

# DEVELOPMENT OF AN OPTIMIZATION TECHNIQUE OF A WARM SHRINK FITTING PROCESS FOR AN AUTOMOTIVE TRANSMISSION PARTS

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**ABSTRACT**—A fitting process carried out in the automobile transmission assembly line is classified into three classes; heat fitting, press fitting, and their combined fitting. Heat fitting is a method that applies heat in the outer diameter of a gear to a suitable range under the tempering temperature and assembles the gear and the shaft made larger than the inner radius of the gear. Its stress depends on the yield strength of a gear. Press fitting is a method that generally squeezes gear toward that of a shaft at room temperature by a press. Another method heats warmly gear and safely squeezes it toward that of a shaft. A warm shrink fitting process for an automobile transmission part is now gradually increased, but the parts (shaft/gear) assembled by the process produced dimensional change in both outer diameter and profile of the gear so that it may cause noise and vibration between gears. In order to solve these problems, we need an analysis of a warm shrink fitting process in which design parameters such as contact pressure according to fitting interference between outer diameter of a shaft and inner diameter of a gear, fitting temperature, and profile tolerance of gear are involved. In this study, an closed form equation to predict the contact pressure and fitting load was proposed in order to develop an optimization technique of a warm shrink fitting process and verified its reliability through the experimental results measured in the field and FEM, thermal-structural coupled field analysis. Actual loads measured in the field have a good agreement with the results obtained from theoretical and finite element analysis and also the expanded amounts of the outer diameters of the gears have a good agreement with the results.

**KEY WORDS** : Warm shrink fitting process, Automotive transmission, Contact pressure, Fitting load, Interference, Finite Element Method (FEM)

## NOMENCLATURES

- $r_1$  : inner radius of shaft  
 $r_2$  : outer radius of shaft or inner radius of gear  
 $r_3$  : outer radius of gear  
 $k$  : radial coefficient,  $r_3/r_2$   
 $E, \nu$  : Young's modulus and Poisson's ratio  
 $p$  : contact pressure occurred by shrink fitting  
 $F_{SF}$  : shrink fitting force  
 $\delta_{SF}$  : the amount of fitting interference obtained by calculation  
 $\delta_D$  : the amount of interference used in the field  
 $\delta_H$  : displacement by heating

## 1. INTRODUCTION

Automotive transmission is the apparatus to convert

generated power of an engine to torque according to velocities and to transfer it. During traveling, torque and angular velocity are changed by velocity variation, and then gears are used to transmit motion from one rotating shaft to the wheels. These gears used on transmission are installed on shaft. In these days, a warm shrink fitting process has been gradually increased to squeeze gears heated warmly toward the outer diameter of a shaft (Ko *et al.*, 2006).

On this process, although the dimensional tolerance of each part should be very precise, it is very difficult to set up the process conditions like temperature and time of shrink fitting and to precisely forecast quality of products as the amount of expansion or profile variation of gears, and contact force between the inner diameter of gear and the outer diameter of shaft and so on. Because the recent researches have been confined to how to improve the efficiency of a transmission (Moon *et al.*, 2005; Rao and Tjandra, 1994; Sen and Aksakal, 2004; Sung and Kim,

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2002), few studies have been done on fabricating processes, and the research for the shrink fitting process is also restricted to the design of die sets (Besterfield *et al.*, 2003; Chen *et al.*, 2004).

In this paper, an closed form equation for prediction of contact pressure and fitting load was proposed in order to establish an optimization technique of a warm shrink fitting process and verified its reliability was verified with the experimental results measured in the field and by thermal-structural coupled field analysis.

## 2. THEORETICAL ANALYSIS

A closed form equation was proposed to calculate the contact pressure induced in assembled parts (gear and shaft). Equation (1) was derived from Figure 1 (Kurt, 1985; Laue and Stenger, 1981; Ugral and Fenster, 1981; Zhang *et al.*, 2000).

The assembled transmission parts (Figure 1(a)) can be separated to two parts, gear and shaft (Figure 1(b)). If the inner pressure ( $p_i$ ) acting on the shaft and radius ( $r_i$ ) of a shaft and the outer pressure ( $p_o$ ) acting on the gear assume zero, Equation (2) can be obtained by considering two cases: shaft subjected to outer pressure; and gear subjected to inner pressure.

The first term of Equation (2) is the amount of contraction occurred on the outer diameter of a shaft by

shrink fitting and the second term is the amount of expansion on the inner diameter of a gear. The total displacement  $\delta_{SF}$  is equal to the amount of shrink fitting interference.

$$u = \frac{1 - \nu r_1^2 p_i - r_2^2 p_o}{E(r_2^2 - r_1^2)} + \frac{1 + \nu r_1^2 r_2^2 (p_i - p_o)}{E(r_2^2 - r_1^2)} \frac{1}{r} \quad (1)$$

$$\delta_{SF} = \frac{r_2^2 p_o r}{E_a (r_2^2 - r_1^2)} \left[ (1 - \nu_a) + (1 + \nu_a) \frac{r_2^2}{r^2} \right] + \frac{r_1^2 p_i r}{E_b (r_3^2 - r_2^2)} \left[ (1 - \nu_b) + (1 + \nu_b) \frac{r_3^2}{r^2} \right] \quad (2)$$

$P_i$  and  $P_o$  are equal to the contact pressure ( $P$ ), and since the materials of a gear and a shaft are same, Young's Modulus ( $E_a$  and  $E_b$ ) and Poisson's Ratio ( $\nu_a$  and  $\nu_b$ ) are also same. And if the ratio of outer radius of gear ( $r_3$ ) over inner radius of shaft ( $r_2$ ) is substituted to radial coefficient ( $k$ ), Equation (3) can be obtained from Equation (2).

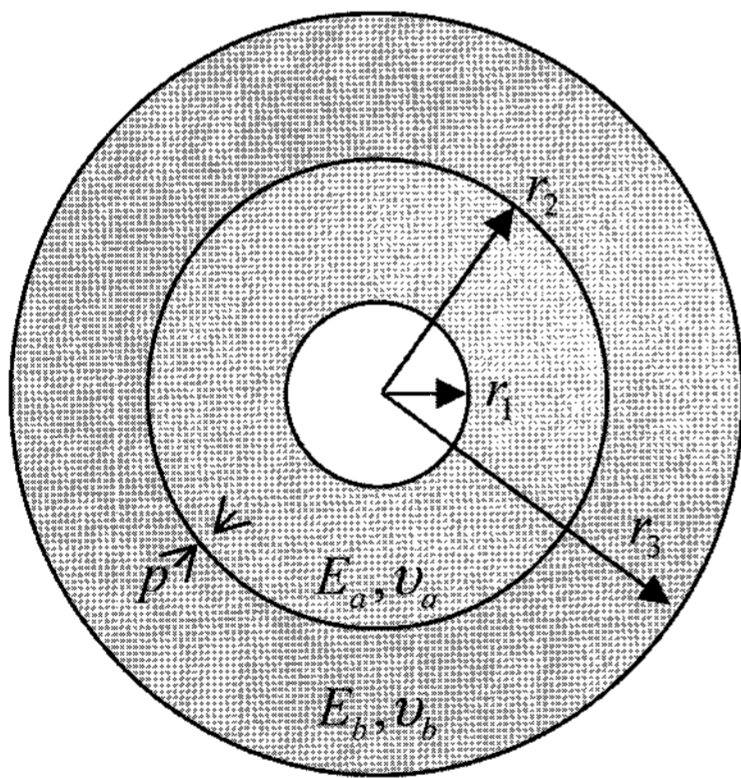
$$\delta_{SF} = \frac{2r_2 k^2}{E(E^2 - 1)} p \quad (3)$$

If materials of a gear and a shaft are same, Equation (3) can be used for calculating the amount of interference and contact pressure on the shrink fitting process, otherwise Equation (2) can be used.

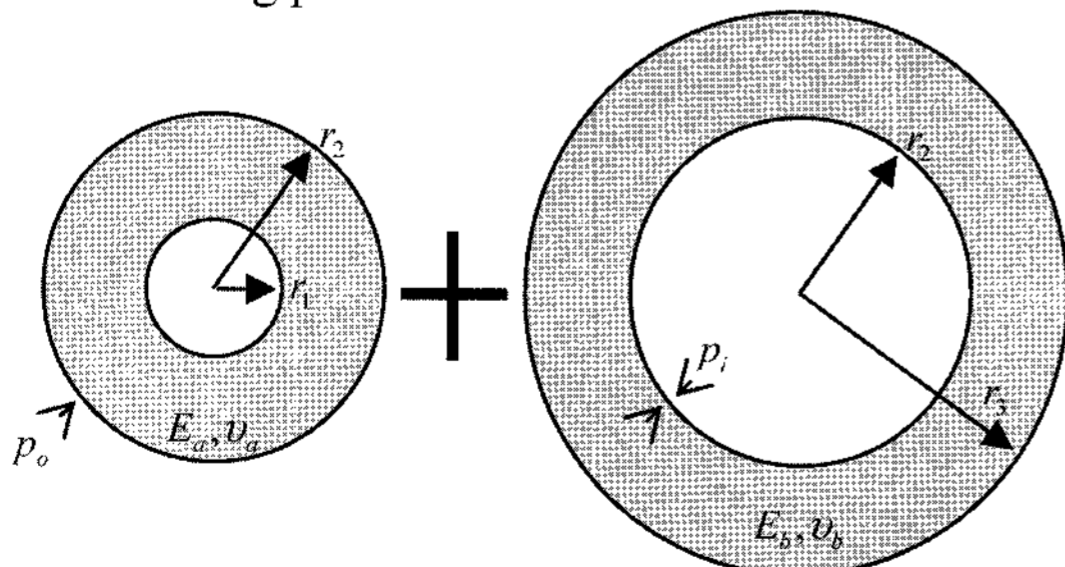
## 3. PROCESS DIVISION FOR FINITE ELEMENT ANALYSIS

FE Analysis has been carried out for verification of the suggested theoretical closed form equation suggested. The warm fitting process for automotive transmission parts can be divided into several sub-processes as shown in Figure 2: heating, positioning, shrink fitting, and cooling processes.

The heating process is to heat machined gears up to the preheating temperature in a low frequency induction furnace, the positioning process is to carry the heated gear to a shaft by an automated equipment, the shrink fitting process is to squeeze the positioned gear toward



(a) Contact pressure acting equal on both gear and shaft by shrink fitting process.



(b) Free body diagram for shaft and gear.

Figure 1. A Equilibrium model for shaft and gear.

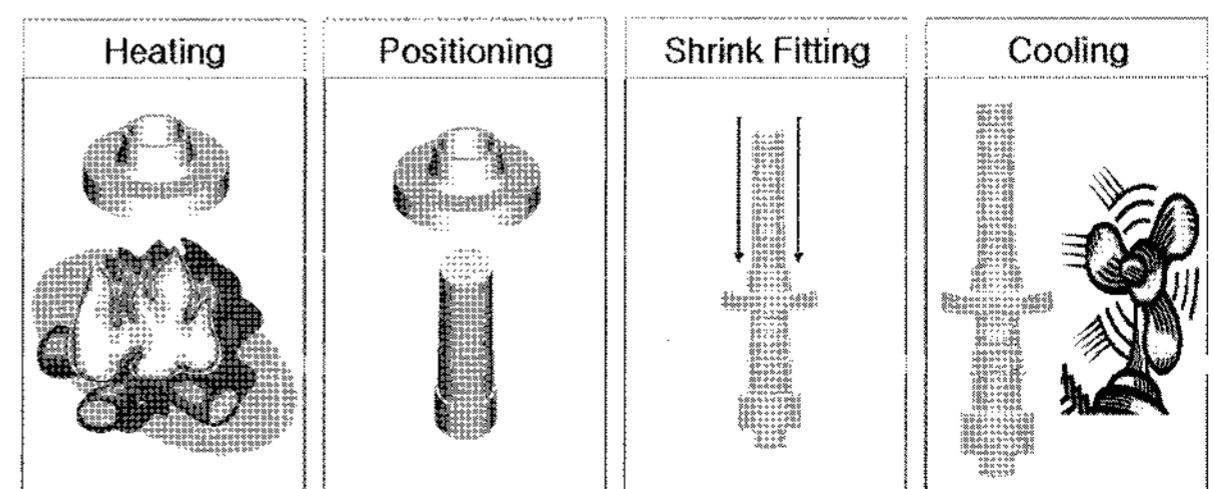


Figure 2. Process division of warm shrink fitting.

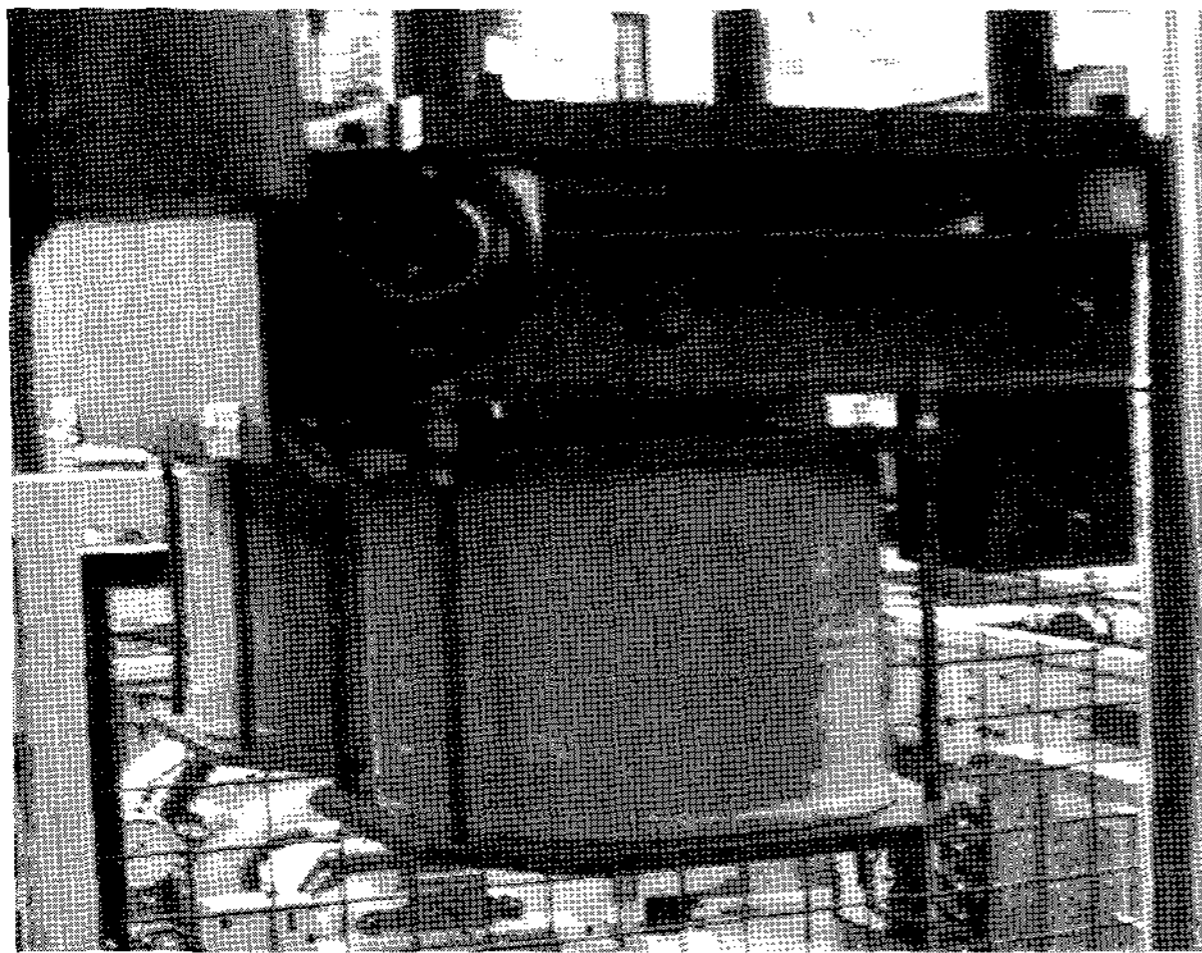


Figure 3. Low frequency induction furnace used on heating process.

the outer diameter of the shaft, and the cooling process is to refrigerate the gear and the shaft on the air just before the warm shrink fitting process of a next gear.

### 3.1. Heating Process

A machined gear is heated up to the appropriate pre-heating temperature in the low frequency induction furnace. For this analysis, convection on the surface of the gear was considered as boundary condition, and local conduction between the gear and the supporter was ignored. The thermal-structural coupled field method, that is to carry out structural analysis with the results of thermal analysis, was applied to analyze this process.

### 3.2. Positioning Process

The heated gear in the furnace is carried to the shaft by the automated equipment. At this point there are the various heat transfers like local conduction between the gear and the automated equipment, and convection and conduction between the gear and the shaft.

In spite of these complex heat transfers, this process was disregarded because the positioning process was completed very quickly within 1–2 seconds. The temperature measured just before the shrink fitting process was only considered on the heating process.

### 3.3. Shrink Fitting Process

The moment ram of the equipment is dropped, the preheated gear positioned in the shaft is squeezed toward the outer diameter of the shaft. Heat transfer was ignored in analysis of the shrink fitting process because of the short time process within 0.5 second.

Figure 4 shows the equipment and the indicator to display displacement-load diagram in the shrink fitting process.

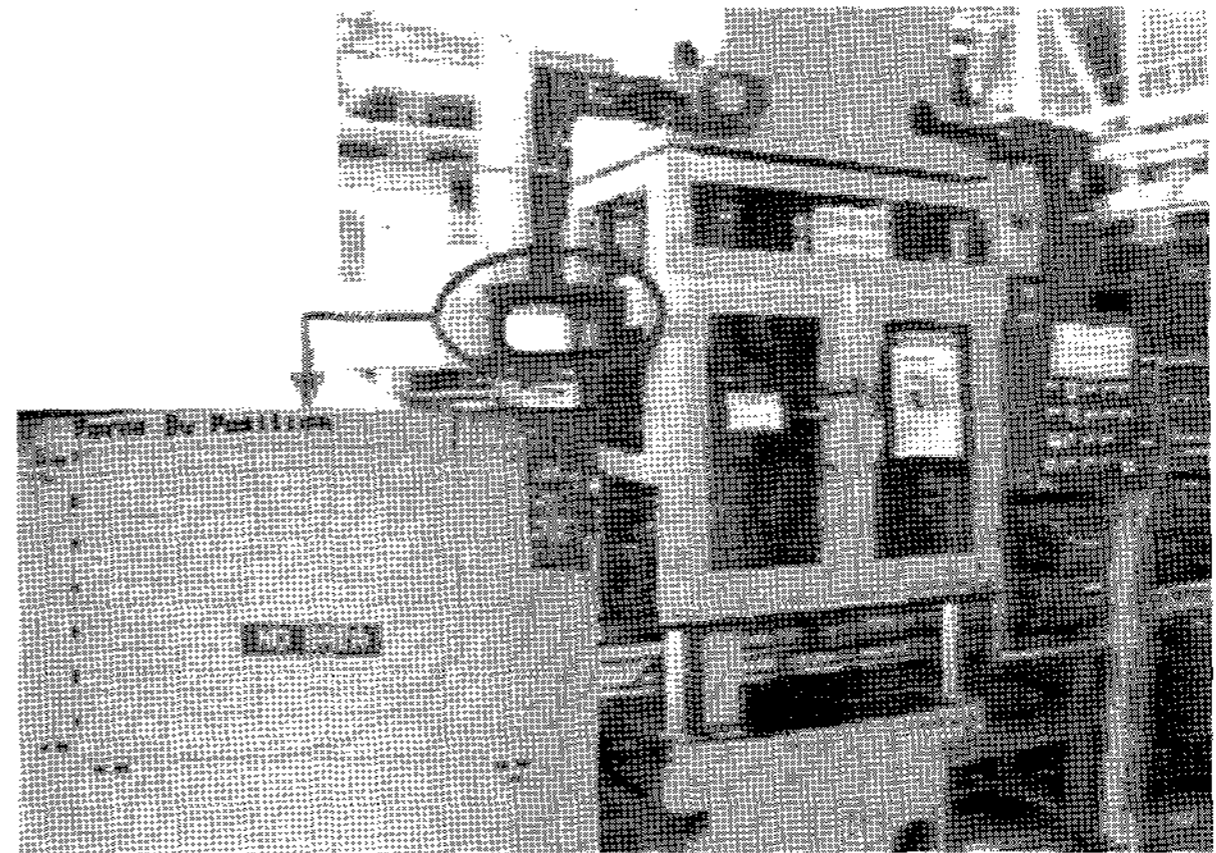


Figure 4. Manufacturing equipment with indicator displaying load-stroke diagram.

### 3.4. Cooling Process

After the shrink fitting process, the gear and shaft is carried out by a conveyor belt, and cooled down in the atmosphere temperature. Conduction and convection between the gear and the shaft, and the thermal-structural coupled field analysis were considered in analysis of this process.

## 4. SIMULATION RESULTS AND DISCUSSIONS

### 4.1. Contact Pressure and Friction Coefficient

#### 4.1.1. Contact pressure

The research targets are 3rd–5th gears currently used on 1,500–2,000cc automobiles, and Figure 5 shows a 3rd simple gear drawing. The diameter of a shaft to be squeezed by the gear is 30 mm, and the amount of

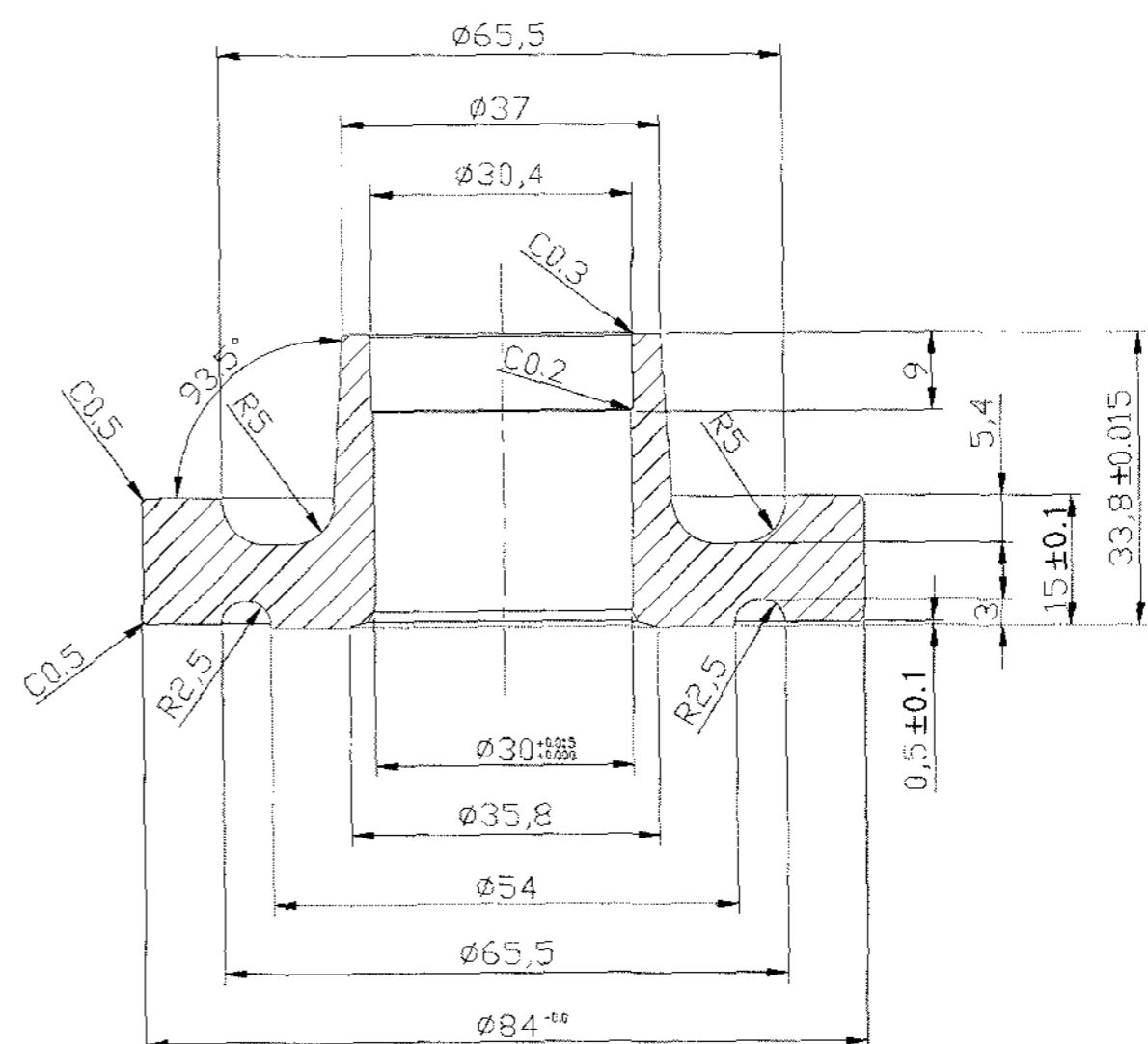


Figure 5. A 3rd gear sample to be used in analysis.

interference is 0.094–0.083 mm.

From all variables given in Figure 5 and the material property of the gear, Equation (4) and (5) can be easily derived. The amount of interference in the field can be calculated from Equation (6) by considering the amounts of maximum fitting interference and heat expansion. The contact pressure induced in the shrink fitting process of the 3rd gear is obtained by substituting Equation (6) to Equation (4) and was calculated as 17.535 kgf/mm<sup>2</sup>.

$$\delta_{SF} = 0.001645p \tag{4}$$

$$\delta_H = \alpha \cdot \Delta T \cdot L = 0.0201 \tag{5}$$

$$\delta_{SF} = \delta_i - \delta_H = 0.0269 \tag{6}$$

#### 4.1.2. Friction coefficient

A friction coefficient was calculated, instead of a ring compression test, by the closed form equation suggested and the shrink fitting load measured, because the shrink fitting load could be measured by the indicator as shown in Figure 4. The loads measured are shown in Figure 6 and the mean shrink fitting load is 1,260 kgf in the 3rd gear.

Since a relationship of the contact pressure and the shrink fitting load is expressed as Equation (7), a friction coefficient caused in the gear and the shaft can be calculated by substituting each variable to Equation (7) and is about 0.05.

$$F_{SF} = \mu \cdot A \cdot p \tag{7}$$

### 4.2. Results of Finite Element Analysis

#### 4.2.1. Heating process

The tool used in this study is ANSYS classic 8.0 widely used on the engineering fields like thermal, structural, vibration, and so on. Figure 7 shows the result of FE analysis carried out as the thermal-structural coupled field method in the heating process of the 3rd gear of

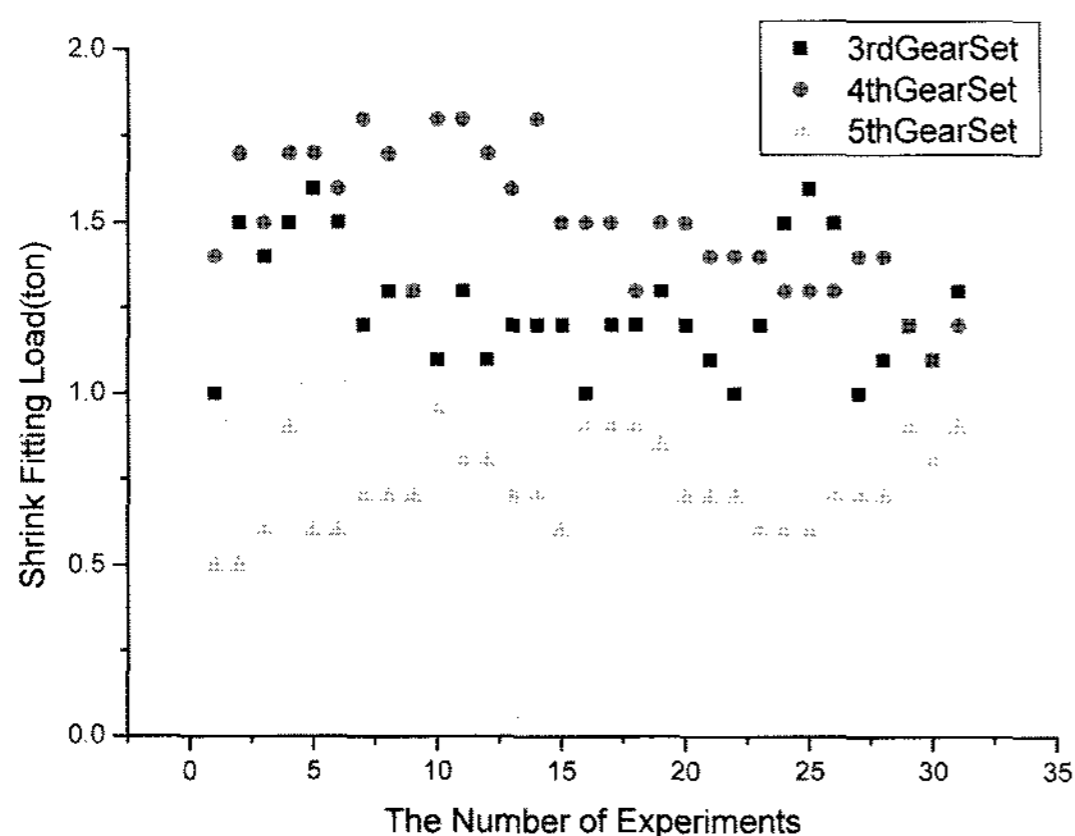


Figure 6. loads measured in shrink fitting process carrying out in the field.

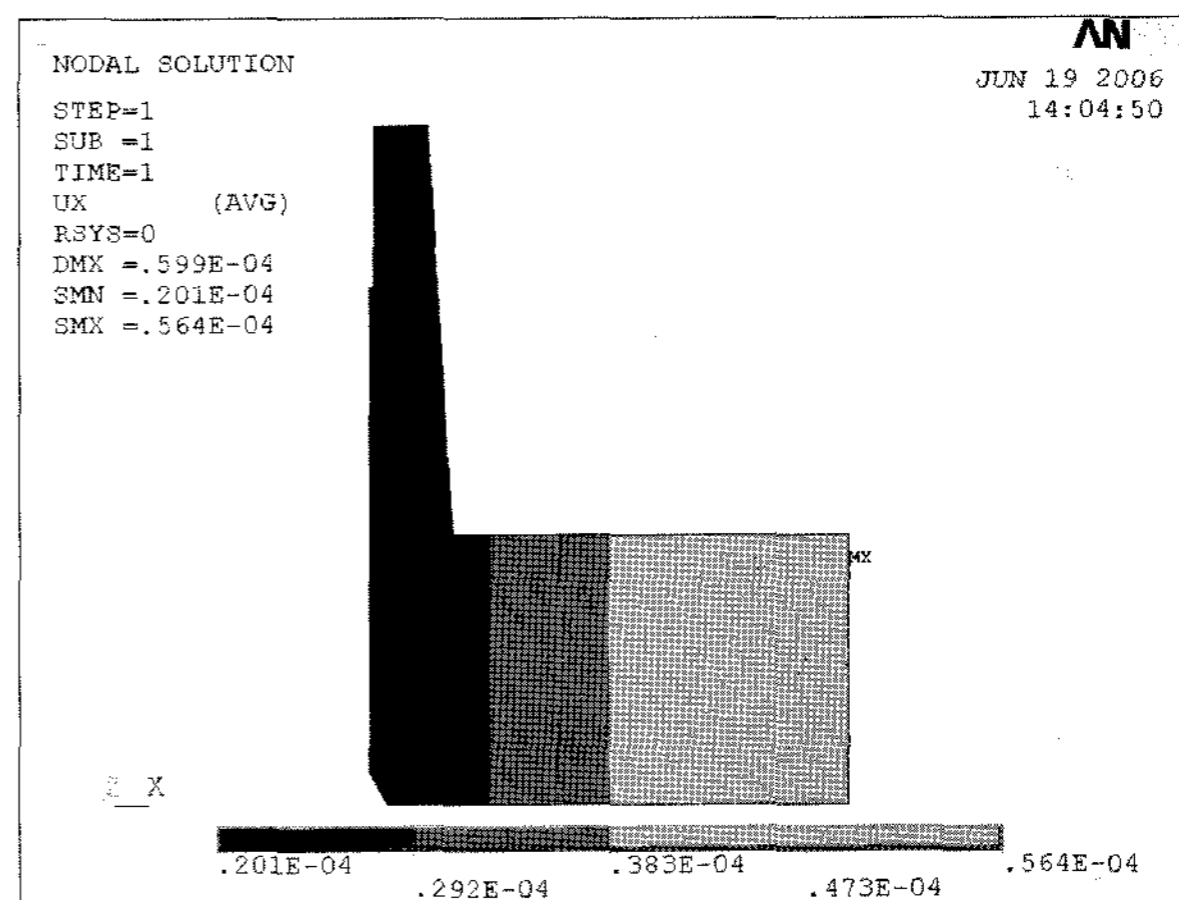


Figure 7. The amount of heat expansion obtained from FEM simulation for 3rd gear in case of maximum interference.

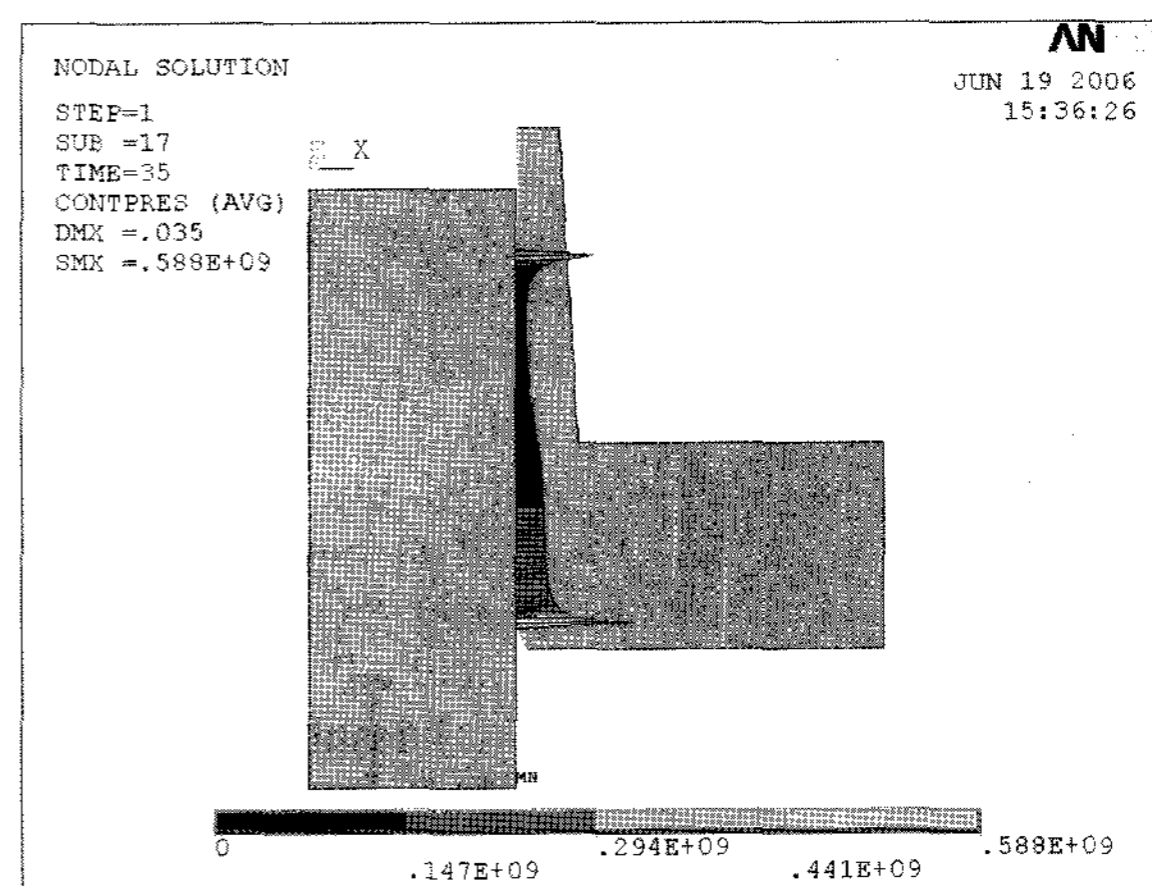


Figure 8. Contact pressure obtained from FEM simulation for 3rd gear in case of maximum interference.

automotive transmission parts.

The maximum amount of heat expansion on the outer diameter of the gear is 56.4 μm, the minimum on the inner diameter is 20.1 μm.

#### 4.2.2. Shrink fitting process

Figure 8 shows the result of the shrink fitting process carried out with that of the previous section, and the results of the theoretical analysis regarding contact pressure and shrink fitting load compared with experimental data are shown in Table 1–3.

The experimental results were a good agreement with those of the theoretical and finite element analysis, and that means there are no inferior goods in measured samples. The results obtained from the proposed equation are larger about 30% than those of FEM analysis, and the difference is caused by derivation of the equation based

Table 1. Comparison of fitting load of 3rd gear sets.  
(Unit: kgf)

Interference	FEM Analysis	Theoretical Analysis	Experimental Data
Max.	1,664.70	2,236.87	1,650.00
Min.	548.23	841.25	1,000.00
Average	1,106.47	1,593.06	1,260.00

Table 2. Comparison of fitting load of 4th gear sets.  
(Unit: kgf)

Interference	FEM Analysis	Theoretical Analysis	Experimental Data
Max.	1,970.84	2,647.79	1,800.00
Min.	846.59	1,211.39	1,100.00
Average	1,408.72	1,934.59	1,522.97

Table 3. Comparison of fitting load of 5th gear sets.  
(Unit: kgf)

Interference	FEM Analysis	Theoretical Analysis	Experimental Data
Max.	1,061.50	1,630.45	950.00
Min.	468.53	561.50	500.00
Average	765.02	1,095.96	730.32

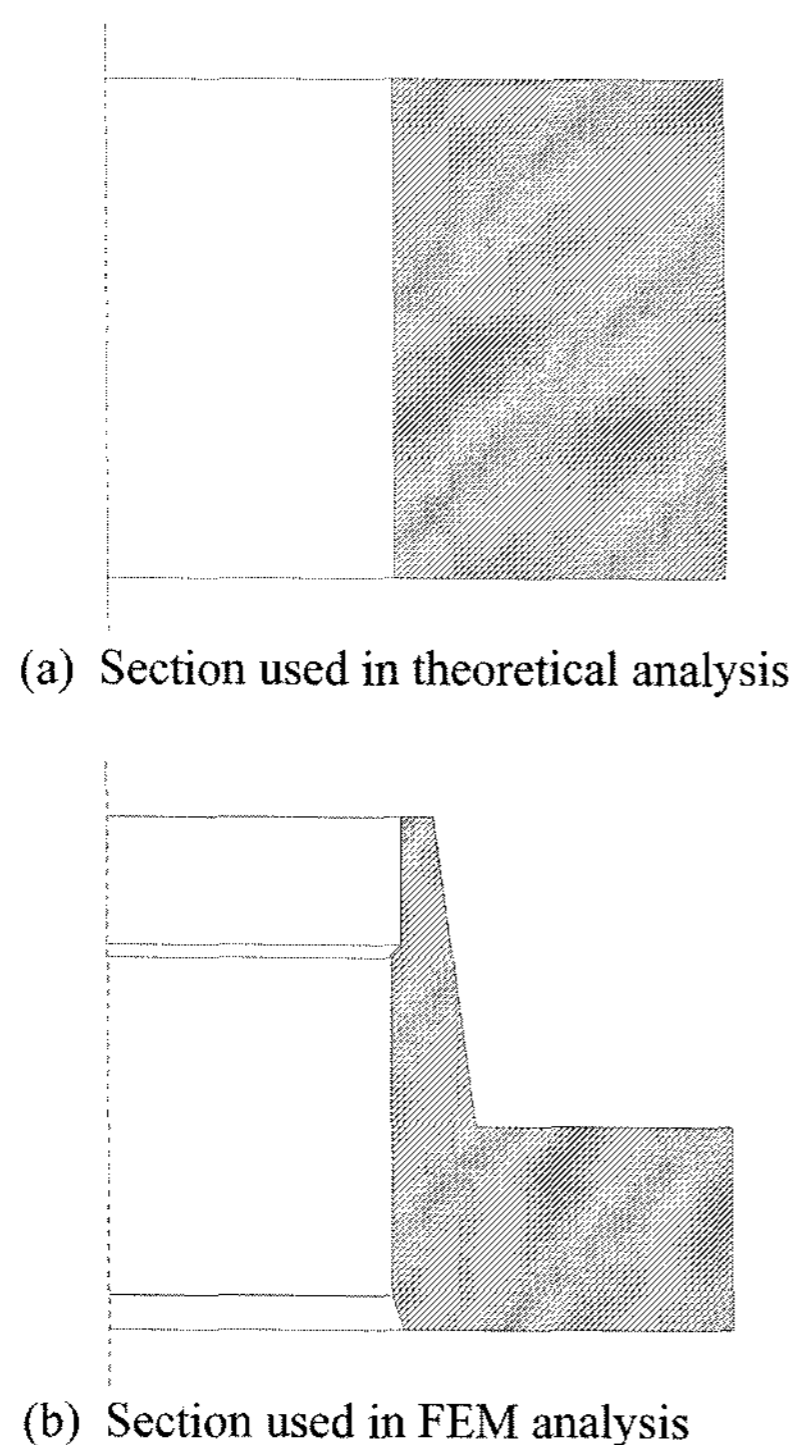


Figure 9. Comparison of model section between theoretical and FEM analysis.

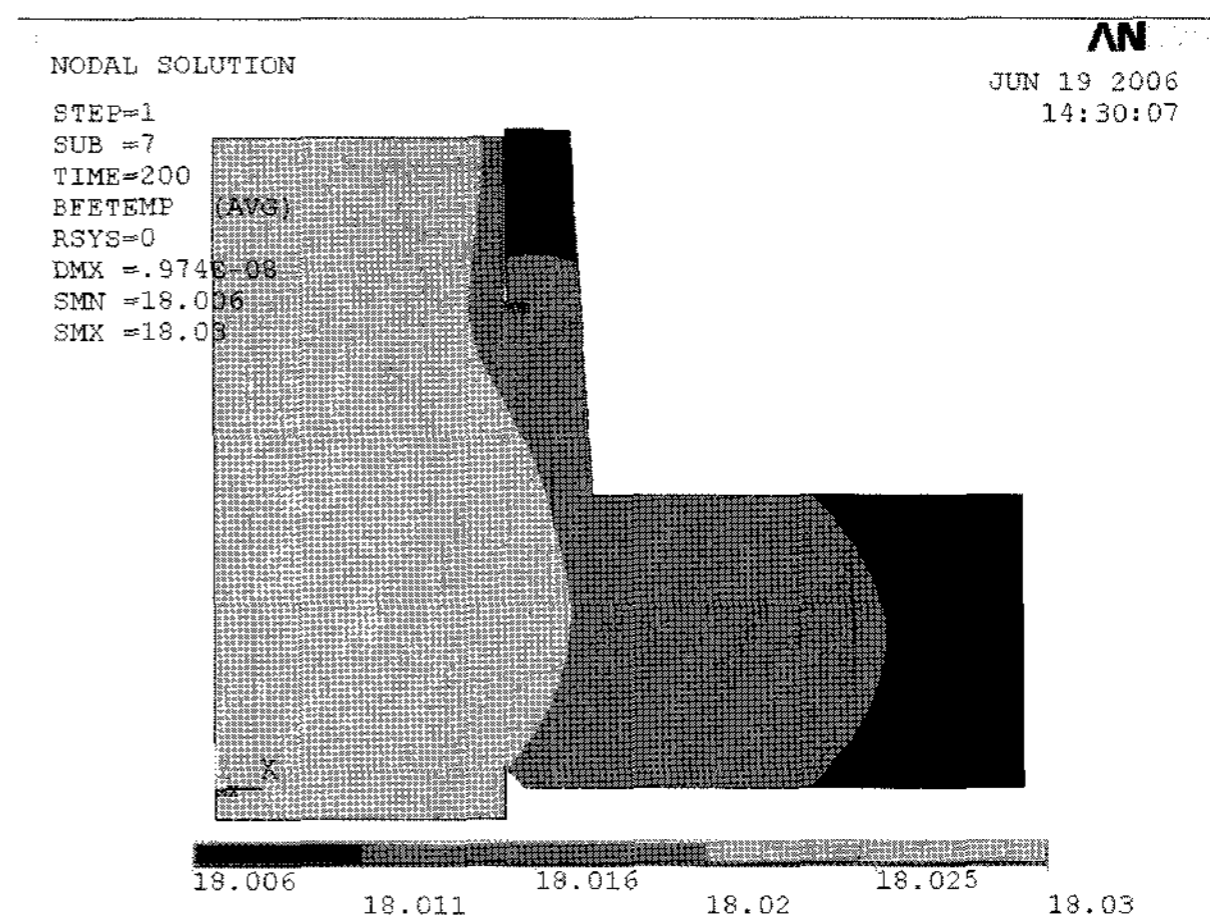


Figure 10. Temperature distribution obtained from FEM simulation for 3rd gear in case of maximum interference.

Table 4. Comparison of the radial profile displacement of gears.  
(Unit:  $\mu\text{m}$ )

	3rd Gear	4th Gear	5th Gear
Max.	8.60	7.48	6.53
Min.	5.44	4.98	4.11
Experimental Data	8.00	6.00	5.00

on the assumption that the section of the gear is rectangular as shown in Figure 9(a).

But the section of the gear used in FEM analysis is like as shown in Figure 9(b). Therefore Equation (3) needs an appropriate coefficient for compensation, and it is reasonable to apply the coefficient, 0.7, to the warm shrink fitting process of the automotive transmission part if Equation (3) uses in the field.

#### 4.2.3. Cooling process

The result carried out in the cooling process with that of the previous section is shown in Figure 10, and the comparison of the radial profile displacements with the experimental data is shown in Table 4.

Figure 10 shows that the gear was nearly cooled down at the room temperature in the end step of the cooling process, and Table 4 shows that the experimental data are between the maximum and minimum values of radial profile displacement of gears.

## 5. CONCLUSIONS

This study carried out the analysis regarding shrink fitting load and amount of expansion of the gear with major process variables, amount of shrink fitting interference, in order to establish an optimization technique of the

warm shrink fitting process for automotive transmission parts.

- (1) The closed form equation was suggested for calculating contact pressure and amount of fitting interference on the warm shrink fitting process, and the results of the suggested equation were in a good agreement with the experimental data measured in the field and with the simulation results using ANSYS classic 8.0.
- (2) The process management can be fulfilled through protecting deformation and breakage of the gear induced in the shrink fitting process on the manufacturing line by obtaining the optimized shrink fitting load.
- (3) Load subjected in the the warm shrink fitting interference in the warm shrink fitting process can be calculated by the closed form equation.

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