

# A New Integral Variable Structure Controller For Incorporating Actuator Dynamics

Jung-Hoon Lee<sup>\*,\*</sup>

## Abstract

In this paper, a new simple integral variable structure controller is designed with incorporating the actuator dynamics. The formulation of the VSS(variable structure system) controller design includes integral augmented sliding surface and the dynamics of the actuator expressed as the state equation. An illustrative example is given to show the effectiveness of the developed controller.

Key words: Actuator dynamics, sliding mode control, variable structure system, integral sliding surface

## 1. Introduction

The VSS is classified into the robust, discontinuous, nonlinear, and deterministic control categories. This algorithm can provide the outstanding means for controlling the uncertain plants[1]. Since the mathematical development of the VSS(variable structure system) by Filippov in 1960[2], the research of the VSS has been mature now because of the development of the general design method for wide spectrum of system types and applications to variety control problems such as stabilization, regulation, tracking, and even identification[3]-[6].

For implementing the control inputs of plants, generally, a certain type of actuators is used. If the dynamics of actuator is low such as in case of large scale systems, it is necessary to incorporate the dynamics of actuators into a design of controllers[7]. As one example of the large scale system, the dynamics of airplanes is highly nonlinear complex and incorporating the first order dynamics of the actuators for flight controls. Until now in general control area, the nonlinearity

property of actuators saturation is considered in the design of controllers in view of the stability and stable gain design[8]. In the research area of the VSS, it is difficult to find the research results incorporating actuator dynamics[9]-[13].

In this paper, an integral VSS incorporating the actuator dynamics is briefly designed for the first time in the VSS research area. The dynamics of actuators is expressed as a state equation. The design of the integral VSS controller is formulated after combining the plant dynamics with actuator one. The integral sliding surface is adapted. An illustrative example is given to show the usefulness of the designed algorithm.

## II. A New Variable Structure Controller

### 2.1 Description of Plants and Actuators

A multi-input uncertain dynamical plant with actuators can be separately described in state equation as [7]

$$\begin{aligned}\dot{X}_p &= (A_p + \Delta A_p)X_p + (B_p + \Delta B)X_a + D_p(t) \\ \dot{X}_a &= (A_a + \Delta A_a)X_a + B_a U\end{aligned}\quad (1)$$

where  $X_p \in R^n$ ,  $X_a \in R^m$ , and  $U \in R^m$  are the state variables of plants, the state variable of corresponding actuators, and control input, respectively. It is assumed that the uncertain  $\Delta A_p$ ,  $\Delta B_p$ ,  $\Delta D_p$ , and  $\Delta A_a$  are bounded and satisfy

\*ERI, Dept. of Control and Instrumentation Engineering, Gyeongsang National University

★ Corresponding author

Manuscript received Feb. 2, 2005 ; revised Dec. 25, 2005

following the matching condition

$$\Delta A_p, \Delta B, D_p \in R(B_p)$$

$$\Delta A_a \in R(B_a) \quad (2)$$

The first equation of (1) stands for the dynamics of the plants controlled by the actuator state. The second state equation of (1) describes the dynamic behaviors of the actuator devices for newly incorporating into the design of the VSS controller. Most of the practical actuators are implemented independently, which means that  $B_a$  has a diagonal structure. Combing both plant and actuator dynamics leads to

$$\dot{X} = (A + \Delta A)X + (B + \Delta B)U + D(t) \quad (3)$$

where

$$A = \begin{pmatrix} A_p & B_p \\ 0 & A_a \end{pmatrix}, \Delta A = \begin{pmatrix} \Delta A_p & \Delta B_p \\ 0 & \Delta A_a \end{pmatrix} \quad (4)$$

$$B = \begin{pmatrix} 0 \\ B_a \end{pmatrix}, D(t) = \begin{pmatrix} D_p(t) \\ 0 \end{pmatrix} \quad (5)$$

For the controller design in order to regulate the state of the plant with respect to a given constant command  $X_{pr}$ , the certain corresponding constant value of the actuator is derived as

$$X_{ar} = (B_p^T B_p)^{-1} B_p^T A_p X_{pr} \quad (6)$$

which is obtained from (1) and steady state

condition. The value of (6) with respect to

implies the reference command for the actuator and will be used in constructing the switching surface for the design of the variable structure systems controller.

## 2.2 Design of Proposed VSS

For the plant (3), the suggested VSS will be designed through two steps, determination of the switching surface and choice of corresponding control function to stabilize the system on the surface. First, the integral sliding surface is proposed in the simplest form for incorporating the dynamics of actuators as

$$S(t) = C_p [X_p(0) - X_p(t)] + C_a [X_a(0) - X_a(t)] + [C_{0p} \ C_{0a}] \int_0^t (X_r - X(\tau)) d\tau \quad (7)$$

where  $X_r$  is the combined reference command value for the plant and actuator as

$$X_r = \begin{pmatrix} X_{pr} \\ X_{ar} \end{pmatrix} \quad (8)$$

where  $X_{pr}$  and  $X_{ar}$  are the reference values for the plant and the actuator, respectively.

By introducing the initial condition and integral term into the right hand side of (6), it is possible to make the switching surface be zero at  $t=0$  so that there is no reaching phase problems. In (6), constant coefficient matrices  $C_p$ ,  $C_a$ ,  $C_{0p}$ , and  $C_{0a}$  are the design parameters with the condition  $\det(C_a B_a) \neq 0$ . For systematic design of the integral surface (6), the ideal sliding surface is derived in the dynamic form from  $\dot{S}(t) = 0$  as

$$\dot{X}_s = -[C_p \ C_a]^* [C_{0p} \ C_{0a}] X_s(t) + [C_p \ C_a]^* [C_{0p} \ C_{0a}] X_r \quad (9)$$

$$[C_p \ C_a]^* = ([C_p \ C_a]^T [C_p \ C_a])^{-1} [C_p \ C_a]$$

where is the dynamic representation of the integral sliding surface, and  $X_s$  is the solution of (9) and identically implies a state set of the ideal sliding surface. Now, by directly applying the well-developed feedback theories to the ideal sliding dynamics (9), for example eigenstructure assignment or optimal control, the coefficient matrix of the surface (7) or (9) can be chosen effectively.

Then, as the second design step, the corresponding control input for stabilizing the sliding surface to be zero is suggested with composing of the continuous term, discontinuous term, and continuous feedback of the switching surface as

$$U = K_r \cdot X_r - K_{eq} \cdot X - \Delta K \cdot X - K_{s1} \cdot S - K_{s2} \left[ \frac{s_i}{|s_i| + \delta_i} \right], \quad i=1,2,\dots,m \quad (10)$$

where each gain designed as follows:

$$K_r = [C_a \ B_a]^{-1} [C_{0p} \ B_{0a}] \quad (11)$$

$$K_{eq} = [C_a \ B_a]^{-1} [C_p A_p + C_{0p} \ C_p B_p + C_a A_a + C_{0a}] \quad (12)$$

$$\Delta K = [C_a \ B_a]^{-1} [\Delta K_p \ \Delta K_a] \quad (13)$$

$$\text{TRIANGLE } K_{rj} \begin{cases} > \max \left[ \sum_{l=1}^n C_{rl} \Delta A_{rlj} \right] \text{ if } X_j \cdot s_i > 0 \\ < \min \left[ \sum_{l=1}^n C_{rl} \Delta A_{rlj} \right] \text{ if } X_j \cdot s_i < 0 \end{cases} \quad (14)$$

$$\Delta K_{aj} \begin{cases} > \max \left[ \sum_{l=1}^n C_{al} \Delta B_{rlj} + C_{al} \Delta A_{alj} \right] \text{ if } X_j \cdot s_i > 0 \\ < \min \left[ \sum_{l=1}^n C_{al} \Delta B_{rlj} + C_{al} \Delta A_{alj} \right] \text{ if } X_j \cdot s_i < 0 \end{cases} \quad (15)$$

$$K_{s1i} > \max[D_i(t)] \quad (16)$$

$$K_{s2i} > 0 \quad (17)$$

$$i = 1, 2, \dots, m, \quad j = 1, 2, \dots, n$$

The constant gains, (11) and (12), are directly determined according to the choice of the integral sliding surface, the gains from (13) to (17) are the design parameters in the present control input, specially (13) is discontinuous term. And (16) and (17) are the constant gains for continuous feedback of the switching surface. Until now, the integral VSS for incorporating the dynamics of the actuators is presented for the first time. The derivative of the integral sliding surface as the dynamics of the surface resulted from the above control input is derived from  $\dot{S}(t) = 0$  as

$$\begin{aligned} \dot{S}(t) &= C_p[\dot{X}_p(0) - \dot{X}_p(t)] + C_a[\dot{X}_a(0) - \dot{X}_a(t)] \\ &\quad + [C_{0p} \quad C_{0a}](X_r - X(t)) \\ &= -(C_p A_p + C_{0p})X_p - (C_p B_p + C_a A_a + C_{0a})X_a \\ &\quad + C_{0p}X_{pr} + C_{0a}X_{ar} - C_a B_a U - C_p \Delta A_p X_p \\ &\quad - (C_p \Delta B_p + C_a \Delta A_a)X_a - C_p D_p(t) \end{aligned} \quad (18)$$

By applying the above control input into (18), the dynamics of the sliding surface is simply rearranged as

$$\begin{aligned} \dot{S}(t) &= C_p \Delta A_p X_p - (C_p \Delta B_p + C_a \Delta A_a)X_a \\ &\quad - C_p D_p(t) - \text{TRIANGLE} K_p X_p - \text{TRIANGLE} K_a X_a \\ &\quad - K_{s1} S(t) - K_{s2} \frac{S}{|S| + \delta} \end{aligned} \quad (19)$$

### 2.3 Stability Investigation

To prove the stability of the closed system and the existence condition of the sliding mode, take a Lyapunov candidate function as

$$V(t) = 1/2 S^T(t) \cdot S(t) \quad (20)$$

Differentiating (20) with respect to time and from (19), the derivative of (20) becomes

$$\begin{aligned} \dot{V}(t) &= S^T(t) \cdot \dot{S}(t) = \sum_{i=1}^m s_i(t) \cdot \dot{s}_i(t) \\ &= C_p \Delta A_p X_p S + (C_p \Delta B_p + C_a \Delta A_a)X_a S - C_p D_p(t) S \\ &\quad - [\Delta K_p X_p S + \Delta K_a X_a S + K_{s1} S^2 + K_{s2} \frac{S^2}{|S| + \delta}] \end{aligned} \quad (21)$$

Substituting (13)–(17) into (21), (22) can be obtained

$$\dot{V}(t) < 0 \quad \text{and} \quad \dot{S}_i \cdot \dot{S}_i, \quad i = 1, 2, \dots, m \quad (22)$$

when which completes the proof the stability of the closed system and the existence condition of the sliding mode at the same time.

From the results of the stability proof, the suggested control input (10) with the gain (11)–(17) can stabilize the integral sliding surface (6). to be zero.

## III. Example

### 3.1 Description of Plant and Actuator

To show the effectiveness of the algorithms simple, an example of the multimode flight control problems[7] is considered. The simple flight dynamics of aircraft and actuator is separately described as

$$\dot{X}_p = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -0.8693 & 42.223 \\ 0 & 0.9933 & -1.341 \end{bmatrix} X_p + \begin{bmatrix} 0 & 0 \\ -17.251 & -1.5766 \\ -0.1689 & -0.2518 \end{bmatrix} X_a \quad (23)$$

$$\dot{X}_a = \begin{bmatrix} -20 & 0 \\ 0 & -20 \end{bmatrix} X_a + \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix} X_a \quad (24)$$

where each variable is defined as

$$\begin{aligned} X_p &= \begin{bmatrix} \theta \\ q \\ \alpha \end{bmatrix} = \begin{bmatrix} \text{pitch} \\ \text{pitch rate} \\ \text{angle of attack} \end{bmatrix} \quad X_a = \begin{bmatrix} \delta_e \\ \delta_f \end{bmatrix} = \begin{bmatrix} \text{elevator deflection} \\ \text{flaperon deflection} \end{bmatrix} \\ U &= \begin{bmatrix} \delta_{ec} \\ \delta_{fc} \end{bmatrix} = \begin{bmatrix} \text{elevator deflection command} \\ \text{flaperon deflection command} \end{bmatrix} \end{aligned} \quad (25)$$

The equation of (23) is the most simple longitudinal dynamics of aircraft and the equation of (24) is the corresponding actuator dynamics. The longitudinal dynamics of aircrafts pitch attitude, pitch rate, and angle of attack are controlled by the actuators, elevator and flaperon[7].

### 3.2 Design if VSS

For the pitch pointing mode (PPM) control problem, a command of the flight attitude is given as

$$X_{pr} = [2^\circ \ 0 \ 2^\circ \ \text{RIGHT}] \quad (26)$$

and for the vertical translation mode (VTM) control problem, a command of the flight attitude is given as

$$X_{pr} = [0 \ 0 \ 2^\circ \ \text{RIGHT}] \quad (27)$$

Using the ideal sliding dynamics (8), the design parameters for the integral sliding surface (6) are selected as

$$C_p = \begin{bmatrix} 4 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix} \text{RIGHT}, \quad C_a = \begin{bmatrix} -0.3 & 0 \\ 0 & -0.1 \end{bmatrix} \text{RIGHT}$$

$$C_{0p} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{RIGHT}, \quad C_{0a} = 0_{2 \times 2} \quad (28)$$

and the chosen gain for the control inputs are as follows:

$$K_r = \begin{bmatrix} -0.1667 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.5 & 0 & 0 \end{bmatrix} \text{RIGHT} \quad (29)$$

$$K_{eq} = \begin{bmatrix} -0.1667 & -0.5218 & -7.2038 & 1.8752 & 0.2628 \\ 0 & -1.9866 & 2.1820 & 0.3378 & -0.4964 \end{bmatrix} \quad (30)$$

$$\text{TRIANGLEK} = 0_{2 \times 5}, \quad K_{s1} = K_{s2} = 0_{2 \times 2}$$

The results of the simulations are depicted in Fig.1 for the pitch pointing mode(PPM) control problem and Fig. 2 for the vertical translation mode(VTM) control problem. As can be shown, each state output is exactly regulated to given command  $X_{pr}$  and each actuator state well follows given each control command. The two sliding surfaces are regulated to near zero.

#### IV. Conclusions

In this paper, a new simple integral variable structure controller incorporating the dynamics of actuators is designed. In formulation of the VSS design, the dynamics of the actuator in form of state equation is included. After the reference command for the actuator is obtained, the integral sliding surface without the reaching phase problems is suggested in error coordinate of the states of the plant and actuator. The corresponding control input is presented in order to generate the sliding mode at every point on the suggested surface. The closed loop stability and existence condition of the sliding mode is proved in detail. Through simulation studies on the flight controls, the usefulness is shown effectively. The presented control technology can be applicable to the control problem where the actuator dynamics is not much faster rather than that of the plant.

#### References

- [1] V. I. Utkin, *Sliding modes and their application in variable structure systems*, Mir Pub. Moscow, 1974.
- [2] A. F. Filippov, "Differential equations with discontinuous right-hand side", *Mathematicheskii Sbornik*, Vol.51, no.1 pp.99-128,1960.
- [3] B. Drazhenovic, "The invariance conditions in variable structure systems," *Automatica*, Vol.5, pp.287-295, 1969.
- [4] V. I. Utkin, "Variable structure systems with sliding mode," *IEEE Trans. on Automatic Control*, Vol. AC-22, no.2 pp.212-222, 1977.
- [5] R. A. DeCarlo, S. H. Zak, and G. P. Mathews, "Variable structure control of nonlinear multivariable systems:A tutorial," *Proc. IEEE* Vol. 76, no.3 pp.212-232, 1988.
- [6] J. Y. Hung, W. Gao, and J. C. Hung, "Variable structure control:A survey," *IEEE Trans. on Industrial Electronics*, Vol.40 no.1, pp.1-22, 1993.
- [7] K. M. Sobel and E. Y. Shapiro, "Eigenstructure assignment for design of multimode flight control systems," *IEEE Control System Magazine*, May, pp.9-15, 1985.
- [8] A. Saberi, Z.Lin, and A. R. Teel, "Control of linear systems with saturating actuators," *IEEE Trans. on Automatic Control* Vol. 41, no.3 pp.368-378, 1996.
- [9] Utkin, V.I. and Yang, K.D., "Methods for Constructing Discontinuity Planes in Multidimensional Variable Structure Systems," *Automat. Remote Control*, 1978, 39, no. 10, pp.1466-1470.
- [10] Chern, T.L. and Wu., Y.C., "An Optimal Variable Structure Control with Integral Compensation for Electrohydraulic Position Servo Control Systems," *IEEE Trans. Industrial Electronics*, 39, no. 5, pp.460-463, 1992.
- [11] El-Ghezawi, D.M.E., Zinober, A.S.I., Owens, D.H., and Bilings, S.A., "Computation of the zeros and Directions of Linear Multivariable Systems," *Int. J. Control*, 1982, vol. 36, no.5, pp.833-843.
- [12] Dorling, C.M. and Zinober, A.S.I., "Two Approaches to Hyperplane Design in Multivariable Variable Structure Control Systems," *Int. J. Control*, 1986, 44, no.1, p.65-82.

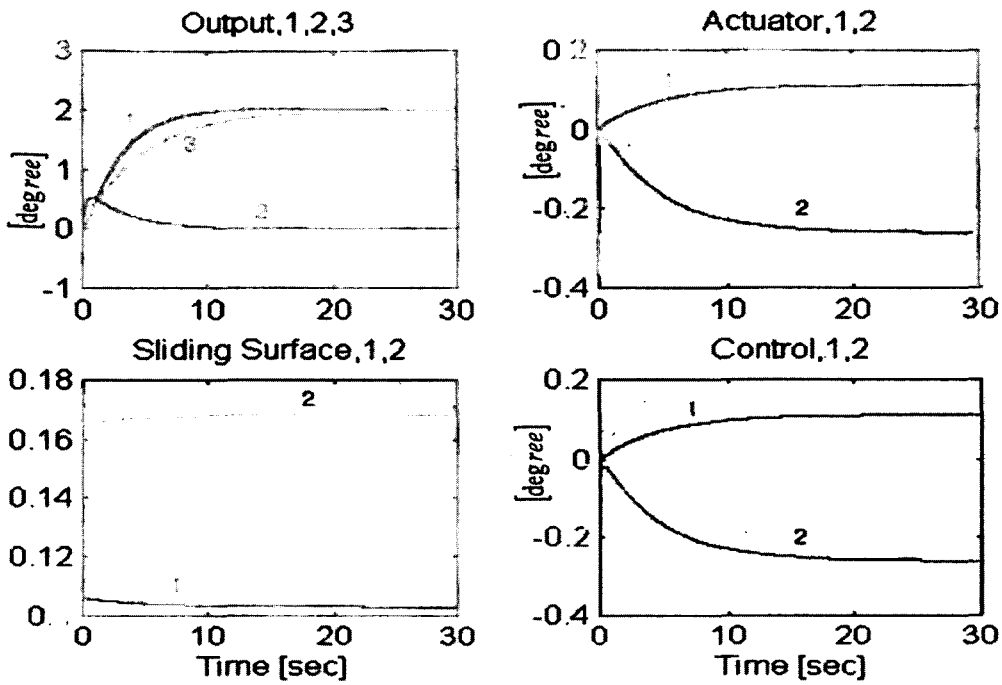


Fig. 1. Pitching pointing mode  $x_{pr} = [2^\circ \ 0 \ 2^\circ]'$

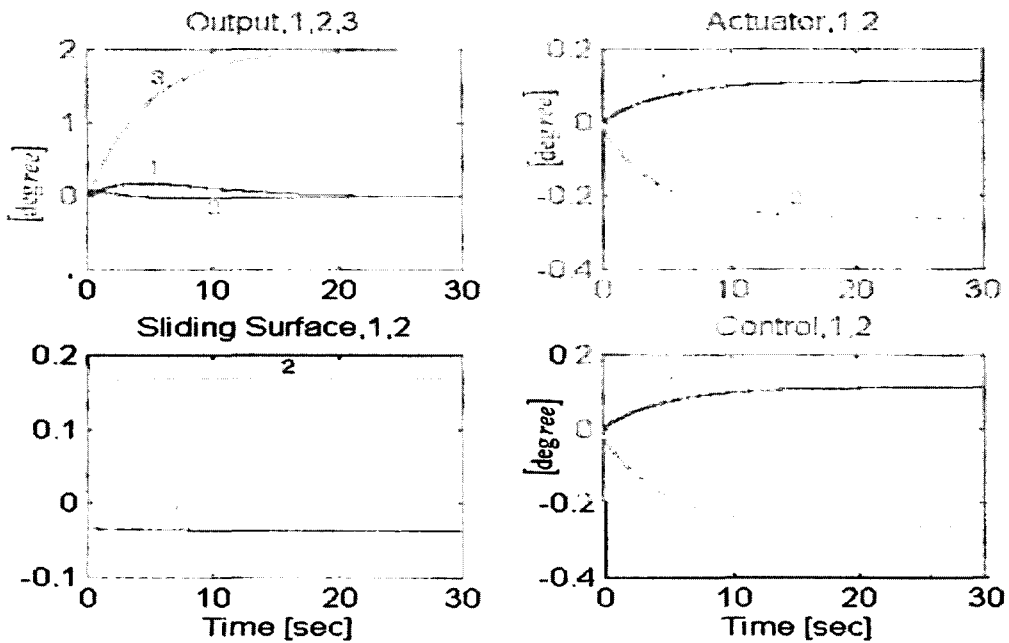


Fig. 2 Vertical translational mode  $x_{pr} = [0 \ 0 \ 2^\circ]'$

- [13] El-Ghezawi, D.M.E., Zinober, A.S.I., and Bilings, S.A., "Analysis and Design of Variable Structure Systems Using a Geometric Approach," *Int. J. Control*, 1983, 38, no.3, pp.657-671.
- [14] Hebertt, S.R., "Differential Geometric Methods in Variable Structure Control," *Int. J. Control*, 1988, 48,
- [15] Chern, T.L. and Wu., Y.C., "Design of Integral Variable Structure Controller and Application to Electrohydraulic Velocity Servosystems," *IEE Proceedings-D*, 1991, 138, No.5, pp.439-444.
- [16] Chang, L.W., "A MIMO Sliding Control with a First-order plus Integral Sliding Condition," *Automatica*, 1991, 27, No. 5, pp.853-858.
- [17] K. S. Yeung, C. C. Cheng, and C. M. Kwan, "A unifying design of sliding mode and classical controllers," *IEEE Trans. On Automatic Control*, vol.38, no, 9. pp.1422-1427,1993.
- [18] C. M.Kwan, "On variable structure output feedback controller," *IEEE Trans. On Automatic Control*, vol.41, no, 11. pp.1691-1693,1996.
- [19] G. Bartolini and A. Ferrara, "Multi-input sliding mode control of a class of uncertain nonlinear systems," *IEEE Trans. On Automatic Control*, vol.41, no, 11. pp.1662-1666,1996.
- [20] J. D. Wang, T. L. Lee, and Y. T. Juang, "New methods to design an integral variable structure controller," *IEEE Trans. On Automatic Control*, vol.41, no, 1. pp.140-143,1996.
- [21] S. C. Chung and C. L. Lin,, "A general class of sliding surface for sliding mode control," *IEEE Trans. On Automatic Control*, vol.43, no, 1. pp.115-119,1998.

---

## BIOGRAPHY

---

Jung Hoon Lee (Member)



1988 : BS degree in Electronics Engineering,, Kyeongbook National University.

1990 : MS degree in Control and power electronics Engineering, KAIST.

1995 : PhD degree in Control power and electronics Engineering, KAIST.  
1995~2007 ; Associate Professor, Gyeongsang National University.