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ROLL CENTER ANALYSIS OF A HALF-CAR MODEL USING POLE FOR SMALL DISPLACEMENT

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ABSTRACT–In this paper, roll behavior of three planar half car models are compared. The first model is a simple model whose contact point between a wheel and the ground is assumed to be fixed with a revolute joint. The second model is a modified model of the first model, whose wheel tread width can vary. In this model, the instant center of a wheel with respect to the ground, which is crucial to find the roll center, is assumed to be at the contact point of a wheel and the ground. The last model uses the pole of a wheel with respect to the ground for small displacement as the instant center of a wheel with respect to the ground. Loci of the center of gravity point, the fixed and the moving centrodes which are traces of roll center position in the ground and the body frame respectively, wheel contact points, and instant centers of a wheel with respect to the ground are calculated.

KEY WORDS : Roll center, Kinematic Analysis, Instant center, Centrodes, Suspension System

1. INTRODUCTION

The roll center of a vehicle is determined by kinematic properties of the vehicle model such as joint positions of the suspension system. This roll center is called the kinematic roll center. In addition, compliance properties, such as bushing and tires, also affect roll center position. When the kinematic model of a suspension mechanism is constructed, the kinematic roll center is determined. Since overall displacement characteristic of a suspension system is determined by the kinematic roll center, it is a very important factor in the early design stage of a suspension mechanism. Various approaches have been proposed in many researches to determine the kinematic roll center position. In many literatures, a wheel contact point is considered fixed in roll center determination. However, this model isn't realistic since wheel tread width is constant. In another approach, wheels are set free to move in lateral direction and wheel contact points are used as instant centers of wheels with respect to the ground. In this case, behavior of the model may be more realistic but roll center positions using this concept are inappropriate because wheel contact points cannot be instant centers of wheels with respect to the ground after body rolls from its symmetric position.

Gillespie (1992), Milliken and Milliken (1995), Dixon (1996) and Reimpell *et al.* (2001) have dealt with kinematic roll center concepts in their literatures. Morse and Starkey (1996) have suggested a force-based roll center model. Mitchell (1998) has proposed roll centers for asymmetric suspensions and Gerrard (1999) has proposed combined roll center concept.

Some of them use a fixed tread width model and others a use variable tread width model. All of them, however, use the contact point between a wheel and the ground as instant center of a wheel with respect to the ground. Location of the instant center of a wheel with respect to the ground is difficult to find because of insufficient information for relationship between a wheel and the ground. Therefore, location of the instant center of a wheel with respect to the ground is usually assumed to be at the contact point for simplicity in analysis. Since the assumption used in these previous studies is unsuitable theoretically, the results of roll center analysis may have geometrical errors.

In this paper, roll center analysis using pole for small displacement is proposed to allow wheel tread width change and to satisfy kinematic theory, and roll behavior of the proposed method is compared with those of the existing two methods. The first method which is called fixed tread width roll center analysis (FRCA) in this paper, is a simple method which uses fixed wheel tread width model. The second method which is called contact point roll center analysis (CRCA) uses variable wheel

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tread width model and assumes the wheel contact point as the instant center of a wheel with respect to the ground. Proposed method is called PRCA. In this method, pole for small displacement of a wheel relative to the ground is assumed to be the instant center of a wheel with respect to the ground at that instant.

2. INSTANT CENTERS AND THE THEOREM OF THREE CENTERS

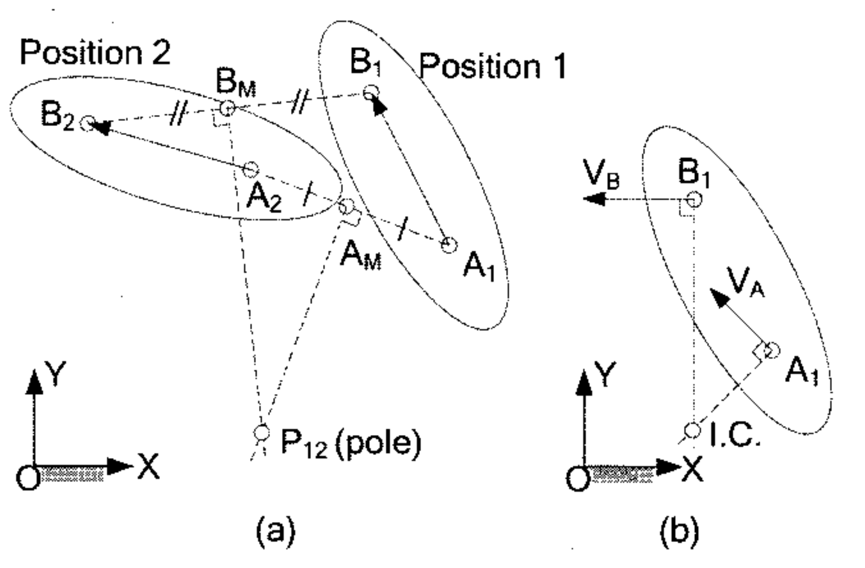
In planar kinematics, when a rigid body moves from one position to other, its displacement can be obtained by pure rotation about a point which is called the pole of the displacement. In instantaneous planar kinematics, an instant center is defined as a point whose relative velocity between two rigid bodies in planar motion is zero. If one rigid body is fixed, the motion of the other body can be described as rotation about the instant center at that instant in the view point of the fixed rigid body. Figure 1 shows concepts of the pole and the instant center of a rigid body in the view point of the fixed coordinate system. When a rigid body displaces infinitesimally, then the pole of the displacement becomes the instant center (Hunt, 1978).

For instantaneous relationship among three rigid bodies, the following Aronhold-Kennedy theorem of three centers (Hunt, 1978) holds: "For three bodies in relative planar motion, their three relative instant centers are always collinear and the directed distances to two of the instant centers from the third are inversely proportional to the (directed) relative angular speeds about the two centers."

In this paper, instant centers of rigid bodies in motion are found by Aronhold-Kennedy theorem and by the fact that the pole and the instant center are coincident in infinitesimal motion.

3. ROLL CENTER

The roll center is the point in the vertical plane which passes through the wheel center points, and in which transverse force can be exerted on the body without kinematic roll angles occurring, thus its location directly influences cornering and motion of the body (Gillespie 1992). It is directly related to vehicle stability. Lee *et al.* (2006) have suggested AGCS system to enhance vehicle stability, which changes suspension geometry when vehicle is in roll motion.



(a) (b)

Figure 1. The pole and the instant center (I.C.).

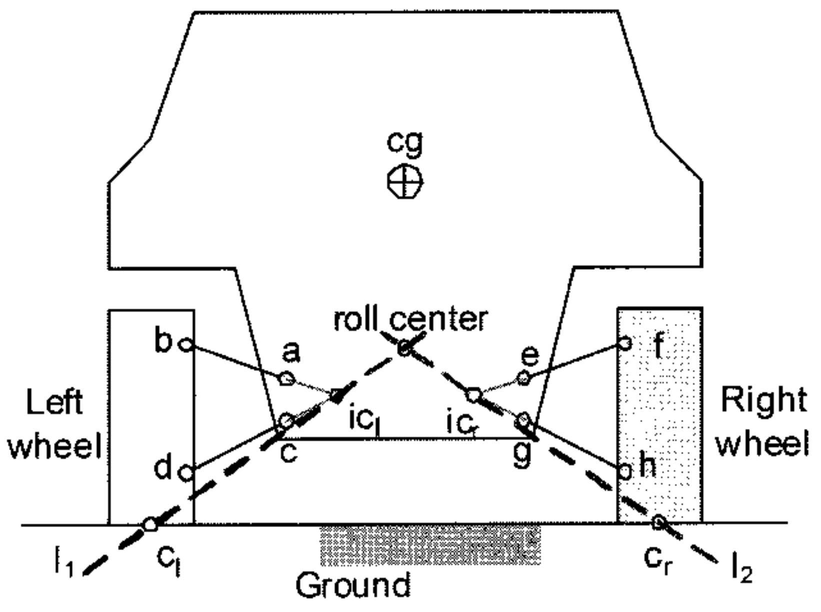


Figure 2. The roll center of a planar car model (rear view).

The position of the roll center of a planar car model with independent suspension systems can be found kinematically using Aronhold-Kennedy theorem since the roll center is the instant center of a vehicle body with respect to the ground. For example, Figure 2 shows a planar car model which has double wishbone type suspension systems. If the instant center of a wheel with respect to the ground (ic_{ground}) is assumed to be at wheel contact points, the roll center of this model is located at the point of intersection between l_1 and l_2. Here, l_1 is the line which passes through the wheel contact point, c_l, and the instant center, ic_l, which is the instant center of the left wheel with respect to the body that is at the intersection of line ab and line cd. l_2 is the line for the right suspension system which is obtained by a similar method used for the left-hand side of the suspension system. As seen in the above example, the position of ic_{ground} is an important factor for determination of the roll center. In a planar car model with one degree of freedom, ic_{ground} is generally assumed to be at the contact point between a wheel and the ground, a wheel is assumed to be connected to the ground by a revolute joint for simplicity.

In reality, however, the contact point of a wheel and the ground may not be ic_{ground} because displacement of a wheel can include translation which causes tread with variation. As shown in Figure 3, depending upon which points are used as ic_{ground}, positions of the roll center are

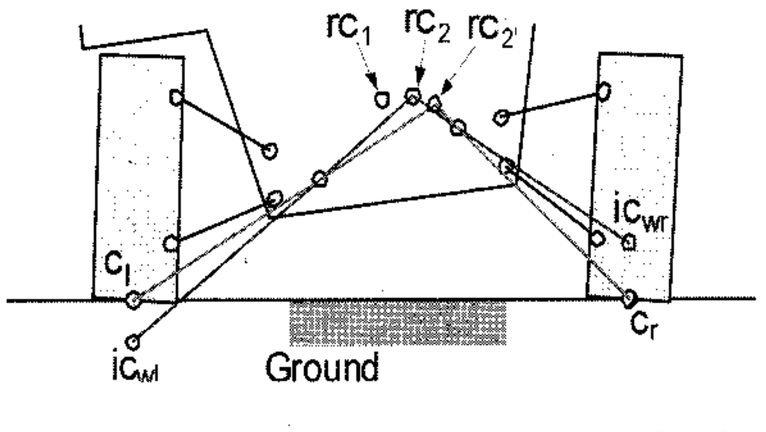


Figure 3. Effects of the instant center of wheels on the position of roll center.

different. In Figure 3, assumed roll center rc_1 at initial position moves to rc_2 when instant centers of wheels with respect to the ground are applied, otherwise if contact points are used to obtain next roll center location, the roll center would move to rc_2'. Hence, different selection of the location of ic_{ground} makes different result on roll center migration.

4. A PLANAR CAR MODELS

A two-dimensional vehicle model is composed with a vehicle body and two wheels which are connected by planar suspension systems as shown in Figure 4(a). Constraints between the body and wheels are definite kinematic constraints but constraints between wheels and the ground are indefinite. For roll motion analysis, a one-degree-of-freedom model is generally used in which translational motions of wheel contact points are fixed as shown in Figure 4(b). But this model, which is used in FRCA in this paper, isn't that realistic since the change of wheel tread width cannot occur during roll motion. For better roll motion estimation, a two-degree-of-freedom model shown in Figure 4(c) which allows wheel tread width variation could be used but the roll center of this model tends to move to the one side which has a sliding joint. A three-degrees-of-freedom model shown in Figure 4(d) can be used for roll motion analysis. This model is used in CRCA which assumes instant centers of wheels relative to the ground always locate at wheel contact points. Relative displacement of a wheel with respect to the ground contains translational displacement in this method. Therefore the instant center of a wheel relative to the ground cannot be on the contact point. From this fact, it is known that roll center positions obtained by CRCA method violates Aronhold-Kennedy theory. Even though CRCA has theoretical error, CRCA is performed to compare with PRCA.

In this paper, a half-car model with variable wheel tread width is proposed as shown in Figure 5 which is the same as in Figure 4(d). To allow variations of wheel tread width, wheels are assumed to be connected to the ground by revolute and sliding joints. To find the position of roll center in its initial symmetric position of the vehicle body, ic_{ground} is assumed to be at contact point which coincides with the revolute joint on the wheel.

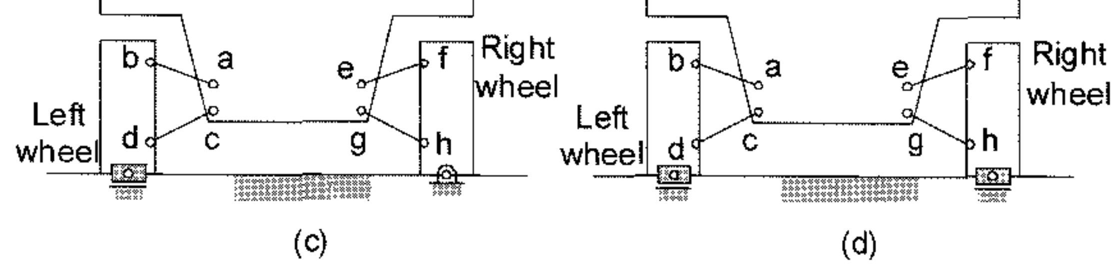


Figure 4. Roll models of planar car.

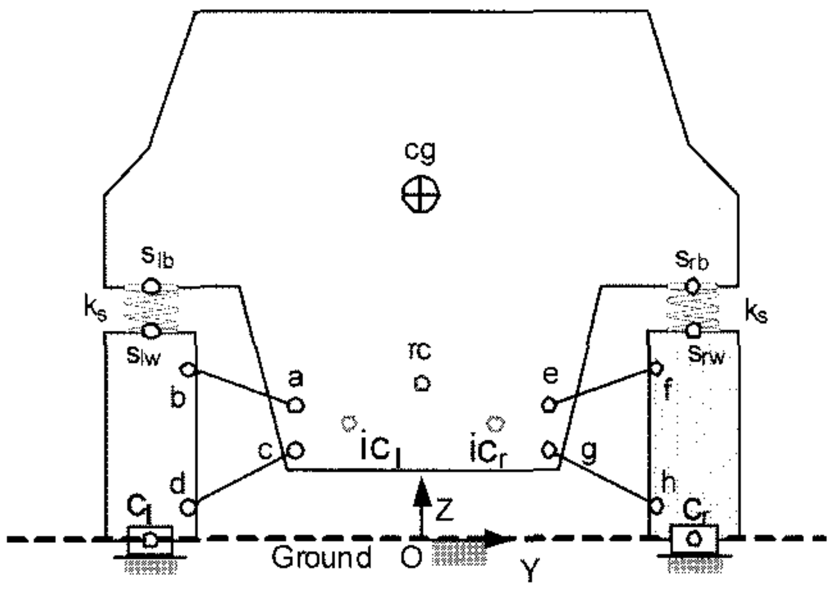


Figure 5. Joints, instant centers of wheels with respect to the body, and the roll center positions of a 3 D.O.F. model.

As the body rolls about roll center with small roll angle, the positions of two wheels are found by the constraints between the wheels and the body. Then, new position of ic_{ground} is assumed to be at the pole of a wheel and the ground which is different from CRCA. Using these procedures repeatedly, roll motion of this half-car model can be analyzed with only one input, that is, roll angle.

5. ANALYSIS PROCEDURE

Figure 5 shows the model used in this paper, where a–h represent joint positions; c_l and c_r are the left and the right wheel contact points; ic_l and ic_r are instant centers of wheels with respect to body (ic_{body}); rc is the roll center; cg is the center of gravity of the body; and k_s is the coefficient of a wheel suspension spring.

As shown in Figure 5, ic_l and ic_r can be found with the aid of Aronhold- Kennedy theorem which states that they must lie on the intersection of lines of link ab and link cd, link ef and link gh, respectively. This can be written as

$$\begin{aligned} \mathbf{ic}_l &= s(\mathbf{a}-\mathbf{b})+\mathbf{b} = t(\mathbf{c}-\mathbf{d})+\mathbf{d} \\ \mathbf{ic}_r &= u(\mathbf{e}-\mathbf{f})+\mathbf{f} = v(\mathbf{g}-\mathbf{h})+\mathbf{h} \end{aligned} \tag{1}$$

In Equation (1), bold-faced letters represent position vectors of joints and instant centers with respect to the origin O, and s, t, u, v represent scalar unknowns. Solution to Equation (1) can be easily found since they are two linear vector equations in four unknowns.

The roll center of the body, rc, is defined as

$$\begin{aligned} \mathbf{rc} &= p(\mathbf{ic}_l-\mathbf{ic}_{wl})+\mathbf{ic}_{wl} \\ &= q(\mathbf{ic}_r-\mathbf{ic}_{wr})+\mathbf{ic}_{wr} \end{aligned} \tag{2}$$

where p and q are parameters, ic_{wl} and ic_{wr} are ic_{ground} for each wheel. In initial position, since ic_{ground} are assumed to be at contact points, c_l and c_r are substituted into ic_{wl} and ic_{wr}, respectively, to determine initial position of rc. Then new position of all points in the vehicle body can be found by rotating the body about the roll center rc by roll angle θ. Hence, displaced positions of the fixed joints in the body are found to be at

$$\begin{aligned}&\mathbf{a'}=\mathbf{R}_\theta(\mathbf{a}-\mathbf{rc})+\mathbf{rc}\\&\mathbf{c'}=\mathbf{R}_\theta(\mathbf{c}-\mathbf{re})+\mathbf{rc}\\&\mathbf{e'}=\mathbf{R}_\theta(\mathbf{e}-\mathbf{rc})+\mathbf{rc}\\&\mathbf{g'}=\mathbf{R}_\theta(\mathbf{g}-\mathbf{rc})+\mathbf{rc}\end{aligned} \tag{3}$$

where

$$\mathbf{R}_\theta=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$

is a rotational matrix of the body and prime on the boldface represents new position.

Forces acting on the cg point are centrifugal force and weight. By these forces, the body roll moment can be written as

$$\mathbf{M}_{roll}=(\mathbf{cg}-\mathbf{rc})\times\mathbf{F}_{ex} \tag{4}$$

where $\mathbf{M}_{roll}$ is the body roll moment, $\mathbf{F}_{ex}$ is a resultant force acting on the cg point

After the body rolls, wheel spring reaction forces occur in accordance with spring deflection. Therefore, there exists the following resistant moment against body roll moment.

$$\mathbf{M}_{resistance}=(\mathbf{s}_{lb}-\mathbf{rc})\times\mathbf{F}_{ls}+(\mathbf{s}_{rb}-\mathbf{rc})\times\mathbf{F}_{rs} \tag{5}$$

where $\mathbf{s}_{lb}$, $\mathbf{s}_{rb}$ are joint positions of wheel springs on the body, and $\mathbf{F}_{ls}$, $\mathbf{F}_{rs}$ are spring forces on each wheel spring which are produced by deflections of springs from their initial equilibrium lengths and can be obtained by

$$\mathbf{F}_{ls}=k_s\delta_{ls}\frac{\mathbf{s}_{lw}-\mathbf{s}_{lb}}{|\mathbf{s}_{lw}-\mathbf{s}_{lb}|},\ \mathbf{F}_{rs}=k_s\delta_{rs}\frac{\mathbf{s}_{rw}-\mathbf{s}_{rb}}{|\mathbf{s}_{rw}-\mathbf{s}_{rb}|} \tag{6}$$

$$\delta_{ls}=|\mathbf{s}_{lw}-\mathbf{s}_{lb}|-|\mathbf{s}'_{lw}-\mathbf{s}'_{lb}|,\ \delta_{rs}=|\mathbf{s}_{rw}-\mathbf{s}_{rb}|-|\mathbf{s}'_{rw}-\mathbf{s}'_{rb}| \tag{7}$$

The body rolls while $\mathbf{M}_{roll}$ is greater than $\mathbf{M}_{resistance}$. The body roll moment varies from a certain value and roll resistant moment increases from zero while body rolls. The maximum body roll angle occurs when magnitude of $\mathbf{M}_{roll}$ is equal to that of $\mathbf{M}_{resistance}$.

New positions of the left and the right contact points, c'_l and c'_r, can be written as

$$\begin{aligned}&\mathbf{c}'_l=\mathbf{c}_l+\Delta\mathbf{c}_l\\&\mathbf{c}'_r=\mathbf{c}_r+\Delta\mathbf{c}_r\end{aligned} \tag{8}$$

where

$$\Delta\mathbf{c}_l=[dy_l,\ 0]^T$$
$$\Delta\mathbf{c}_r=[dy_r,\ 0]^T$$

dy_l, dy_r are lateral displacements of contact points.

The displacement of each wheel can be described by the combination of rotation about the contact point and translation of the contact point. Hence, new positions of joints on the wheels can be written as

$$\begin{aligned}&\mathbf{b'}=\mathbf{R}_\phi(\mathbf{b}-\mathbf{c}_l)+\mathbf{c}_l'\\&\mathbf{d'}=\mathbf{R}_\phi(\mathbf{d}-\mathbf{c}_l)+\mathbf{c}_l'\\&\mathbf{f'}=\mathbf{R}_\psi(\mathbf{f}-\mathbf{c}_r)+\mathbf{c}_r'\\&\mathbf{h'}=\mathbf{R}_\psi(\mathbf{h}-\mathbf{c}_r)+\mathbf{c}_r'\end{aligned} \tag{9}$$

where $\mathbf{R}_\phi$ and $\mathbf{R}_\psi$ are rotational matrices of the left and the right wheels respectively that can be written as

$$\mathbf{R}_\phi=\begin{bmatrix}\cos\phi & -\sin\phi\\ \sin\phi & \cos\phi\end{bmatrix},\ \mathbf{R}_\psi=\begin{bmatrix}\cos\psi & -\sin\psi\\ \sin\psi & \cos\psi\end{bmatrix}$$

Equation (9) have four unknowns, namely ϕ, ψ, dy_l, and dy_r. These values can be found by substituting Equations (3) and (9) into the following four constraint equations that represent constant link lengths.

$$\begin{aligned}&(\mathbf{a}-\mathbf{b})^T(\mathbf{a}-\mathbf{b})-(\mathbf{a'}-\mathbf{b'})^T(\mathbf{a'}-\mathbf{b'})=0\\&(\mathbf{c}-\mathbf{b})^T(\mathbf{c}-\mathbf{d})-(\mathbf{c'}-\mathbf{d'})^T(\mathbf{c'}-\mathbf{d'})=0\\&(\mathbf{e}-\mathbf{f})^T(\mathbf{e}-\mathbf{f})-(\mathbf{e'}-\mathbf{f'})^T(\mathbf{e'}-\mathbf{f'})=0\\&(\mathbf{g}-\mathbf{h})^T(\mathbf{g}-\mathbf{h})-(\mathbf{g'}-\mathbf{h'})^T(\mathbf{g'}-\mathbf{h'})=0\end{aligned} \tag{10}$$

Then, next pole positions between each wheel and the ground, $\mathbf{ic}'_{wl}$ and $\mathbf{ic}'_{wr}$, can be determined with the aid of the followings.

$$\begin{aligned}&\mathbf{c}_l'=\mathbf{R}_\phi(\mathbf{c}_l-\mathbf{ic}'_{wl})+\mathbf{ic}'_{wl}\\&\mathbf{c}_r'=\mathbf{R}_\psi(\mathbf{c}_r-\mathbf{ic}'_{wr})+\mathbf{ic}'_{wr}\end{aligned} \tag{11}$$

Substituting these new pole positions into Equation (2) yields new roll center of the vehicle body.

Repeating the above procedure until $|\mathbf{M}_{roll}|$ is equal to $|\mathbf{M}_{resistance}|$, the roll center for each step can be found, and connecting these roll centers, the fixed centrode of the vehicle body with respect to the ground, which is a trace of the roll center, is obtained. The mating moving centrode of the vehicle body which rolls on the fixed centrode can be determined by inverting the fixed centrode at each step about the roll center.

6. RESULTS

The rear view of the model used in this paper is shown in Figure 6. The opposite direction of forward driving is selected as positive X direction, and point coordinates (0,

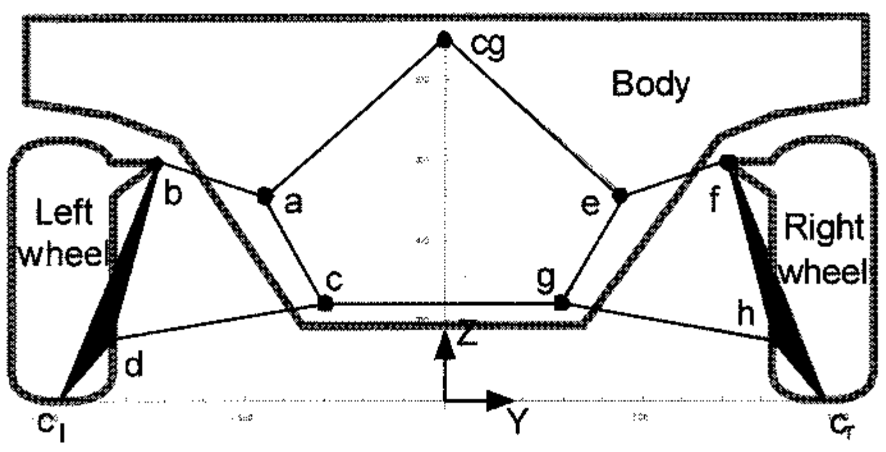


Figure 6. The analysis model.

Table 1. Initial joints and wheel contact points coordinates.

Left wheel			Right wheel		
Joint	Y	Z	Joint	Y	Z
a	–450	510	**e**	450	510
b	-720	600	**f**	720	600
c	-300	240	**g**	300	240
d	-840	150	**h**	840	150
s_{lb}	–450	900	s_{rb}	450	900
s_{lw}	-750	300	s_{rw}	750	300
Initial wheel contact points					
c_l	-960	0	c_r	960	0

Table 2. Analysis information of vehicle models.

Weight	1000 N
Lateral acceleration	0.3G
Wheel spring coefficient	50 N/mm

900) is set as the center of gravity to obtain information of position of the body. Initial position of joints and wheel contact points are shown in Table 1. Other information for this analysis is in Table 2. Centrifugal force is acting on the cg point along negative Y direction. Roll angle step is set to be 0.001 (rad.) for the three-degrees-of-freedom model. In this example, the body rolls counterclockwise since center of gravity is always located above roll center position.

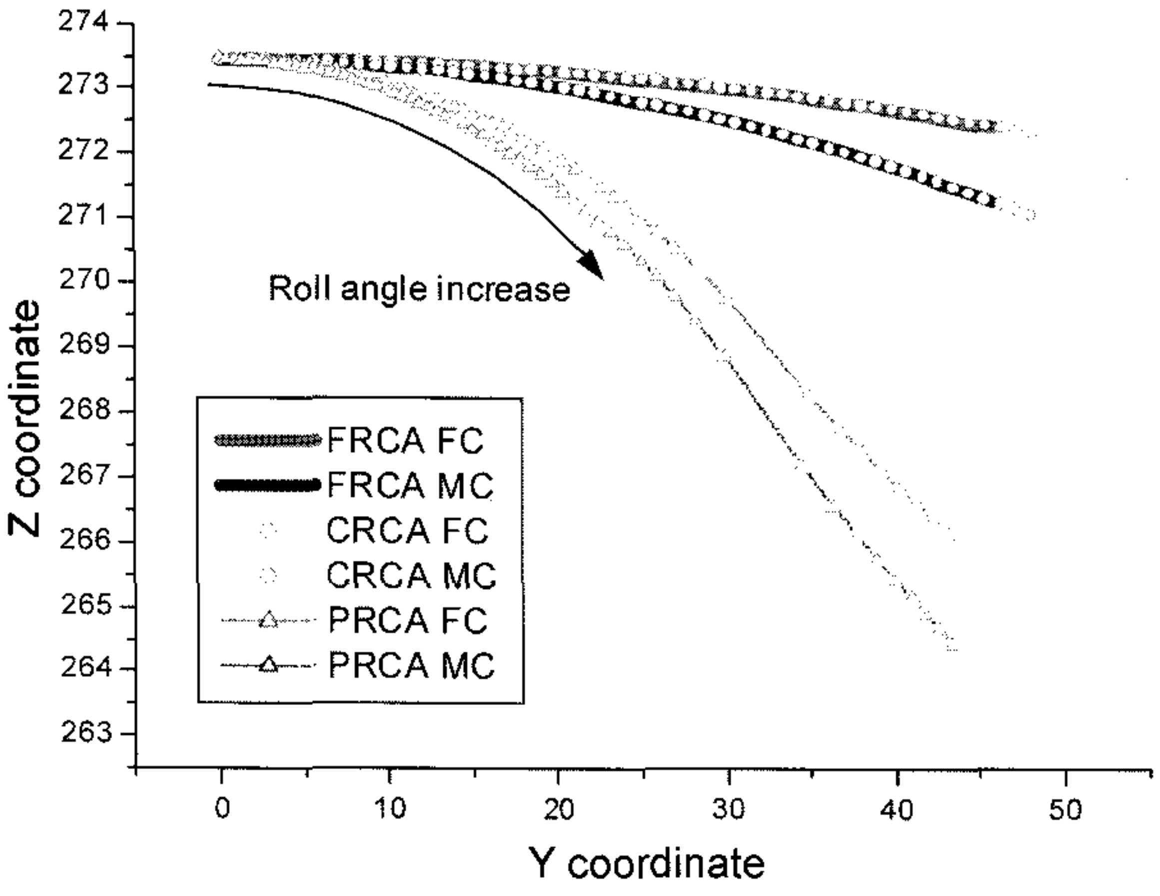


Figure 7. The fixed and the moving centrodes.

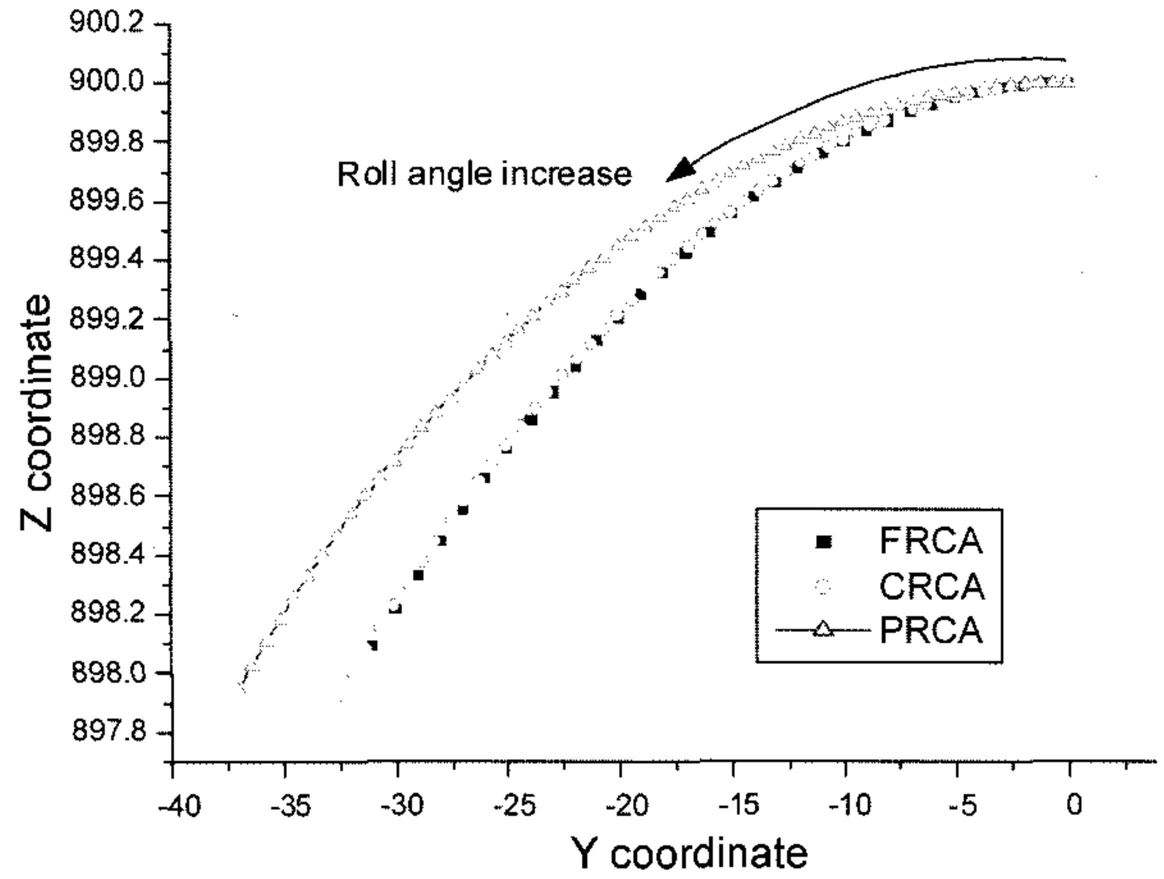


Figure 8. The trace of the cg point.

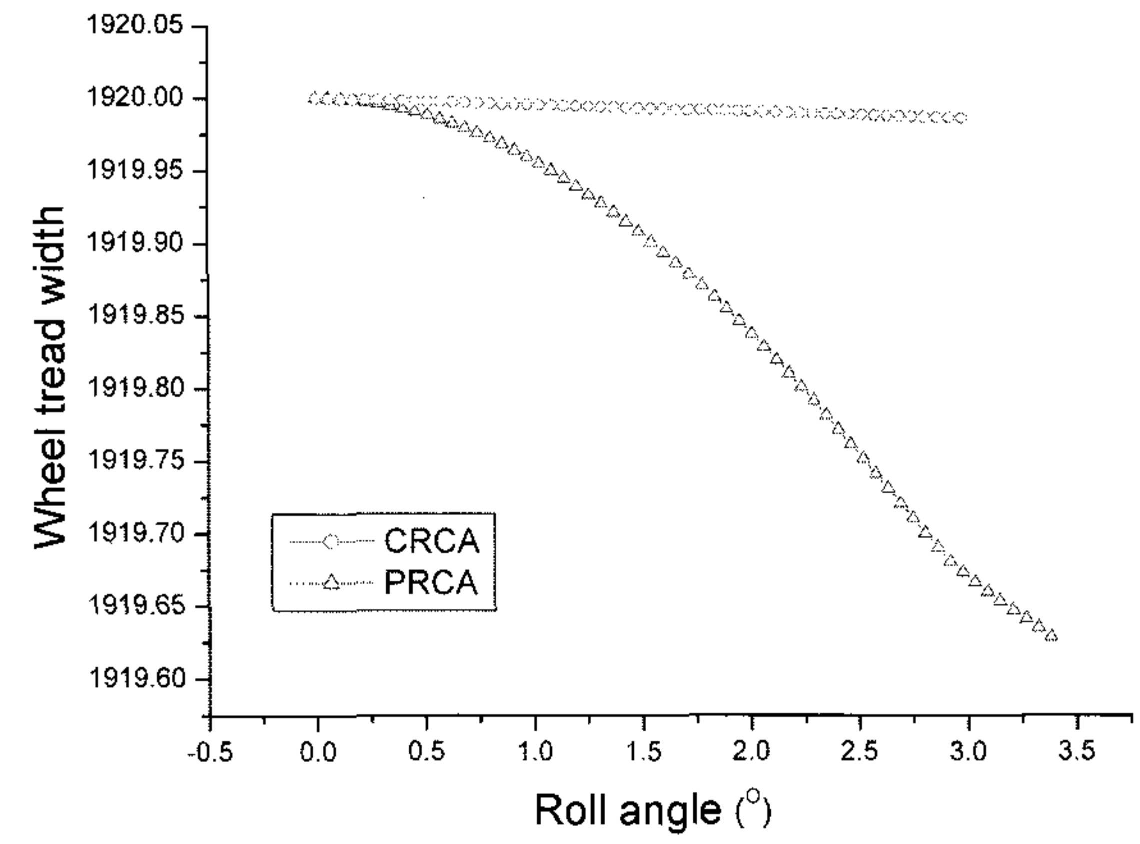
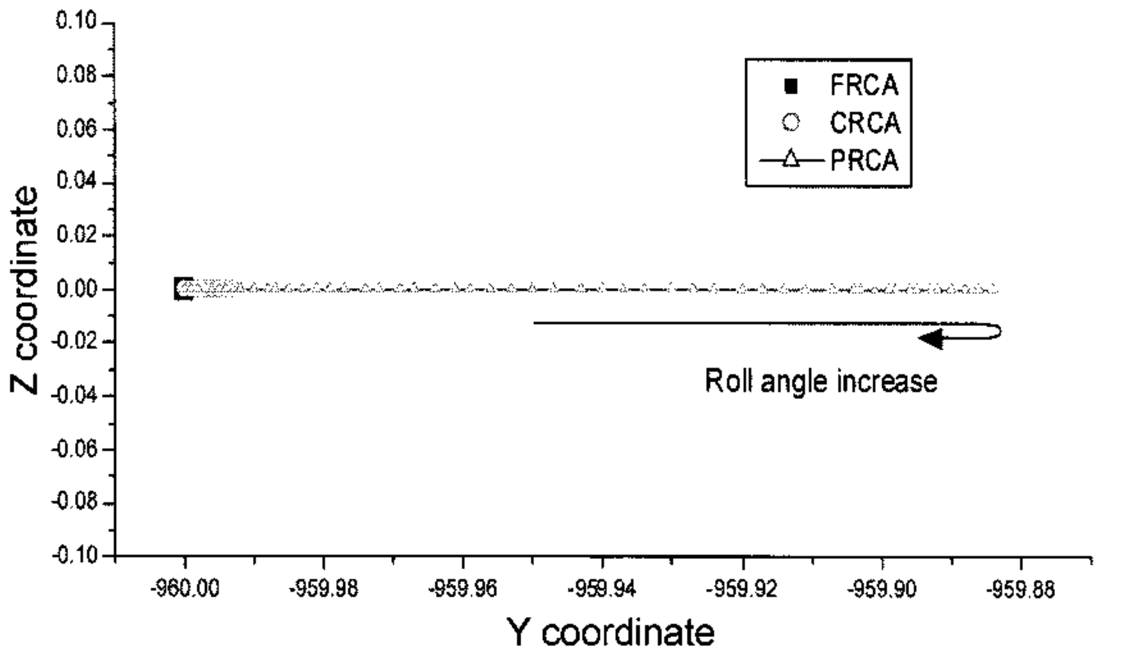
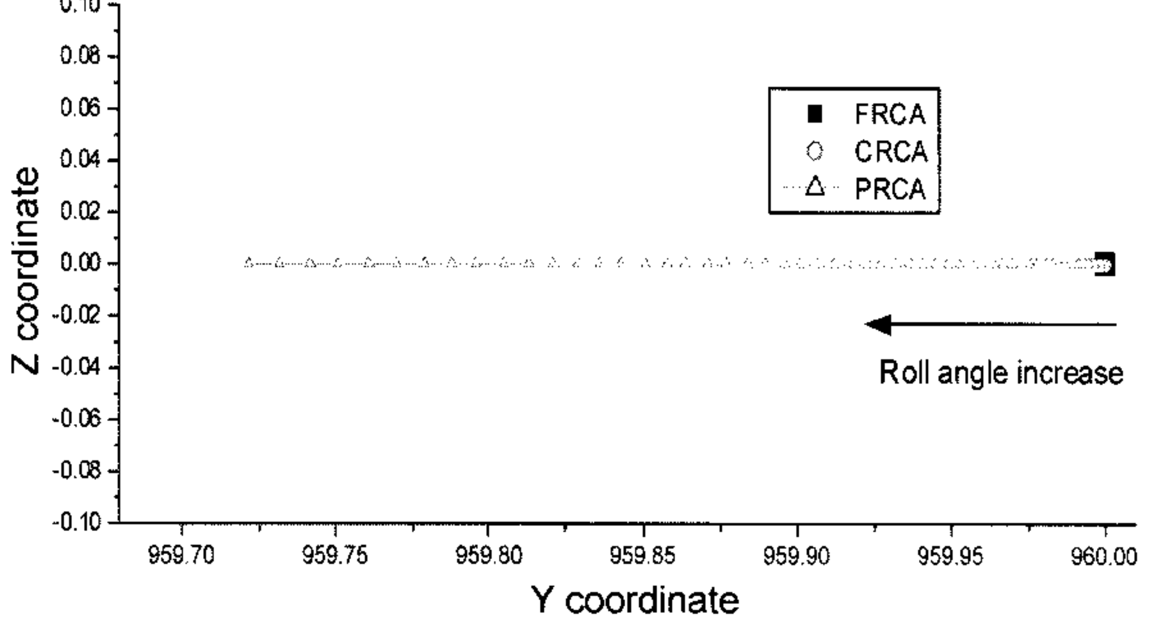


Figure 9. Wheel tread width variations.

In Figures 7–12, analysis results are shown for three models. Figure 7 shows the fixed centrodes and the moving centrodes of three models. The fixed (FC) and the moving centrodes (MC) of FRCA are similar to those of CRCA, on the other hand those of PRCA are quiet different. As shown in Figure 8, loci of the cg point of FRCA and CRCA are almost same. Locus of the cg point of PRCA is above those of FRCA and CRCA. These two figures show that the distance between the roll center and the cg point in PRCA which is related to the roll moment acting on a vehicle body increases faster than those in FRCA and CRCA. Figure 9 shows wheel tread width change. In this figure, the amount of change in wheel tread width of PRCA is much larger than that of CRCA. Figure 10 shows the left and the right wheel contact point variation of the analyzed models. The contact point of the analysis model varies in the plane as the vehicle body moves, while that of the constant width model does not. As shown in Figure 10, the range of variation of the wheel contact point of CRCA is very small, therefore the results in Figure 7 and 8 have to be similar. This means that the cg point rise occurs due to jacking force which acts on the cg point upward because of wheel tread width decrease. In FRCA or CRCA, the cg point rise cannot be calculated by roll center analysis only. They require jacking force analysis because wheel tread width is fixed or its change does not occur enough to the influence on

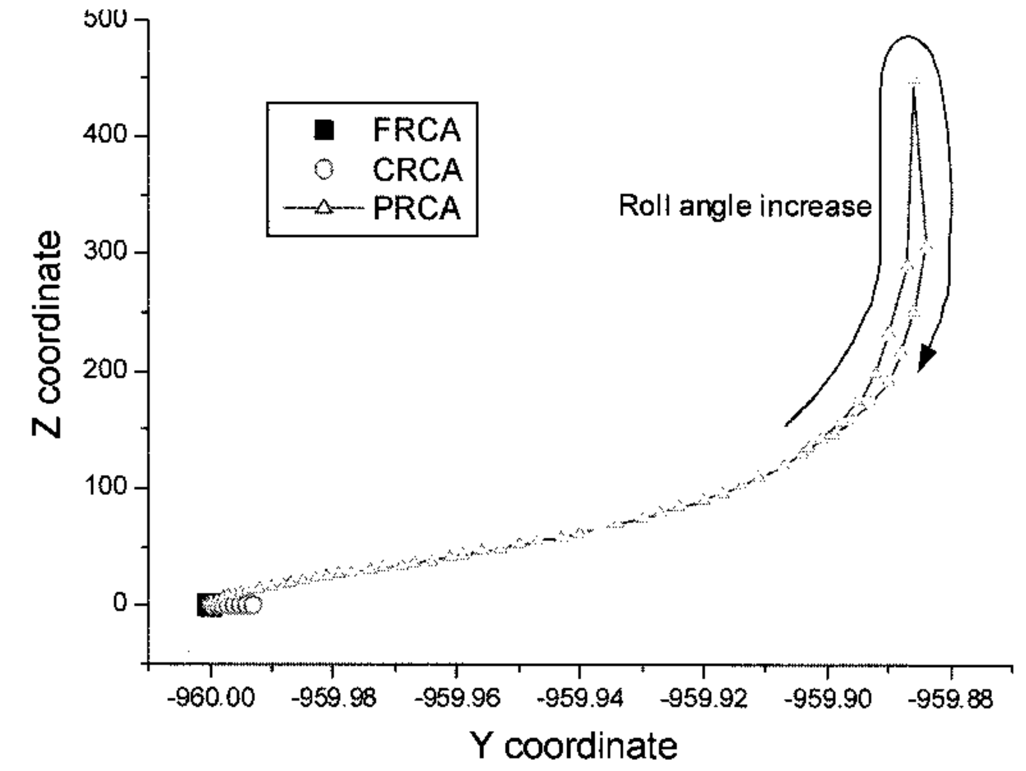


(a) Left wheel

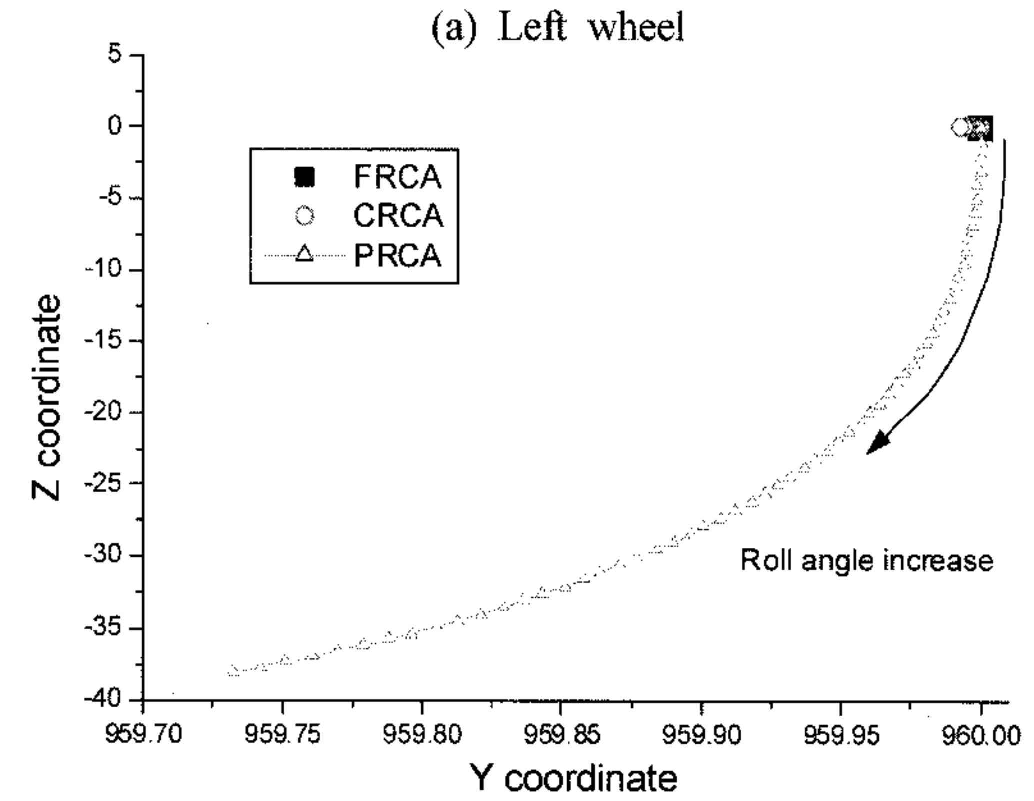


(b) Right wheel

Figure 10. Traces of wheel contact points.



(a) Left wheel



(b) Right wheel

Figure 11. Loci of instant centers of wheels with respect to the ground.

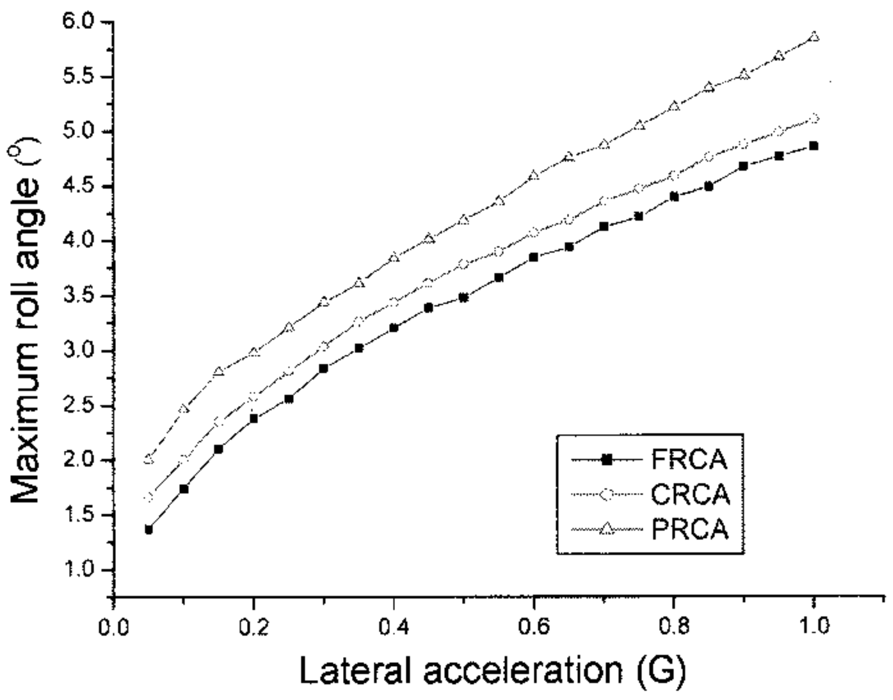


Figure 12. Maximum roll angles due to lateral accelerations.

vertical change of the cg point. Loci of instant center of left and right wheel with respect to the ground are shown in Figure 11. In Figure 12, maximum roll angles are presented with respect to various lateral accelerations. The maximum roll angle value of PRCA is always the greatest at each lateral force value. This means that the roll moment acting on the vehicle body in PRCA is bigger than those of other models.

From these results it can be said that locations of instant centers of wheels with respect to the ground are important factors in the roll center analysis of a half-car model. Analysis results are not realistic if locations of instant centers of wheels with respect to the ground are contact points between them.

7. CONCLUSIONS

An analysis method that determines the roll center of a planar half-car model is proposed. In general, wheels of conventional kinematic half-car models are considered to be connected to the ground by fixed revolute joints, hence they have constant wheel tread widths. On the other hand, wheel contact points of the proposed planar vehicle model can vary in the plane, thus, using the proposed model and analysis method, variation of wheel tread widths can be considered.

Kinematically, the roll center is an instant center of vehicle body with respect to the ground in planar motion. If two sets of instant centers, specifically ic_{body} and ic_{ground}, are known, the roll center can be easily determined. The ic_{body} can be readily determined if suspension type is given. To determine ic_{ground} for the model proposed in this paper, the pole of a wheel with respect to the ground for small roll displacement is used.

The results of the proposed method are compared with those obtained using FRCA, CRCA. FRCA isn't realistic because the change of wheel tread width is not allowed. CRCA, which is most common method, allows the change of wheel tread width but the roll center position obtained by this method violates Aronhold-Kennedy theory.

The proposed method, PRCA, is based on kinematic theories and allows wheel tread width change. This method does not include any dynamic properties. It provides only trends of position changes of points of interest such as the roll center, the cg point, wheel contact points, and so on. The smaller size of roll step angle are used, the more accurate analysis results are obtained.

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