

# Analysis of the Crankshaft Behavior on In-plane and Out-plane Mode at the Firing Stage

엔진 운전시 크랭크샤프트의 면내·외 모드의 거동 해석

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## ABSTRACT

This paper presents a method for analysis of the mechanical behavior of a crankshaft in a four-cylinder internal combustion engine. The purpose of the analysis was to study the characteristics of the shaft in which the pin and arm parts were assumed to have a uniform section in order to simplify the modal analysis. The results of natural frequency transfer function and mode shape were compared with those obtained by experimental work. The results obtained from the comparison showed a good agreement with each other and consequently verified the analysis model. Furthermore, a prediction of crankshaft characteristics under the firing condition, by using the model, was performed. This study describes a new method for analyzing the dynamic behavior of crankshaft vibrations in the frequency domain based on the initial firing stages. The new method used RMS values to calculate the energy at each bearing journal and counter weight shape modification under the operating conditions.

## 요 약

4-실린더 엔진의 작동 시 크랭크샤프트의 기계적 거동을 해석하는 방법에 관한 연구 논문이다. 이 해석의 목적은 모드 해석을 단순화 하기 위해 Pin 과 Arm을 일정하게 가정하고, 이를 통해 단순화된 크랭크샤프트의 특성을 연구하는 것이며, 해석을 통하여 얻어진 전달 함수에서의 고유진동수와 모드 형상을 실험을 통한 모드 해석과 비교하였다. 시뮬레이션을 통한 결과와 실험을 비교한 결과 해석치와 실험치의 값이 일치함을 확인할 수 있었고 이를 통하여 해석 모델을 검증하였다. 또한 검증된 모델을 통하여 엔진 작동 시 크랭크샤프트의 특성을 해석하고자 하였다. 초기 연소 조건에 기초하여 주파수 영역에서 크랭크샤프트의 동적 거동을 해석하기 위한 새로운 방법을 기술하였다. 새로운 기법은 엔진의 작동 조건에서 저널 베어링과 밸런서의 형상 변경을 통하여 얻어진 에너지 값을 계산하기 위해서 RMS값을 이용하였다.

## 1. Introduction

The demands on a modern engine system are increasing rapidly. High performance engines, low fuel consumption, low noise, and

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vibration are several of the factors a customer demands simultaneously. These demands pressurize engineers to upgrade engine designs and require optimization of engine components.

Since the crankshaft is one of the key components in the engine system identification of dynamic behavior plays as important role in the design. Thus, an accurate prediction for the dynamic characteristics of the system using finite element method is essential for modern equipment.

The crankshaft, supported by the crankcase bearing, receives the combustion gas force and gas torque force exerted from the pistons through the connecting rod in each cycle. These gas forces are due to the gas from the exploding fuel/air mixture impinging on top of the piston surface while the gas torque is due to the gas force acting on the moment arm about the crankshaft center. Generally, these latter forces are higher and act cyclically.

Therefore, it was required to study the behavior and characteristics and, finally, to identify the dominant forces in each frequency domain, whereby the equations of motion of the system are expressed in terms of frequency and integrated in the frequency domain to obtain the frequency response. The advantage of this method was that the high frequency components of the systems can be eliminated and the computation can be reduced. Also, interpretation of the results was simpler than with the 'time history' solution. For non-linear equations, a sinusoidal random input describing the function approach could be used.

Previous crankshaft analyses were both by modal analyses and experiments<sup>(1-3)</sup> and were not given a proper detailed explanation. There is a need to contrast more especially involvement with the operating stage. Consideration of the load history in time

domain<sup>(4)</sup> under running conditions is obviously important, but the high frequency components of the system cannot be eliminated since the range of the time domain is not limited. The analysis using the rotating coordinate system<sup>(5)</sup> under operating conditions, investigated at the initial firing, with consideration of several cases dealing with the pulley and flywheel, may be obtained by instructive knowledge about the behavior. But in this study the bending by dynamic behavior modes, effected mainly at the bearing journal, are more preferable since journal bearings are the most sensitive section to be considered since they receive a repetitive load history.

As new engines are continuously researched and developed, a process is required from the beginning of engine design stages, and engine designers need to know the precise vibration behavior of the crankshaft system. For these needs, a simple modeling method and computation approach for the crankshaft system<sup>(6)</sup> have been reported. The modeling and analyses of crankshaft vibrations in the frequency domain including the effects of the rotation of the crankshaft under operating conditions are given in reference.<sup>(5)</sup>

This study describes a new method for analysis of the dynamic behavior of the crankshaft vibrations in the frequency domain based on initial firing stage where the force is given at the first crankpin. The new method includes the effect of RMS value for the calculated energy at each bearing journal under the firing conditions. There are three main objectives for this study: (a) to verify the highest load case under the firing stage (b) to identify the response at journal bearing location using the RMS value, and (c) to estimate the transferable energy corresponding to the modifying counter weight.

The modifications of counter weights are

based on an assumed shape where the RMS value has been investigated during modal analysis diagnostics.

## 2. Modeling of the Crankshaft

A crankshaft system is one of the most complex structure systems. Therefore, the crankshaft system is being simplified as much as possible without violating the originality of the system towards vibration analysis. For the case of modal analysis, the crankshaft was idealized by a set of jointed structures consisting of round and blocks of rectangular beam element cross-sections. In order to simplify the analysis, pin and arm parts were assumed to have a uniform section. In the case of total engine, the flywheel, bearing

journals, and pulley were taken into account. Details of the modeling method can be summarized as follows. Since the flywheel was relatively thin, the flywheel was idealized by triangular plate elements. However, the original mass and the moment of inertia of the flywheel are preserved in the  $x$ ,  $y$  and  $z$  directions since the flywheel has sufficient rigidity in these directions. The reciprocating masses consist of masses of the pistons and the mass of the small end in the connecting rod was attached at the crankpin end. The crank journal bearings were idealized by three set of linear springs and dashpots in the  $y$  and  $z$  directions. The front pulley and the crank gear were idealized by a set of lumped masses and moment inertia attached at their centers of gravity. The engine block was idealized by a rigid body supported with linear springs in the  $x$ ,  $y$  and  $z$  directions to the ground. The stiffnesses were used from the data given in the previous report by Khang.<sup>(7)</sup> The idealized crankshaft model is shown in Fig. 1 and the idealized engine model is shown in Fig. 2. The whole model of the system is then analyzed using commercial software MSC NASTRAN/PATRAN.

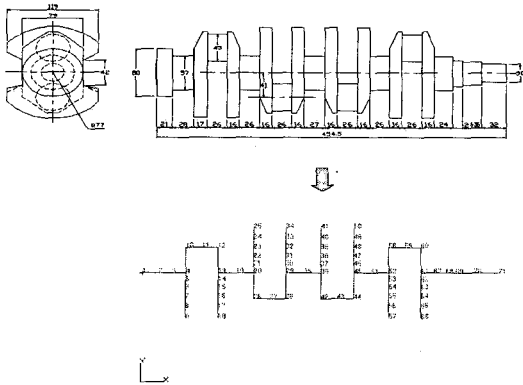


Fig. 1 Idealized model of the crankshaft

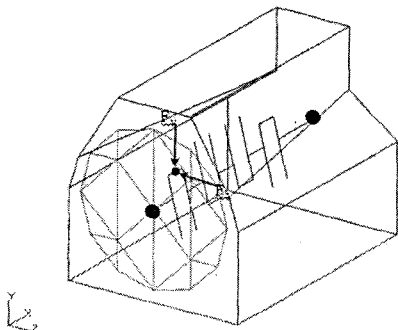


Fig. 2 Idealized model of the engine

## 3. Theory

### 3.1 Finite Element Method.

The crankshaft system may be presented by the sum of the  $n$  dividable finite elements. Fig.

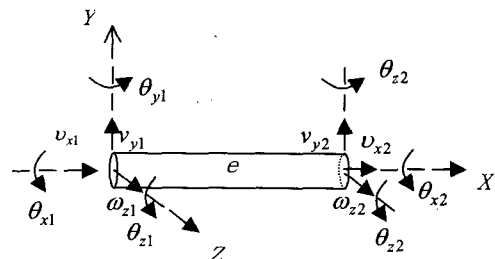


Fig. 3 Degree of freedom of the finite element

3 shows the node points of an element  $e$ . Each node point has six degrees of freedom which consist of three linear displacements  $v_x, v_y, v_z$  and three rotational displacements  $\theta_x, \theta_y, \theta_z$  and can be presented by

$$\{\mathbf{u}\}_e = \{v_{x1}, v_{y1}, w_{z1}, \theta_{x1}, \theta_{y1}, \theta_{z1}, v_{x2}, v_{y2}, w_{z2}, \theta_{x2}, \theta_{y2}, \theta_{z2}\} \quad (1)$$

The kinetic energy for  $n$  degrees of freedom system can be expressed as

$$KE_e = \frac{1}{2} \int \{\dot{\mathbf{u}}\}^T \{\dot{\mathbf{u}}\} \rho dV \quad (2)$$

Where

$\rho$  - Density (mass per unit volume)

$\{\dot{\mathbf{u}}\}$  - Velocity vector

In the finite element, the system is divided into elements and each element of the displacement vector  $\{\mathbf{u}\}$  is expressed in terms of the nodal displacements  $\{\mathbf{q}\}$ , using shape the function  $N$ . However, in dynamic analysis, the elements  $\{\mathbf{q}\}$  are dependent on time, while  $N$  represents the shape function defined on a master element where the velocity vector is the given by

$$\{\dot{\mathbf{u}}\} = \mathbf{N}\{\dot{\mathbf{q}}\} \quad (3)$$

Substituting Eq. (3) into Eq. (2), the kinetic energy in element  $e$  is

$$KE_e = \frac{1}{2} \{\dot{\mathbf{q}}\}_e^T \left[ \int \rho \mathbf{N}^T \mathbf{N} dV \right] \{\dot{\mathbf{q}}\}_e \quad (4)$$

where the bracketed term denotes the element mass matrix

$$\mathbf{m}_e = \int \rho \mathbf{N}^T \mathbf{N} dV \quad (5)$$

Thus, the element kinetic energy can be written as

$$KE_e = \frac{1}{2} \{\dot{\mathbf{q}}\}_e^T \mathbf{m}_e \{\dot{\mathbf{q}}\}_e \quad (6)$$

The strain energy term is considered when obtaining the stiffness matrix. The strain energy for an element is

$$U_e = \frac{1}{2} \int \{\sigma\}^T \varepsilon A dx \quad (7)$$

for  $\sigma = E\mathbf{B}\{\mathbf{q}\}$  and  $\varepsilon = \mathbf{B}\{\mathbf{q}\}$ , where

$\sigma$  - Stress in terms of nodal value

$\varepsilon$  - Strain in terms of nodal value

$\mathbf{B}$  - Element strain-displacement matrix

$E$  - Young's modulus

Thus

$$\begin{aligned} U_e &= \frac{1}{2} \int \{\mathbf{q}\}^T [\mathbf{B}]^T E [\mathbf{B}] \{\mathbf{q}\} A dx \\ &= \frac{1}{2} \{\mathbf{q}\}_e^T \left[ \int [\mathbf{B}]^T E [\mathbf{B}] A dx \right] \{\mathbf{q}\}_e \end{aligned} \quad (8)$$

The bracketed term shows the stiffness element

$$\mathbf{k}_e = \int [\mathbf{B}]^T E [\mathbf{B}] A dx \quad (9)$$

Thus, the potential energy,  $PE$  of elastic system in finite element method can be expressed as

$$PE = \frac{1}{2} \{\mathbf{q}\}_e^T \mathbf{k}_e \{\mathbf{q}\}_e - \{\mathbf{q}\}_e^T \{\mathbf{f}_e\} \quad (10)$$

Using the Lagrangian,  $L = KE - PE$ , the equations of motion are obtained without consideration of damping as

$$\mathbf{m}_e \{\ddot{\mathbf{q}}\}_e + \mathbf{k}_e \{\mathbf{q}\}_e = \{\mathbf{f}\}_e \quad (11)$$

If damping is considered, the equations of motion in Eq. (11) can be rewritten as

$$\mathbf{m}_e \{\ddot{\mathbf{q}}\}_e + \mathbf{c}_e \{\dot{\mathbf{q}}\}_e + \mathbf{k}_e \{\mathbf{q}\}_e = \{\mathbf{f}\}_e \quad (12)$$

### 3.2 Force Applications to Engine Crankshaft

The gas force,  $F_g$  acting on the piston center, where the magnitude is  $F_g = \pi D^2 P_g / 4$ , where  $P_g$  is gas pressure and  $D$  is bore of cylinder. The horizontal is a reactive gas force of the piston on the cylinder wall. This force trying to rock the ground plane is called the reaction gas torque, the primary components of inertia force in vertical and horizontal direction respectively.

### 3.3 Root Mean Square (RMS) And Power Spectral Density Function

In order to identify the energy transferred that is excited by a load case and to predict the energy distributions on the bearing journal, RMS values are calculated and the instructive knowledgeable results obtained for the behaviors of the system.

#### (1) Root Mean Square

The RMS value for a signal  $f(t)$  is defined as the square root of the mean value of quantity  $f(t)$  at a proper average time  $T$ . The RMS value is given by

$$\psi_s = \sqrt{\lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} f^2(t) dt} \quad (13)$$

$\psi_s$  - Signal directly related to energy.

#### (2) Power Spectral Density Function

Auto-correlation function  $R_f$  offers information related with to the characteristics of random variables in the time domain. The power density function  $S_f$  offers similar information

in the frequency domain.

$$\begin{aligned} R_f(\tau) &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} f(t)f(t+\tau)dt \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} S_f(\omega)e^{i\omega\tau} d\omega \end{aligned} \quad (14)$$

At the time domain, in the case of  $\tau = 0$

$$\begin{aligned} R_f(0) &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} f^2(t)dt = \psi_s^2 \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} S_f(\omega)d\omega \end{aligned} \quad (15)$$

It was clear that, the area under the frequency response curve in the frequency domain is equal to the RMS value at the time domain.

## 4. Experiment

For the experiment, the crankshaft was divided into 30 points, while impact hammer was in place, and the response of acceleration was measured. The crankshaft was supported on a sponge approximately 90 mm thickness in order to allow it to be in a free-free end condition states for modal testing. Exciting the crankshaft with an impact hammer (5800A4 Dytran) at one point and measuring the responses at all 30 measuring points using an accelerometer (3100B Dytran), allowed the transfer function of the crankshaft to be

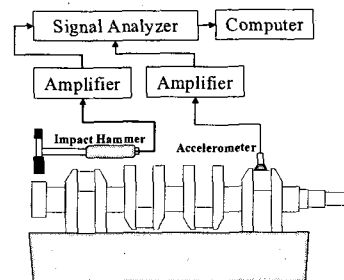


Fig. 4 Experimental set-up

obtained by means of an FFT(fast Fourier analyzer(SA-390). Figure 4 shows the experimental set-up and the measurement system.

### 5. Results

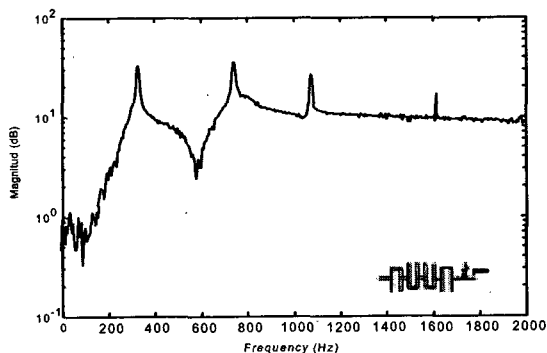
#### 5.1 Frequency Response, Natural Frequencies and Mode Shapes

Exciting the crankshaft with an impact hammer at one impact point and measuring the responses at all of the points, the transfer function of the crankshaft is obtained by means of the FFT analyzer and some signal processing. Two spectra cases of frequency responses from this crankshaft for experiment and simulation are shown in the Figs. 5(a), (b), (c) and (d) respectively. Figures 5(a) and (b) are the response in the in-plane case and the Figs. 5(c) and (d) are the out-plane case.

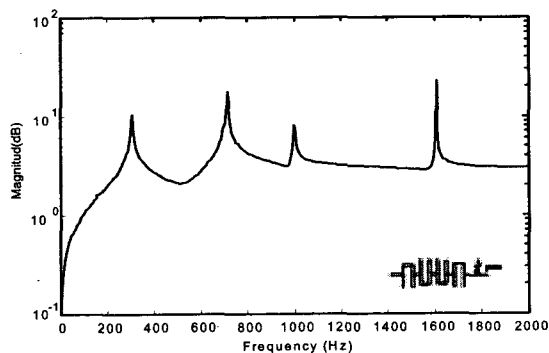
From the spectra, the natural frequencies of the first 10 modes under 2000 Hz occur for four in-plane modes at 312.8 Hz, 725.5 Hz, 1007.1 Hz and 1615.4 Hz and six out-plane at 364.3 Hz, 824.4 Hz, 1133.5 Hz, 1362.7 Hz, 1814.1 Hz, and 1981.3 Hz, and correspond to each of the peaks shown in Figs. 5(a), (b),

Table 1 Comparison of natural frequency between experimental and computer simulations

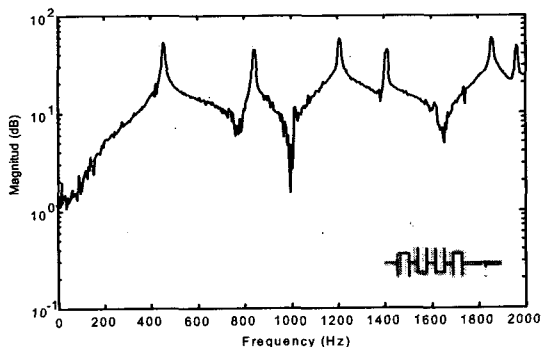
Mode No	Simulation (Hz)	Experiment (Hz)	Error (%)
1	312.8	335	-6.6
2	364.3	460	-20.8
3	725.5	750	-3.3
4	824.4	805	2.4
5	1007.1	1080	-6.7
6	1133.5	1215	-6.7
7	1362.7	1415	-3.7
8	1615.4	1610	0.3
9	1814.1	1865	-2.7
10	1981.3	1970	0.6



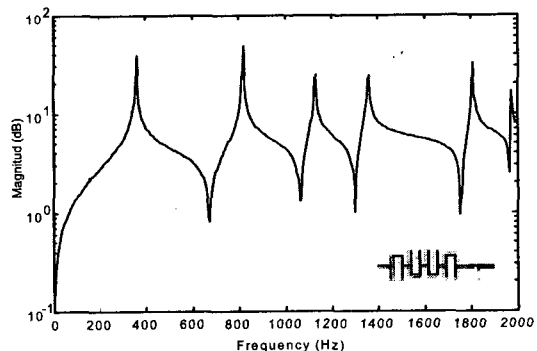
(a) The in-plane mode(experiment)



(b) The in-plane mode(simulation)



(c) The out-plane mode (experiment)



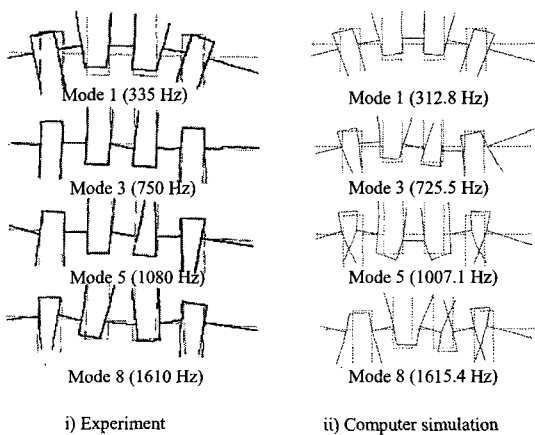
(d) The out-plane mode(simulation)

Fig. 5 Comparison of frequency response between experimental and computer simulations

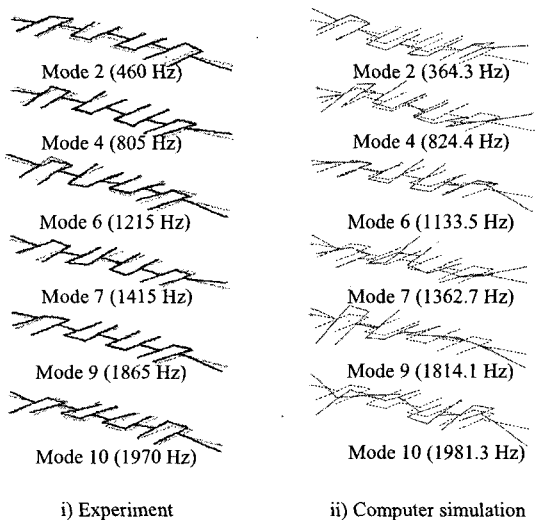
(c) and (d) respectively. In order to verify these results, theoretical modeling was performed and the results were compared.

Table 1 shows a comparison of the natural frequencies between theoretical and experimental results. The percentage relative errors are very small except mode 2 (first out-plane mode). This is because of the rigidity of the crankshaft it self and also the complexity of structure, especially at the

overlap between the journal and the pin crank, where the thickness of the overlap section has a large influence on changing the natural frequency. The thickness is used in the modeling for each crank throw's thicknesses at the crankshaft. A miss-calculation of the thickness will lead to the changes in the natural frequency. However, the mode can still be categorized into their prevailing vibration, which is a bending vibration. Figures 6(a) and (b) show the mode shape comparison between experiments and computer simulation for both in-plane and out-plane modes.



(a) The in-plane mode(experiment & simulation)



(b) the out-plane mode(experiment & simulation)

Fig. 6 Comparison of mode shapes between experimental and computer simulations

### 5.2 Firing Load Cases Response

Modeling had been verified, the crankshaft was idealized with applicable engine model systems. In order to identify the excitation forces in the in-plane and out-plane direction, three types of load cases were considered. Load case 1 is a vertical direction representing as in-plane excitation, Load case 2 is a horizontal direction representing as out-plane excitation and load case 3 is a result of the combination Load case 1 with Load case 2 which is the in-plane and out-plane direction of excitation. The characteristics of the responses identifying the load cases 3 and 1 and load cases 3 and 2 were compared and measured. Table 2 gives a description of the load case three types of load case were considered.

Firstly, the force acting on the firing condition was investigated. Normally, the forces are expanded into a series of harmonic force components by Fourier analysis in the time domain, but, since the forces are involved with nonlinear equation, a sinusoidal random input describing function approach was used.<sup>(8)</sup> Next, from Figs. 7(a) and (b), it can be seen that for load case 3, in the vertical and lateral directions the exciting response is comparably higher than for load case 1 in the vertical

direction. Load case 3 also maintained a higher level response than load case 2. Therefore to identify the average response excited at each bearing journal only load case 3 was selected to be used for the next analysis.

**5.3 RMS Value At The Bearing Journals**  
Based on the above results, the average

Table 2 Load case

Type	Description
Load case 1	Vertical direction
Load case 2	Lateral direction
Load case 3	Vertical and lateral direction

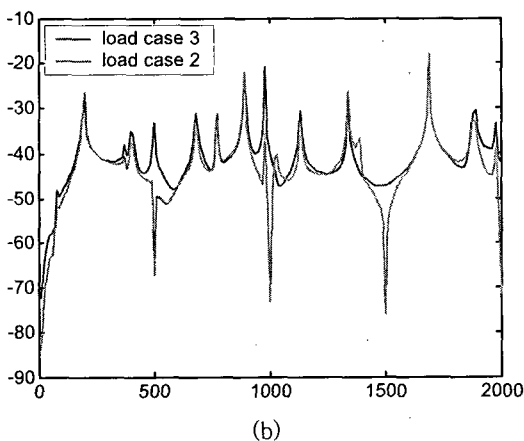
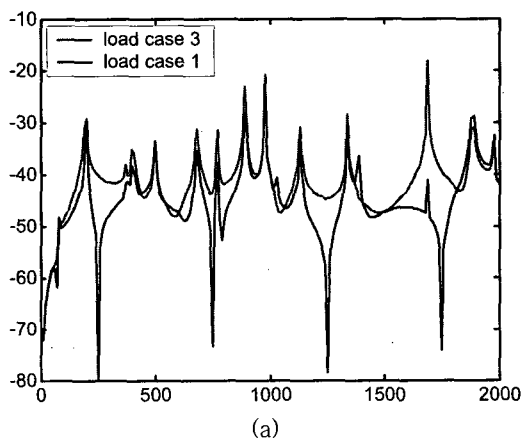


Fig. 7 Comparison of responses between (a) load cases 3 and 1 (exciting at the first crank pin) and (b) load cases 2 and 3 (exciting at the first crank pin)

excited load response at the bearing journals was identified using RMS values. The RMS value is very simple and makes it easier to identify the average of energy transferred at a certain given load. The highest RMS values provide the highest energy, but the disadvantage of this energy is that it can reduce the lubrication film thickness in the bearing journal and cause wear. Figure 8 shows the level of RMS values for each bearing journal at the crankshaft. The location of bearing journal 2 was identified as having the highest energy calculated using the RMS value. The RMS value decreased linearly until the energy at bearing journal 5 was calculated. The highest RMS value at bearing journal 2 occurred because the effects of alternating energy donated by the flywheel located the left-hand side created a bending mode shape.

**5.4 Effects Of The Counter Weight Modification**

The crankshaft could not to be modified

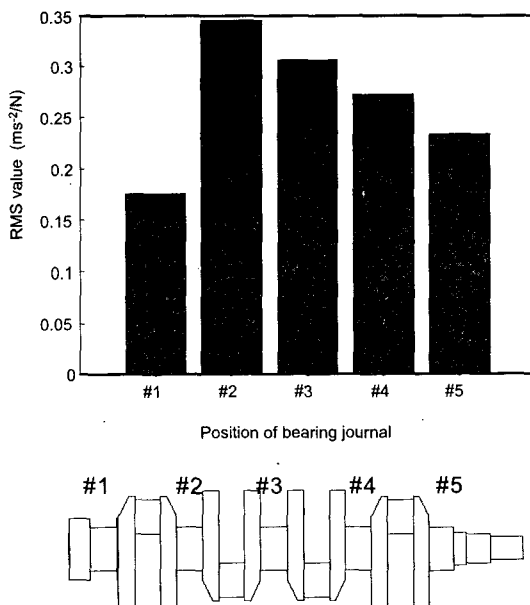


Fig. 8 Comparison of the RMS values at each bearing journal



easily, and moreover the experimental approach with a new modification structure in the crankshaft would be very expensive to implement in both time and money. To overcome this problem, a theoretical approach was used which required considerably less time and money to predict the behaviors of the crankshaft system faster and easier through the energy stored in the shaft system. Ideally, the crankshaft counter weights are given a step by step modification in the same way as the additive stiffness process shown in Fig. 9. The responses due to this counter weights modification of the counter weights are measured at each step on bearing journal 2. The RMS values were calculated to identify the transferable energy and the lowest RMS value was used to provide the suitable shapes for the crankshaft counter weights. Figure 10 shows the levels of RMS values for each case, with a modification for three repeatable

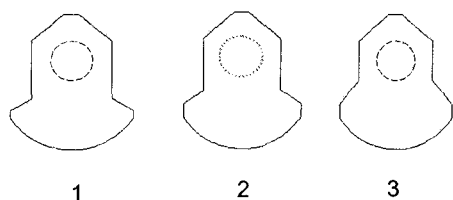


Fig. 9 Counter weight modeling (1-original)

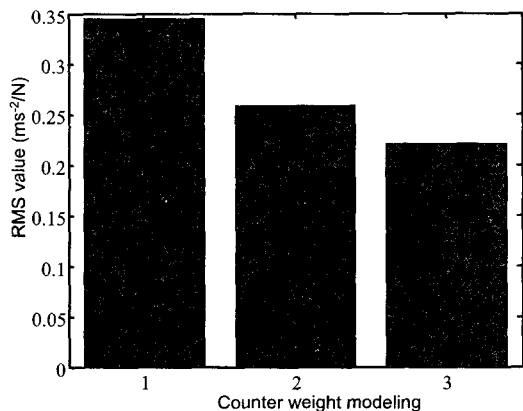


Fig. 10 Comparison of the RMS value corresponding to the counter weight shape

counter weights. It was found that modification of the counter weight could give beneficial effects to the energy characteristics and transferable energy at a level of RMS value that decreased linearly. As a result, the shape of the optimum counter weights was found from a given shape with the lowest RMS value. However, in order to obtain a perfect dynamic behavior, a further studies regarding on a static and dynamic balancing should be carry out with a new obtained counter weight. These will give better information and provides a perfect procedure in a dynamic analysis.

## 6. Conclusions

The crankshaft for an automotive internal combustion inline engine was investigated and the conclusions are given as follows :

(1) The in-plane and out-plane calculated natural frequencies, mode shapes and frequency responses show that the results give good agreement with the experimental results. Thus the modeling of the crankshaft was verified.

(2) The characteristics of a load case were identified and by using both vertical and lateral directions forces it was verified that the vertical and lateral forces gave the higher load.

(3) The RMS value was found to be a simple method to identify the energy transfer in vibration and was dominantly excited at bearing journal 2.

(4) The modification to the counter weight produced lower RMS values and it was easy to reach an optimum design of crankshaft counter weights.

## References

- (1) Kang, Y., Sheen, G. J. and Tseng, S. H., 1998, Modal Analyses and Experiments for Engine

Crankshafts, *Journal of Sound and Vibration*, Vol. 214, No. 3, pp. 413~430.

(2) Lee, J. Y., Aminudin, A. and Oh, J. E., 2003, The Three-dimensional Crankshaft Analysis using Transfer Matrix Method. Proc. of Malaysian Science and Technology Congress, Cititel, Midvalley, Kuala Lumpur, Malaysia.

(3) Kim, K. S., Oh, J. E., Lee, J. Y. and Kim, M. B., 1991, Two Dimensional Vibration Crankshaft Analysis Using Transfer Matrix, *KSME*, Vol. 15, No. 2, pp. 55~462.

(4) Morita T. and Okamura H, 1995, Simple Modeling and Analysis for Crankshaft Three-dimensional Vibrations, Part 2 : Application to an Operating Engine Crankshaft, *Transactions of the ASME*, Vol. 117, pp. 80~86.

(5) Tetsuji, M. and Hideo, M., Analysis of Crankshaft Three-dimensional Vibrations in a Rotating Coordinate System, SAE 951292, pp. 473~480.

(6) Okamura H., Shinno A., Yamanaka T., Suzuki A. and Sogabe K., 1995, Simple Modeling and Analyses for Crankshaft Three-dimensional Vibration, Part I: Background and Application to

Free Vibrations, *Transactions of the ASME*, Vol. 117, pp. 70~79.

(7) Khang, U. J., Eigenmode Sensitivity and Structure Modification for Improvement of Dynamic Characteristics of Engine Mount System, 1991, Ms Thesis, Department of Precision Mechanical Engineering, Hanyang University.

(8) Rao, V. D., 2000, "Vehicle Dynamics," Narosa Publishing House, India.

(9) Kazuomi, O. and Mitsuo N., 1980, Relation Between Crankshaft Torsional Vibration and Engine Noise, SAE, 790365, pp. 1291~1298.

(10) Martin, S., Frank, K., Herald, S. and Rolf, I., Combustion Supervision by Evaluating the Crankshaft Speed and Acceleration, SAE 2000 World Congress, 2000-01-0558, pp. 1~8.

(11) Mourelatos, Z. P., 2000, An Efficient Crankshaft Dynamic Analysis Using Substructuring with Ritz Vectors, *Journal of Sound and Vibration*, Vol. 238, No. 3, pp. 495~527.

(12) MSC/NASTRAN User's Manual, 2001 Volume 1 & II The MacNeal Schwendler Corp., Los Angeles, California.