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IMM Method Using Kalman Filter with Fuzzy Gain

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Abstract

In this paper, we propose an interacting multiple model (IMM) method using intelligent tracking filter with fuzzy gain to reduce tracking errors for maneuvering targets. In the proposed filter, the unknown acceleration input for each sub-model is determined by mismatches between the modelled target dynamics and the actual target dynamics. After a acceleration input is detected, the state estimates for each sub-filter are modified. To modify the accurate estimation, we propose the fuzzy gain based on the relation between the filter residual and its variation. To optimize each fuzzy system, we utilize the genetic algorithm (GA). The tracking performance of the proposed method is compared with those of the adaptive interacting multiple model(AIMM) method and input estimation (IE) method through computer simulations.

Key Words : Interacting multiple model, Maneuvering target tracking, Intelligent tracking filter, Fuzzy system, Genetic algorithm

1. Introduction

The Kalman filter has been widely used in communication. The design of a Kalman filter relies on having an exact dynamic model of the system under consideration in order to provide optimal performance. The accurate modelling of a maneuvering target is one of the most important problems when the Kalman filter is used for target tracking. However, there exists a mismatch between the modelled target dynamics and the actual target dynamics. If the system model of a maneuvering target is not correct, tracking loss will occur easily. These problems have been studied in the field of state estimation. Later, the various techniques were investigated and applied[1-5]. The interesting one of them is the interacting multiple model (IMM) approach[6]. In the algorithm, a parallel bank of filters are blended in a weighted-sum form by an underlying finite-dimensional Markov chain so that a smooth transition between sub-models is achieved. However, to realize a target tracker with an outstanding performance, a prior statistical knowledge on the maneuvering target should be supplied, i.e., the process noise variance for each sub-model in IMM should be accurately selected in advance by the domain expert who should fully understand the unknown maneuvering characteristics of the target, which is not an easy task.

An approach to resolve this problem is the adaptive interacting multiple model (AIMM) algorithm, where the acceleration is estimated via the two-stage Kalman filter and the model is selected based on the estimated acceleration[7]. However, in the method, the acceleration levels to construct the multiple models should also be predesigned in a trial and error manner, which significantly affect the tracking performance of the maneuvering target. Another important problem is that there is no direct measurement of acceleration available. As usual, significant delay and error arise in estimating acceleration from the noisy measurement of the position and the velocity. Until now, no tractable method coping with the problem has been proposed. It remains a theoretically challenging issue in the maneuvering target tracking, and thereby should be fully tackled.

Motivated by the above observations, we propose an intelligent tracking filter with fuzzy gain to reduce the additional effort required in conventional methods. The algorithm improves the tracking performance and establishes the systematic tracker design procedure for a maneuvering target. The complete solution can be divided into two stages. First, when the target maneuver occurs, the acceleration level for each sub-model is determined by the using the fuzzy system based on the relation between the non-maneuvering filter residual and the maneuvering one at every sampling time. Second, to modify the accurate estimation, the target with maneuver is updated by using the fuzzy gain based on the fuzzy model. Since it is hard to approximate adaptively this time-varying variance and fuzzy gain owing to the highly nonlinear, a fuzzy system is applied as the universal approximator to compute it. To optimize each fuzzy system, we utilize the genetic algorithm (GA). On the other hand, the GA has shown to be a flexible and robust optimization tool for many nonlinear optimization problems, where the relationship between adjustable parameters and the resulting objective functions. Then, multiple models are represented as the acceleration levels

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estimated by these fuzzy systems, which are optimized for different ranges of acceleration input. Finally, the tracking performance of the proposed method is compared with those of the input estimation(IE) algorithm method and the IMM algorithm method through computer simulations.

2. Target model and IMM

2.1 Target model

The linear discrete time model for a maneuvering target and a non-maneuvering target are described for each axis as

$$x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1} \quad (1)$$

$$x^{*k} = Ax_{k-1} + w_{k-1} \quad (2)$$

$$A = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \quad B\begin{bmatrix} T^2/2 \\ T \end{bmatrix}$$

where, x_k is the state vector, the position and velocity of target, T is the time sampling, u_{k-1} is unknown maneuver input and w_{k-1} is the process noise, and zero mean white known covariance Q.

The measurement equation is

$$z_k = H_k x_k + v_k \quad (3)$$

where, $H = [1 \ 0]^T$ is the measurement matrix, and v_k is the measurement noise, and zero mean white known covariance R_k. Both Q_k and R_k are assumed to be uncorrelated..

2.2 IMM

Traditionally, target tracking problems are solved using linearized tracking filters, target maneuvers are often described by multiple linearized models. Here the IMM method has a limited number of sub-models for each axis, and each sub-model is represented as the estimated acceleration or the acceleration levels distributed symmetrically about the estimated one. In the case of N sub-models for each axis, the set of multiple models is represented as

$$M_I = \{ \hat{a}_k, \ \hat{a}_k \pm \varepsilon_1, \ldots, \ \hat{a}_k \pm \varepsilon_{(N-1)/2} \}$$

where $\hat{a}_k$ is the estimated acceleration and $\pm\varepsilon_{(N-1)/2}$ is the predetermined acceleration interval. he IMM algorithm is obtained by a weighted sum of the estimates from sub-models in accordance with the probability of each model being effective. However, it requires predefined sub-models with the different dimensions or process noise levels in consideration of the properties of the maneuvers.

3. Tracking algorithm using the fuzzy system

3.1 Fuzzy model of the unknown acceleration input

When the target maneuver occurs in (1), the standard Kalman filter may not track the maneuvering target because the original process noise variance Q cannot cover the acceleration $\hat{u}_k$. To treat $\hat{u}_k$ simply, the state prediction of the system can be determined by the Kalman filter based on the fuzzy system. The filter is based on non-maneuvering (2), which can be derived by assuming a recursive estimator of the form.

$$\hat{x}^{*k|k-1} = A\hat{x}_{k-1|k-1} \quad (4)$$

In the standard Kalman filter, the residual of the estimation and its variation on non-maneuvering filter are defined as

$$v^*_k = z_k - \hat{z}_k^* \quad (5)$$

$$\Delta v^* = v^*_k - v_{k-1} \quad (6)$$

In order to decrease the tracking error, we propose the GA-based fuzzy Kalman filter algorithm. This algorithm is the estimation of the unknown acceleration input by a fuzzy system using the non-maneuvering filter residual(5) and the difference between non-maneuvering residual and maneuvering residual (6).

The unknown acceleration input $\hat{u}_k$ is inferred by fuzzy system, of which the jth fuzzy IF-THEN rule is represented.

$$Rule\ j: IF\ x_1\ is\ A_{1j}\ \text{and}\ x_2\ is\ A_{2j},\ THEN\ y\ is\ \hat{u}_j \quad (7)$$

where two premise variables x_1 and x_2 are the v^*_k and the Δv^*_k respectively. A consequence variable y is the acceleration input $\hat{u}_j$. The A_{ij} are fuzzy sets, and throughout this paper, it has the Gaussian membership function with the center c_{ij} and the standard deviation σ_{ij} as follows:

$$\mu_{ij}(x_i) = \exp\left[-\frac{1}{2}\left(\frac{x_i - c_{ij}}{\sigma_{ij}} \right)^2 \right] \quad (8)$$

$\hat{u}_k$ is approximated in the following form:

$$\hat{u}_k = \frac{\sum_{j=1}^{M} \hat{u}_j \left(\prod_{i=1}^{2} \mu_{ij}(x_{ij}) \right)}{\sum_{j=1}^{M} \left(\prod_{i=1}^{2} \mu_{ij}(x_{ik}) \right)} \quad (9)$$

We utilize the GA, in order to optimize the parameters in both the premise part and the consequence part of the fuzzy system simultaneously. The optimization process is performed for the purpose of minimizing the tracking errors according to the universal approximation theorem [10]. Obviously the fuzzy system should be designed such that the difference between the actual acceleration input and the estimated one is minimized

$$E=\sum_k (u_k-\hat{u}_k) \tag{10}$$

3.2 IMM algorithm with fuzzy gain

In the preceding section, our proposed discussion is to detect the unknown acceleration input. Once it is detected, a modification is necessary. Since the magnitude of the acceleration input is unknown, we can use the estimate $\hat{u}_k$ to modify the non-maneuver state when we detect the maneuver. The system equation is first modified to contain additive process noise. According to the Kalman filter, the predicted state of the target with maneuver can be written as

$$\hat{x}_{k|k-1}=\hat{x}_{k|k-1^*}+B\hat{u}_{k-1}+w \tag{11}$$

One cycle of the proposed IMM algorithm is summarized as follows: The mixed state estimate $\hat{x}_{k-1|k-1^{0m}}$ and its error covariance $P^{0m}_{k-1|k-1}$ are computed from the state estimates and their error covariances of sub-filters as follows:

$$\hat{x}_{k-1|k-1^{0m}}=\sum_{n=1}^{N}\mu^{n|m}_{k-1|k-1}\hat{x}_{k-1|k-1^n} \tag{12}$$

$$P^{0m}_{k-1|k-1}=\sum_{n=1}^{N}\mu^{n|m}_{k-1|k-1}\left[P^n_{k-1|k-1}+\left(\hat{x}^n_{k-1|k-1}-\hat{x}^{0m}_{k-1|k-1}\right)\cdot\left(\hat{x}_{k-1|k-1^n}-\hat{x}_{k-1|k-1^{0m}}\right)^T\right] \tag{13}$$

where the mixing probability $\mu^{n|m}$ and the normalization constant α^m are

$$\mu^{n|m}_{k-1|k-1}=\frac{1}{\alpha^m}\phi^{nm}\mu^n_{k-1} \tag{14}$$

$$\alpha^m=\sum_{n=1}^{N}\phi^{nm}\mu^n_{k-1} \tag{15}$$

Each sub-model provides the model state estimate update using the estimated acceleration $\hat{u}_{k^m}$ from $\hat{x}_{k-1|k-1^{0m}}$ and $P^{0m}_{k-1|k-1}$ are used as inputs to the sub-filter matched to the m sub-model to compute $\hat{x}_{k|k^m}$ and $P_{k|k^m}$.

$$\hat{x}_{k|k-1^{m*}}=A\hat{x}_{k|k^{0m}} \tag{16}$$

$$\hat{u}^{m_k}=\frac{\sum_{j=1}^{M}\hat{u}_j\left(\prod_{i=1}^{2}\mu^{m_{ij}}(x_{ij})\right)}{\sum_{j=1}^{M}\left(\prod_{i=1}^{2}\mu^{m_{ij}}(x_{ik})\right)} \tag{17}$$

Because of the modified maneuver state $\hat{x}_{k|k-1^m}$, the measurement residual is defined. The modified fuzzy Kalman filter is corrected by the new update equation method. This filter is implemented by

$$\hat{z}_{k|k-1^m}=H_k\hat{x}_{k|k-1^m} \tag{18}$$

The residual of the estimation by using the equation (3) and (18) is defined as:

$$\tilde{z}^m_k=z^m_k-\hat{z}^m \tag{19}$$

Consider a double-input single-output fuzzy system with the linguistic rules.

$$Rule\ j: IF\ x_1\ is\ A_{1j}\ \text{and}\ x_2\ is\ A_{2j},\ THEN\ y\ is\ \bar{\gamma}_j \tag{20}$$

where two input x_1 and x_2 are the filter residual and change rate of the filter residual, respectively, and consequent variable y is the fuzzy correction gain γ_j, $A_{ij}(i\in 1,2$ and $j\in 1,2,\ldots,M)$ is fuzzy set, it has the Gaussian membership function.

That is, we assume that the fuzzy system and we are going to design of the following form.

$$\bar{\gamma}_j=\frac{\sum_{j=1}^{M}\gamma_j\left(\prod_{i=1}^{2}\phi_{ij}(x_{ij})\right)}{\sum_{j=1}^{M}\left(\prod_{i=1}^{2}\phi_{ij}(x_i)\right)} \tag{21}$$

According to the approximation theorem by the GA, the fuzzy gain γ_k is optimized. The first measurement fuzzy gain is defined as

$$\gamma^F_k=\left[\bar{\gamma}_k\ \ \bar{\gamma}_k\right]^T \tag{22}$$

So, the state estimator under the fuzzy correction gain (15) is then written as

$$\hat{x}^{Fm}_{k|k-1}=\hat{x}^m_{k|k-1}+\gamma^{Fm}_k \tag{23}$$

In the second stage, the measurement correction is the Kalman gain. The new update equation of the proposed filter can be modified as follows:

$$\begin{aligned}\hat{x}_{k|k}&=\hat{x}^{Fm}_{k|k-1}+K^m_k\left(z^{m_k}-H_k\hat{x}^{Fm}_{k|k-1}\right)\\&=\hat{x}^m_{k|k-1}+\gamma^{Fm_k}+K^m_k\left[z^{m_k}-H_k(\hat{x}^m_{k|k-1}+\gamma^{Fm}_k)\right]\\&=\left(I-K^m_kH_k\right)\left(\hat{x}^m_{k|k-1}+\gamma^{Fm}_k\right)+K^m_kz^m_k\end{aligned} \tag{24}$$

At the same time, the covariance matrix

$$P^m_{k|k}=P^m_{k|k-1}-K^m_kS^m_kK^{mT}_k \tag{25}$$

The innovation covariance is defined as

$$S^m_k=HP^m_{k|k-1}H^T+R \tag{26}$$

Secondly, the measurement correction is the Kalman gain. We can find the optimal Kalman filter gain.

$$K^m_k=P^m_{k|k}H^TS^{-m}_k \tag{27}$$

3.3 Identification of fuzzy model using the GA

To approximate the unknown acceleration input $\widehat{u}_{k^m}$ and the fuzzy gain γ_k^m, the GA is applied to optimize the parameters in both the premise and the consequence parts. The GA represents the parameters for the given problem by the chromosome S which may contain one or more substrings. Each chromosome, therefore, contains a possible solution to the problem. Because the objective of a target tracker is to minimize the error, the fuzzy system should be designed such that the following objective function can be minimized:

$$\sqrt{(\sum position\ \ error)^2+(\sum velocity\ \ error^2)} \tag{28}$$

Since the GA guides the optimal solution for the purpose of maximizing the fitness function value, it is necessary no map the objective function to the fitness function form by

$$f=\lambda\frac{1}{error+1}+(1-\lambda)\frac{1}{rule+1} \tag{29}$$

where λ is a positive scalar which adjusts the weight between the objective function and the rule number. Each individual is evaluated by a fitness function. Furthermore, since it is strongly desired that we reduce the number of the fuzzy IF-THEN rules in a hardware implementation, and if a computation resource point of view is strongly desired, then we should use the fitness function.

4. Simulation Results

To evaluate the proposed filtering scheme, a maneuvering target scenario was examined and the theoretical analysis from the previous section show how to determined and updated for the maneuvering target model. For comparison purposes, we also simulated conventional the input estimation method (IE) and the adaptive interacting multiple method (AIMM) methods. We assumed that the target moves in a plane and its dynamics is given by (1). For convenience, the maximum target acceleration is assumed to be $0.1km/s^2$, and the sampling period T is $1s$. The fuzzy identified off-line for the acceleration input with the fuzzy gain $-0.01<u_k^1<0.01km/s^2$ are showed in Table2, for in Table3, and for $-0.1<u_k^3<0.01km/s^2$ in Table4.

The initial parameters of the GA are presented in Table 1. The target is assumed to be an incoming anti-ship missile on the $x-y$ plane [11]. The initial position of the target is assumed to be $x_0=72.9km$, $y_0=3.0km$, and its velocity components are assumed to be $0.3km/s$ along the $-150°$ line to the $x-axis$.

Table 1. The initial parameters of the GA

Parameters	Values
Maximum Generation	300
Maximum Rule Number	50
Population Size	500
Crossover Rate	0.9
Mutation Rate	0.01
λ	0.95

The target has lateral accelerations as shown in Fig. 1, and the corresponding target motion is illustrated in Fig. 2. The standard deviation of the zero mean white Gaussian measurement noise is $R=0.5^2$ and that of the random acceleration noise is $Q=0.001^2$. After the unknown target acceleration is determined, the modified filter is implemented by the measurement correction method.

The standard deviations of the bias filter, and the bias-free filter for the two-stage Kalman estimator are $0.01km/s^2$ and $0.001km/s^2$, respectively, which are used only for the AIMM algorithm. The model transition probability from the nth sub-model to the mth one for the IMM and AIMM methods are assumed to be

$$\Phi=\begin{cases}0.97 & \text{if } n=m\\ \frac{1-0.97}{N-1} & otherwise\end{cases} \tag{30}$$

where N is the total number of sub-models. We further assume that the initial motion of the target is similar to that of the first sub-model, so the initial model probability for each sub-model is chosen as

$$\mu_m(0)=\begin{cases}0.3 & \text{if } n=1\\ \frac{1-0.6}{N-1} & otherwise\end{cases} \tag{31}$$

Table 2. Fuzzy rules identified for u_k^1

Rules	c_1	σ_1	c_2	σ_2	u_k^1
1	-0.25645	3.5869	0.47503	0.24968	0.0088927
2	0.65658	1.7919	1.6407	0.56984	-0.009695
3	-1.8646	2.8599	0.034797	0.13294	0.0096743
4	0.69148	0.358	0.861	0.54361	0.0059062
5	1.0725	0.69088	-1.9845	0.57467	-0.003966
6	-0.90388	1.2187	0.57824	3.6104	-0.004285

The simulation results with 100 Monte-Carlo simulations shown in Fig 3. Fig3 shows that the simulation results of the proposed method are compared with those of the IE method and AIMM.

Table 3. Fuzzy rules identified for u_k^2

Rules	c_1	σ_1	c_2	σ_2	u_k^2
1	-1.1299	0.48115	-0.18401	3.4154	0.043957
2	0.67182	0.27258	1.416	2.096	0.011209
3	-0.8938	0.91197	-1.8105	1.796	0.018979
4	-1.8985	2.3218	-0.38648	0.2721	0.040269
5	1.6011	1.7384	0.43168	0.4040	0.042398
6	1.2013	1.5141	-1.8945	4.3522	0.051333

Table 4. Fuzzy rules identified for u_k^3

Rules	c_1	σ_1	c_2	σ_2	u_k^3
1	-1.0392	2.3006	-0.08718	1.0627	-0.044163
2	-1.0222	1.8149	-1.4291	0.159	-0.015613
3	1.7659	1.0216	-0.21199	0.084052	-0.045228
4	2.3135	1.4002	-1.085	4.356	-0.047337
5	1.3201	1.0207	1.9074	0.68719	-0.052096
6	1.1125	1.8671	-1.3362	0.059977	-0.044519
7	-0.2845	2.3505	2.1331	1.3257	-0.054298
8	-0.6785	3.353	-0.76277	1.3307	-0.013949

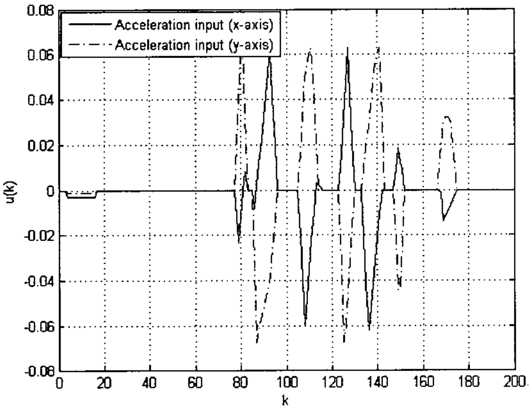


Fig. 1. The acceleration input

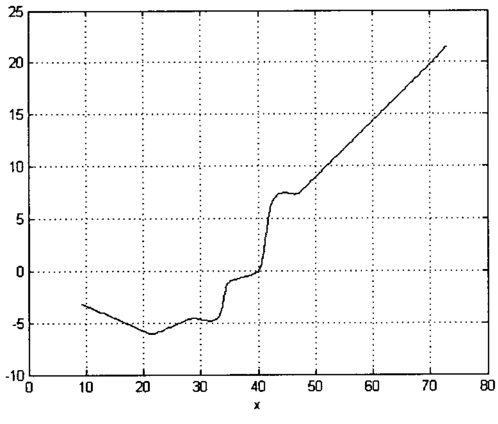
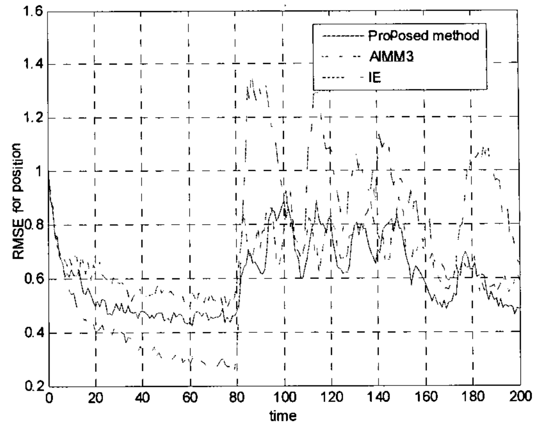
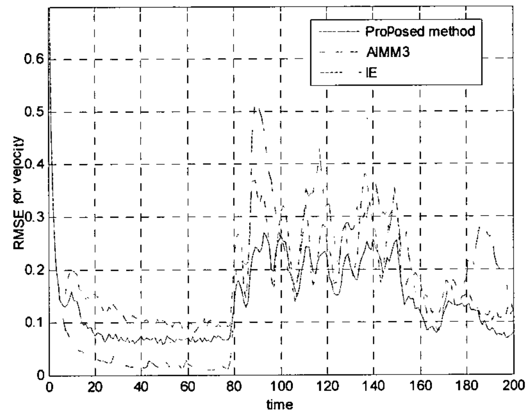


Fig. 2. The target motion



(a) Normalized position error



(b) Normalized velocity errors

Fig. 3. Comparisons of normalized position and velocity errors for proposed algorithm and IE and IMM method.

4. Conclusions

In this paper, we have developed IMM tracking algorithm with fuzzy gain. In the proposed method, the unknown acceleration level for each sub-model was determined by proposed method which is the estimation of the acceleration input by a fuzzy system using the relation between maneuvering filter residual and non-maneuvering one. And then, modified sub-model are corrected by the new update equation method which is a fuzzy system using the relation between the filter residual and its variation. The GA was utilized to optimize a fuzzy system. In computer simulation, we have shown the proposed filter can effectively treat a target maneuver with only one filter by comparing with IE and AIMM.

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