D \tag{7}$$

$$d_j = T_j^1 \ A_j \ T^2 N \ I_N \ D \tag{8}$$

By combination of Eqs.(7) and (8), the relative displacement of spring-α is given as

$$d_\alpha = T_i^1 \ A_i \ T^2 \ I_M \ D + T_j^1 \ A_j \ T^2 \ I_N D \tag{9}$$

In Eq.(9), I is the unit matrix of each cylindrical shell element, and D is the expanded column vector of rigid displacements in the global coordinate system, The internal forces of spring-α is written as

$$\begin{Bmatrix} f_A \\ f_S \\ f_N \end{Bmatrix}_\alpha = \begin{bmatrix} k_A & 0 & 0 \\ 0 & k_S & 0 \\ 0 & 0 & k_N \end{bmatrix} \begin{Bmatrix} d_A \\ d_S \\ d_N \end{Bmatrix}_\alpha \tag{10}$$

$$f_\alpha = K_\alpha d_\alpha \tag{11}$$

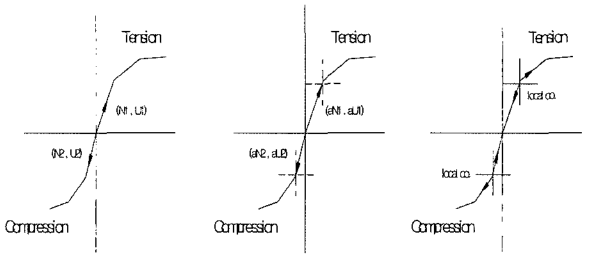
Substitution of Eq.(9) into Eq.(11) yields the internal forces of spring-α as follows

$$f_\alpha = K_\alpha T_i^1 A_i T^2 I_M D + K_\alpha T_j^1 A_j T^2 I_N D \tag{12}$$

4. Elasto-plastic Analysis Procedure

In the rigid-bodies spring model, cylindrical shell structures are composed of a set of finite rigid cylindrical shell elements for the analysis. The internal forces of structures are distributed over the spring system which connects the rigid cylindrical shell elements. The connection springs assumed to be have the characteristics of elasto-plastic material. When the internal force is backward in plastic range(the 2nd grade and the 3rd grade), the elastic grade(the 1st grade)is selected. The essential information required in the analysis of structures by the rigid element spring model is to obtain the spring stiffness with the mechanical properties of continuum.

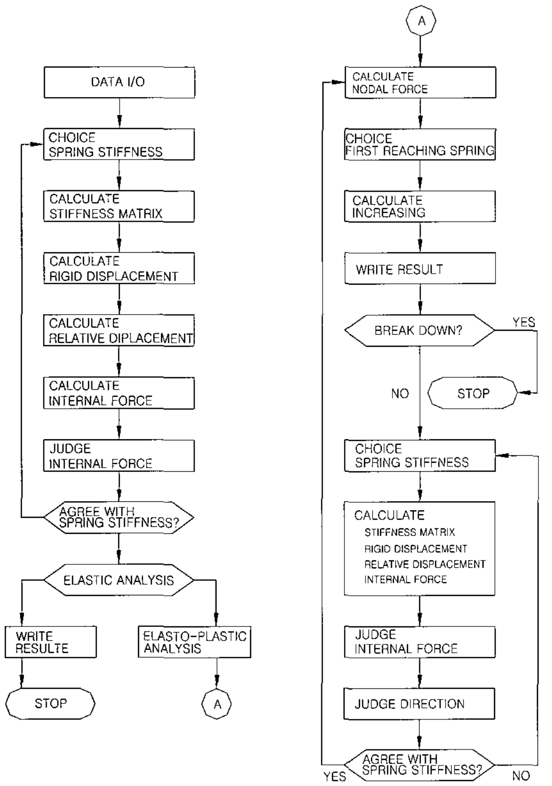
The spring stiffness can be obtained by the experimental testing in general, and differs according to the constitutive material of structures. Because the reinforced concrete cylindrical shell structures differs the tensional stiffness from the compressive stiffness, it is needed to distinguish between the stress-strain relationship of tension and that of compression. Because it is previously not decided to choose the compressive stiffness and tensional stiffness for the axial force of spring, the tensional stiffness is used in calculated at first. If the sign of the relative displacement in results is negative, it is converted to the compressive stiffness. And this procedure is repeated until the sign of spring stiffness is consistent with that of the relative displacement.



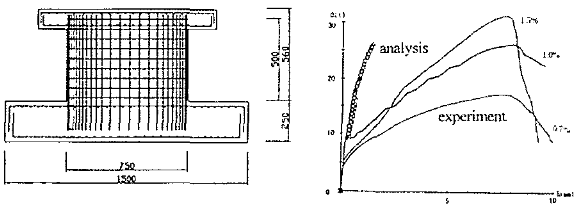
〈Fig. 4〉 Procedure elasto-plastic analysis

5. Elasto-plastic Analysis Results of Cylindrical Shell under Horizontal Load

An analytical model of cylindrical shell is a cylinder under horizontal load as shown in <Fig. 6>(a). <Fig. 6>(b) show the analytic and experimental result of load-deflection curve. The solution domain of circular cylindrical shell is analyzed by considering a half part of the analytical model. The

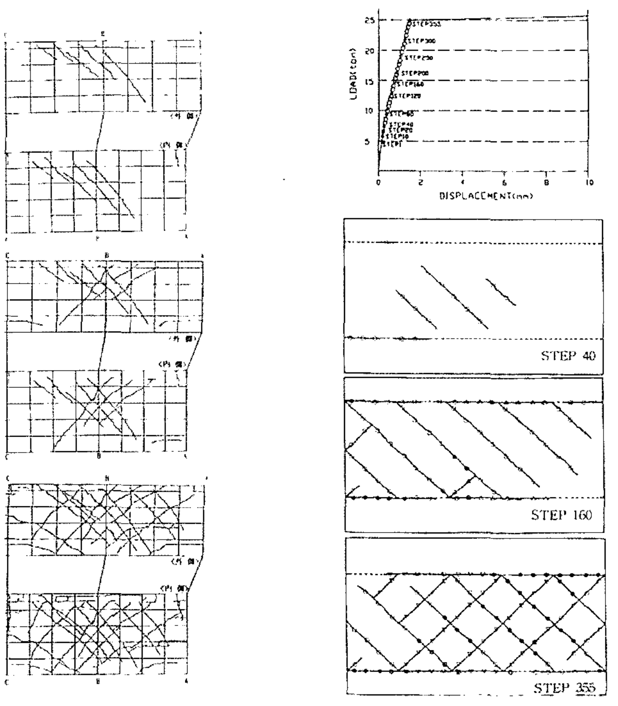


〈Fig. 5〉 Flow chart of elasto-plastic analysis



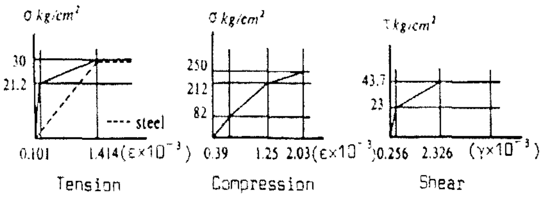
(a)Model (b) Load displacement curve

〈Fig. 6〉 Circular cylindrical shell



(a)Experimental results (b)Numerical results

〈Fig. 7〉 Circular cylindrical shell under horizontal load



〈Fig. 8〉 Tri-linear model

〈Table. 1〉 Elasto-plastic results by rigid-bodies spring model

Step	Load (tonf)	Displacement (mm)	
Step 1	5.158	0.185	Bending crack
Step 10	6.120	0.206	
Step 20	6.436	0.219	
Step 40	7.121	0.251	
Step 80	9.031	0.366	
Step 120	12.404	0.616	Arrived 3rd grade
Step 160	14.270	0.755	Shear crack
Step 200	15.773	0.875	
Step 250	18.345	1.064	
Step 300	21.040	1.264	
Step 350	24.210	1.507	

〈Table. 2〉 Experimental results

	C-0.7%	C-1.0%	C-1.5%
Section area of specimen(cm^2)	353.4	353.4	353.4
Concrete compression strength(kgf/cm^2)	413	401	345
Concrete tension strength(kgf/cm^2)	31.0	26.4	27.6
Yield strength of reinforced bars(kgf/cm^2)	6618	6618	6618
Shear crack strength(kgf/cm^2)	7.17	8.72	4.19
Bending crack strength(kgf/cm^2)	5.92	3.12	4.44
Ultimate shear strength(kgf/cm^2)	17.71	26.4	31.94

pattern of yield lines and fracture is shown in <Fig. 4>. The analytical model is composed of 29 rigid elements and 240 springs. <Fig. 7>(a) show the crack development and yield line in the experimental result. <Fig. 7>(b) show the elesto-plastic limit analysis of circular cylindrical shell under horizontal load by rhombus-shaped cylindrical shell element.

6. Conclusion

In this paper, it is proposed how the rigid element spring model can be applied to the elasto-plastic analysis of cylindrical shell structures. The procedures of elasto-plastic analysis on the discrete analysis of the cylindrical shell is proposed. The the curve of load-displacement and the pattern of yield lines and fracture are shown in cylindrical shell structures. Analyzing the results of some numerical analysis of cylindrical shell structures, the present authors believe that the effect of slip movement, the initiation and growth of cracks and the characteristics of displacement discontinuity can be easily treated by the spring system which is assumed to be the elasto-plastic material taking tri-linear model.

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