

## Mixed-state Hall effect of MgB<sub>2</sub> thin films

Boyeon Kim<sup>a</sup>, Soon-Gil Jung<sup>a</sup>, Kyeong-Hee Moon<sup>a</sup>, W. N. Kang<sup>\*,a</sup>  
Eun-Mi Choi<sup>b</sup>, Heon-Jung Kim<sup>b</sup>, Sung-Ik Lee<sup>b</sup>, Hyeong-Jin Kim<sup>c</sup>

<sup>a</sup>Department of Physics, Pukyong National University, Busan 608-737, Korea

<sup>b</sup>National Creative Research Initiative Center for Superconductivity, Department of Physics,  
Pohang University of Science and Technology, Pohang 790-784, Korea

<sup>c</sup>National Creative Research Initiative Center for Semiconductor Nanorods, Department of Materials  
Science and Engineering, Pohang University of Science and Technology, Pohang 790-784, Korea

## MgB<sub>2</sub> 박막의 혼합상태에서의 홀 효과

김보연<sup>a</sup>, 정순길<sup>a</sup>, 문경희<sup>a</sup>, 강원남<sup>\*,a</sup>  
최은미<sup>b</sup>, 김현정<sup>b</sup>, 이성익<sup>b</sup>, 김형진<sup>c</sup>

### Abstract

We have measured the Hall resistivity ( $\rho_{xy}$ ) and the longitudinal resistivity ( $\rho_{xx}$ ) on superconducting MgB<sub>2</sub> thin films in extended fields up to 18 T. We found a universal scaling behavior between the Hall resistivity and the longitudinal resistivity, which is independent of the temperature and the magnetic field. At a wide magnetic field region from 1 to 18 T, a universal power law of  $\beta = 2.0 \pm 0.1$  in a scaling relation,  $\rho_{xy} = A\rho_{xx}^\beta$ , was observed in c-axis-oriented MgB<sub>2</sub> thin films. These results can be well interpreted by using recent models.

*Keywords* : MgB<sub>2</sub>, Hall effect, scaling, quasiparticle

### I. Introduction

Since 1965, the first observation [1] of the Hall effect in the superconducting state, numerous experimental results have been reported for type II superconductors [2-8]. However, the origin of the Hall effect due to vortex motion still remains an unresolved subject [8-11] for more than three decades.

In this situation, the fully convincing empirical law of vortex matter is required as an important breakthrough for understanding vortex dynamics. If a transport current is applied, the vortices move perpendicular to the current direction due to the Lorentz force  $F_L = \mathbf{J} \times \mathbf{B}$ , where  $\mathbf{J}$  is the applied current density and  $\mathbf{B}$  is the average magnetic induction. In this scenario, we cannot detect the Hall voltage, as was speculated before 1965, because the electric field caused by the vortex motion is parallel to the current direction according to the Josephson

\*Corresponding author. Fax : +82 51 611 6357  
e-mail : wnkang@pknu.ac.kr

relation  $\mathbf{E} = \mathbf{B} \times \mathbf{v}$ , where  $\mathbf{v}$  is the average velocity of the vortices. In order to reconcile this inconsistency, Nozieres and Vinen [12] suggested the Magnus force as a possible origin of the longitudinal component of the vortex velocity, whereas Kopnin *et al.* [13] considered the vortex-traction force by a transport supercurrent. Another unusual feature of the mixed-state Hall effect is a sign anomaly [3-8, 14, 15], that is believed to be strongly related to the Hall force, but this also is not yet well understood.

In the meanwhile, an interesting Hall scaling behavior has been observed by several groups [2-7]. Furthermore, a scaling behavior between  $\rho_{xy}$  and  $\rho_{xx}$  has been found in most the high-temperature superconductors (HTS). The puzzling scaling relation,  $\rho_{xy} = A\rho_{xx}^\beta$ , with  $\beta = 2$  has been observed for  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$  crystals and  $\text{Tl}_2\text{Ba}_2\text{Ca}_2\text{Cu}_3\text{O}_{10}$  films. Other similar studies have reported  $\beta = 1.5 - 2$  for  $\text{YBa}_2\text{Cu}_3\text{O}_7$  (YBCO) films, YBCO crystals, and  $\text{HgBa}_2\text{CaCu}_2\text{O}_6$  films. Even  $\beta = 1$  was reported for heavy-ion-irradiated  $\text{HgBa}_2\text{CaCu}_2\text{O}_6$  thin films

A number of theories have been proposed to explain the scaling behavior between  $\rho_{xy}$  and  $\rho_{xx}$ . The first theoretical attempt was presented by Dorsey and Fisher [16]. They showed that near the vortex-glass transition,  $\rho_{xy}$  and  $\rho_{xx}$  could be scaled with an exponent  $\beta = 1.7$ , and they explained the experimental results of Luo *et al.* [2] for YBCO films. A phenomenological model was put forward by Vinokur *et al.* [10]. They claimed that in the flux-flow region,  $\beta$  should be 2 and independent of the pinning strength. Their result was consistent with the observed exponent in Bi-, Tl- and Hg-based superconductors only for the Hall data measured in high magnetic fields [17]. Another phenomenological model was proposed by Wang *et al.* [11], who claimed that  $\beta$  could change from 2 to 1.5 as the pinning strength increased, which agreed with the results reported for YBCO crystals [5] and  $\text{HgBa}_2\text{CaCu}_2\text{O}_6$  films [6].

The Hall scaling behavior, therefore, is a complicated phenomenon, which seems strongly depend on the type of superconductors. However, from the experimental Hall data reported in previous

papers, one can found a general trend [17]. At higher fields, the scaling range of a magnetic field and a temperature is wide and shows a universal value of  $\beta = 2$ , which is independent of the field, the temperature, and the pinning strength. At lower fields, the scaling range is relatively narrow because the contribution from the hydrodynamic term is comparable to that of quasiparticle term; thus, the value of  $\beta$  does not appear to be constant.

The  $\text{MgB}_2$  superconductor is a very interesting sample for investigating the flux dynamics [18]. Different from HTS,  $\text{MgB}_2$  shows no Hall sign anomaly in the mixed state and has a rather simple vortex phase diagram [18, 19]. The absence of sign anomaly implies that the hydrodynamic contribution is very small or negligible [13, 18]. Thus, the  $\text{MgB}_2$  compound is probably the best candidate for probing whether the Hall scaling is universal or not because we need only consider the quasiparticle term of the Hall conductivity, which is consistent with the universal Hall scaling theory.

In this paper, we report the demonstration of a universal scaling behavior of the Hall resistivity in  $\text{MgB}_2$  superconducting thin films for extended range of magnetic fields up to 18 T, and the results can be well described using recent theories. We confirmed that the scaling exponent  $\beta = 2$  is universal, which is independent of the temperature and the magnetic field. Based on our results, we will discuss that the universal Hall scaling law is also valid for HTS in high fields where the hydrodynamic contribution in the Hall effect is very small compared to the quasiparticle contribution.

## II. Experiment

The  $\text{MgB}_2$  thin films were grown on  $\text{Al}_2\text{O}_3$  substrates under a high-vacuum condition of  $\sim 10^{-7}$  Torr by using pulsed laser deposition and postannealing techniques [20, 21]. The fabrication process and the normal-state transport properties of  $\text{MgB}_2$  thin films are described in detail elsewhere. The X-ray diffraction patterns indicated highly c-

axis-oriented thin films perpendicular to the substrate surface and showed a sample purity in excess of 99%. The critical current density at 15 K and under a self-field condition was observed to be on the order of  $10^7$  A/cm<sup>2</sup>. Standard photolithographic techniques were used to produce thin-film Hall bar patterns, which consisted of a rectangular strip (1 mm × 3 mm) of MgB<sub>2</sub> film with three pairs of sidearms. The narrow sidearm width of 0.1 mm was patterned so that the sidearms would have an insignificant effect on the equipotential. Using this 6-probe configuration, we were able to measure simultaneously the  $\rho_{xy}$  and  $\rho_{xx}$  at the same temperature and magnetic field. To achieve good ohmic contacts, we coated Au film on the contact pads after cleaning the sample surface by using Ar-ion milling. The magnetic field was applied perpendicular to the sample surface by using a superconducting magnet system. We applied very high current density of  $10^4$  A/cm<sup>2</sup> in order to obtain  $\rho_{xy}$  and  $\rho_{xx}$  data for wide range of magnetic fields and temperatures.

### III. Results and Discussion

Figure 1 shows the temperature dependence of  $\rho_{xx}$  for MgB<sub>2</sub> thin films grown on Al<sub>2</sub>O<sub>3</sub> substrates for applied current density of  $10^4$  A/cm<sup>2</sup>. The inset is a magnified view near the critical temperature region. At zero field, the onset transition temperature ( $T_c$ ) was 39 K and had a narrow transition width of  $\sim 0.1$  K, as judged from the 10% to 90% superconducting transition. The normal-state resistivity at 290 K was  $\sim 10$   $\mu\Omega$  cm, indicating an intermetallic nature with a relatively high charge carrier density [22].

Figure 2 shows the magnetic field dependence of  $\rho_{xx}$  for MgB<sub>2</sub> films at various temperatures from 10 to 34 K and at a current density of  $10^4$  A/cm<sup>2</sup>. A very small land positive magnetoresistance can be observed at normal state. At lower temperatures, the superconducting transitions of  $\rho_{xx}$  became broad, indicating that vortices move easily for wide magnetic field ranges due to the strong Lorentz force by applying a very high current density. As the

magnetic field was increased,  $\rho_{xx}$  grew gradually up to an upper critical field, which is different from the

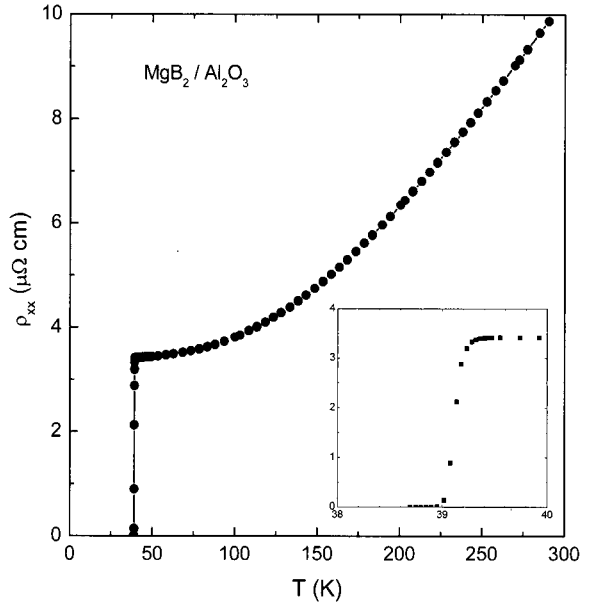


Fig. 1. Temperature dependences of  $\rho_{xx}$  for MgB<sub>2</sub> thin films at current density of  $10^4$  A/cm<sup>2</sup>. Inset is a magnified view near a superconducting transition region.

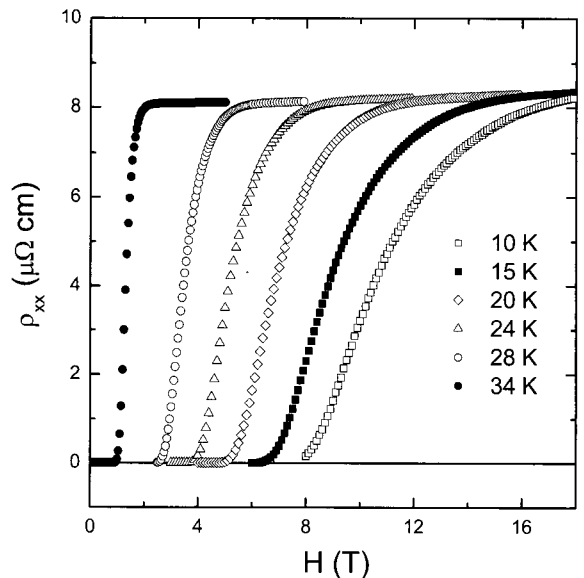


Fig. 2. Magnetic field dependences of  $\rho_{xx}$  curves for MgB<sub>2</sub> thin films at temperatures from 10 to 34 K and at current density of  $10^4$  A/cm<sup>2</sup>.

previous observation measured with low current density in polycrystalline MgB<sub>2</sub> [19].

The magnetic field dependence of  $\rho_{xy}$  is plotted in Fig. 3. The similar trend as seen in the  $\rho_{xx}$ -T curves was observed with increasing temperatures. No sign reversal of the Hall signal was detected for magnetic fields up to 9 T over a wide temperature range from 10 to 34 K. According to the Kopnin et al.'s theory [13], this result implies that in the MgB<sub>2</sub> superconductor, the hydrodynamic contribution for the mixed-state Hall conductivity is very small or negligible compared to the quasiparticle contribution. Thus, this compound is probably the most suitable sample to prove the Hall scaling behavior. The overall feature of the field dependence is quite different from that of HTS, in which a dip structure is observed near  $T_c$  due to the negative contribution by the hydrodynamic term [2-7].

The scaling behavior of  $\rho_{xy}$  and  $\rho_{xx}$  for the field-sweep data of MgB<sub>2</sub> films is plotted in Fig. 4 for various temperatures from 10 to 28 K. The  $\rho_{xy}$  and  $\rho_{xx}$  data were extracted from the field dependence of  $\rho_{xy}$  and  $\rho_{xx}$  data shown in Figs. 2 and 3. The universal Hall scaling with an exponent of  $2.0 \pm 0.1$  is evident

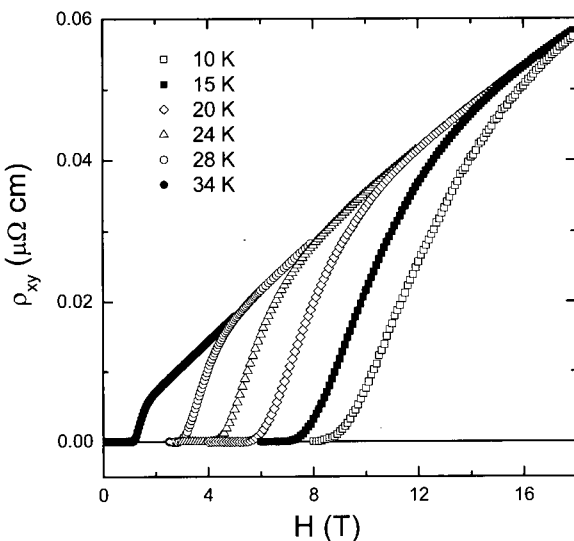


Fig. 3. Magnetic field dependences of  $\rho_{xy}$  curves for MgB<sub>2</sub> thin films at temperatures from 10 to 34 K and at current density of  $10^4$  A/cm<sup>2</sup>.

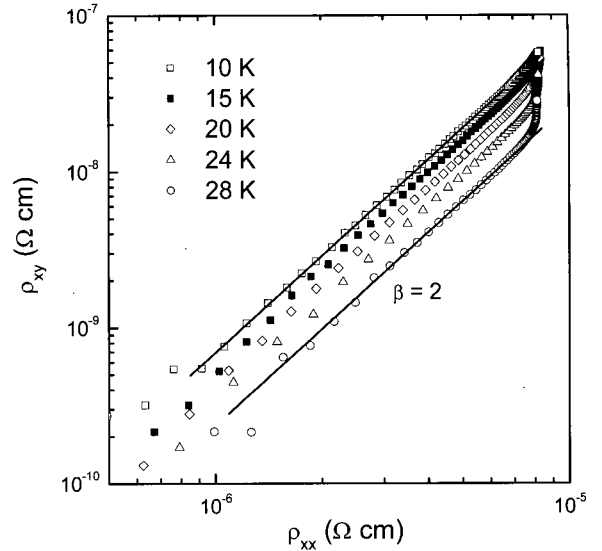


Fig. 4. Scaling behaviors between  $\rho_{xy}$  and  $\rho_{xx}$  for the field-sweep data of MgB<sub>2</sub> thin films measured at temperatures from 10 to 28 K and at current density of  $10^4$  A/cm<sup>2</sup>.

and is independent of the magnetic fields and the temperatures. More strikingly, this universal scaling generally occurs in the regimes of the flux flow, such as the free-flux-flow, the thermally activated flux-flow, and the vortex-glass regions. This result is different from those for HTS where the scaling relation is valid only in the thermally activated flux-flow and vortex-glass regions [2-7]. This provides an evidence for the pinning-strength independence of the Hall scaling. Thus, we have confirmed that the flux-flow  $\rho_{xy}$  is determined by  $\rho_{xx}$  and that the relation is independent of the magnetic field, the temperature, and the pinning strength. This universal behavior of the Hall scaling is our principal finding, and this observation will have serious physical implications of mixed-state Hall behavior, as discussed below.

Vinokur et al. [10] proposed a universal Hall scaling theory based on a force balance equation where the Lorentz force  $f_L$  acting on a vortex is balanced by the usual frictional force  $\eta v$  and the Hall force  $\alpha v \times n$  with  $v$  being the average velocity of vortices and  $n$  being the unit vector along the vortex lines. The coefficient  $\alpha$  is related to the Hall angle by means of  $\tan \Theta = \alpha / \eta$ . If a pinning force of  $\gamma v$ , which

renormalizes only the frictional term but does not affect the Hall force term, is included, the equation of vortex motion is given by

$$\eta \mathbf{v} + \alpha \mathbf{v} \times \mathbf{n} = f_L \quad (1)$$

Using this force balance equation, we can easily calculate the relation between  $\rho_{xy}$  and  $\rho_{xx}$ :

$$\rho_{xy} = A \rho_{xx}^2 \quad (2)$$

where  $A = \alpha / (\Phi_0 B)$  with  $\Phi_0$  being the flux quantum. Equation (2) gives a universal scaling law with  $\beta = 2$ , which is independent of the magnetic field, the temperature, and the pinning strength. Furthermore, this relation can apply to the entire flux-flow region. Our experimental data in Fig. 4 can be interpreted completely by this model. Such a scaling relation is possible because  $A$  is a slowly varying function of temperature and magnetic field while  $\rho_{xx}$  is an exponential function of temperature and magnetic field, as shown in Fig. 2. This excellent consistency between theoretical and experimental results is very important in the field of vortex dynamics since we are able to set up the basic equation of vortex motion, Eq. (1), which should provide a significant direction for future investigations on vortex dynamics. If this theory is to be generally accepted, however, experimental observation of HTS [2-7] with  $\beta$  ranging from 1 to 2 should be explained.

Now, we discuss the spread value of  $\beta$  reported for HTS. An interesting microscopic approach based on the time-dependent Ginzburg-Landau theory has been proposed [13]. According to this model, the mixed-state Hall conductivity ( $\sigma_{xy}$ ) in type II superconductors is determined by the quasiparticle contribution ( $\sigma_{xy}^q$ ) and the hydrodynamic contribution ( $\sigma_{xy}^h$ ) of the vortex cores,  $\sigma_{xy} = \sigma_{xy}^q + \sigma_{xy}^h$ . Since  $\sigma_{xy}^h$  is determined by the energy derivative of the density of states, if that term is negative and dominates over  $\sigma_{xy}^q$ , a sign anomaly can appear. This theory is consistent with experimental data for HTS.

This microscopic theory suggests that Hall scaling can be broken in the case where  $\sigma_{xy}^h$  is comparable to  $\sigma_{xy}^q$  because those terms have opposite signs. Indeed,  $\beta$  was observed to less than 2 at low fields whereas a universal value of  $\beta = 2$  was observed at higher fields where  $\sigma_{xy}^h$  is very small compared to  $\sigma_{xy}^q$ . For example,  $\beta = 2$  was observed in Hg- and Tl-based superconductors for the high field region from 9 to 18 T [17]. In the MgB<sub>2</sub> compound, since no sign anomaly was detected, we can say with confidence that  $\sigma_{xy}^h$  is very small or negligible; thus, universal Hall scaling holds over a wide range of fields. One can conclude from these results, that Hall scaling is universal under conditions where the  $\sigma_{xy}^q$  term dominates the  $\sigma_{xy}^h$  term; thus we can then explain all the reported data related the Hall scaling issues for HTS.

#### IV. Summary

We have found a universal Hall scaling behavior between  $\rho_{xy}$  and  $\rho_{xx}$  in MgB<sub>2</sub> thin films for the extended field range up to 18 T, which is in good agreement with the high-field Hall data from Bi-, Hg- and Tl-based HTS [2-7]. Our Hall data can be completely explained within the context of the universal Hall scaling theory [10]. We also show that  $\rho_{xy}$  can scale as  $\rho_{xx}^2$  in cases where the  $\sigma_{xy}^q$  term dominates  $\sigma_{xy}^h$  term. With a simple phenomenological theory [10] and a microscopic theory [13], we are able to explain the Hall scaling behavior in HTS, which has been debated for a long time. We believe that these results will provide new insight into the future theoretical studies on the vortex dynamics of superconductivity.

#### Acknowledgements

This work was supported by Pukyong National University Research Fund in 2003.

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