Numerals and Pragmatic Interpretations

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Jae-Il Yeom. 2006. Numerals and Pragmatic Interpretations. *Language and Information 10.2*, 47–65. In this paper I address the problems of defining the semantics of numerals and accounting for how pragmatic inferences are made. I basically assume that a numeral n simply means 'λPλx[#(x) = n & P(x)]', as commonly assumed. Even when a numeral n has 'at least' interpretation, a sentence with the number does not entail a sentence with n replaced with $n-1$. But when a sentence with $n-1$ holds, it is possible that a sentence with n or a larger number holds too. This is not based on a semantic relation, but on pragmatic informativeness. In addition to pragmatic strength, the actual reading of a numeral is affected by some background knowledge of generalizations about the world, but the ordering of pragmatic strength among numbers always plays a role in determining unilateral interpretations. In such a case, we can assume that a set of numbers relevant in the context forms a scale. Forming a scale does not necessarily lead to a unilateral interpretation. The bilateral interpretation of a number is possible in the context where it is known whether or not alternative sentences with contextually salient alternative numbers are true. (**Hongik University**)

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1. Introduction

The semantics and pragmatics of numerals have been a hot topic, together with the topic of scalar implicatures, since Grice (1961). He says, "One should not make a weaker statement rather than a stronger one unless there is a good reason for so doing." This idea is embodied in the maxim of quantity. Especially, the first submaxim of the quantity maxim has been incorporated in the Q-principle in Horn (1984) and Levinson (1987). A variant of it is called the principle of volubility. The standard analysis of numerals has been simply part of the topic of scalar implicatures. A numeral n is assumed to have the reading 'at least n' on the basis of the assumption that sentences with larger numbers are stronger than those with

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smaller numbers. A number n gives rise to the reading 'exactly n' by pragmatic inference on the basis of the maxim of quantity.

Consider the following examples.

(1) a. John ate some apples.

b. NOT(John ate all apples).

(2) a. John has three children.

b. John has at least three children.

c. NOT(John has more than three children)

In (1), *all* is stronger than *some*, and the fact that the speaker makes a weaker statement implicates that he or she cannot make the stronger statement. That is, (1a) implicates (1b). Similarly, the sentence with the numeral *three* in (2a) is assumed to have the meaning as in (2b). It is assumed that the speaker follows the maxim of quantity, so it can be inferred that the statement is the strongest one he or she can make. From this reasoning, together with the assumption that the speaker knows the actual state of affairs, (2c) is implicated. This, together with the semantics of the sentence (2b), leads to the meaning that John has exactly three children.

Recently it has been pointed out that numerals are different from other scalar terms. Horn (1992) makes a number of observations in support of a pragmatic enrichment account of the interpretation of cardinals. In addition, he shows that this does not carry over to other scalar cases. For instance, he contrasts the following two examples:

(3) a. A: Are many of your friends linguists?
B: ?No, all of them are.
C: Yes, (in fact) all of them are.

b. A: Do you have three children?
B: No, four.
A: Do you have three children?
C: ?Yes, (in fact) four.

Ordinary scalar terms can be accounted for by traditional mechanisms of calculating scalar implicatures on the basis of the maxim of quantity. Uttering a sentence with *many* implicates the negation of its counterpart with the stronger alternative *all*. When the weaker statement is negated, it is the case that stronger statements do not hold either. B's statement in (3a) is odd because the weaker statement is negated and a stronger statement is asserted. In contrast, when a weaker statement is affirmed, a stronger statement can be asserted as in C's statement. Numerals are different. It is generally assumed that numerals constitute a scale and that a numeral n means 'at least n'. And the utterance of a weaker statement should mean the negation of a stronger statement. It is, however, not the case with numerals. This would be strange if the numeral had the meaning 'at least three'. It casts doubt on the idea that a numeral n has the meaning 'at least n'.[1]

Horn (1996, 316) reinforces the point with further examples:

(4) a. ??Neither of us liked the movie - she hated it and I absolutely loved it.

b. Neither of us have three kids - she has two and I have four.

The two verbs *love* and *like* form a scale <love, like>. If someone loves you, he or she likes you too. Conversely, if someone does not like you, he or she does not love you. The first example is odd because this semantic relation is not satisfied. If having four children entails having three, we would expect the same oddness in the second example. But this example shows that having four children may not entail having three children. Numerals are indeed different from other scalar terms.

A further point in this direction comes from Scharten (1997, 67-68), who notes an erroneous asymmetry in the implicature view: in (5a), B's utterance is considered a case of implicature cancellation, while in (5b) it is a case of repair or self-correction:

(5) a. A: How many pupils are there in your class?
B: 31. No wait, 33.

b. A: How many pupils are there in your class?
B: 31. No wait, 29.

These observations lead to various new analyses. There are still linguists who follow the orthodox Neo-Gricean analysis: Levinson (2000) and Winter (2001), etc. On the other hand, linguists like Sadock (1984), Koenig (1991), Horn (1992), Scharten (1997), Geurts (1998), Breheny (2005), etc. propose to abandon the traditional view: they propose that *five* basically means 'exactly five'. For linguists who take this position, they need to explain how a numeral n has interval readings like 'at least n' and 'at most n'. In determining the readings, contrasts of predicates/arguments, and information structures like the focus/topic contrast are involved. Basically they assume that one reading can be derived systematically from another reading. On the other hand, Carston (1998), Atlas (1992), and Verkuyl & van der Does (1995), etc. have maintained that the meaning of a numeral is underspecified and that the actual meaning is pragmatically determined. Especially, Carston (1998), following Recanati (1989), claims that numerals have a variable for the underspecified reading as part of their meanings, the actual value of which is provided in the context. She denies the idea that numerals are ambiguous. This

[1] One anonymous reviewer suggests that when a numeral has 'exactly' interpretation, it can be the effect of the EXH(austivity) operator. If this is the case, we should say the same thing about other scalar terms than numerals. Moreover, the EXH operator is introduced pragmatically, and if it leads to an absurd state of affairs, it should be easily canceled. This is not what we have observed in the examples above. What is important here is not by what mechanism scalar effects are accounted for, but why and how numerals are different from other scalar terms.

implies that she does not need the systematic relationships between possible readings.

Basically linguists assume that numerals are not ambiguous in three ways, but that there are some basic meanings of numerals and the other reading(s) are derived from them. A problem is that their assumptions and explanations of the meanings of numerals can be easily defeated. This paper is another attempt to account for the semantics of numerals and the pragmatic inferences from them, but I try to avoid the weaknesses previous accounts had on the basis of indisputable assumptions and observations. In doing this, I will show that there is no relationship of entailment between sentences with alternative numbers, even when the numbers have unilateral ('at least' or 'at most') interpretations. Despite the lack of ordering in semantic strength between numbers, a subset of numbers should constitute a scale when the numbers have unilateral readings. An ordering in such a scale must be pragmatic. Background knowledge like generalizations about the world is involved in determining the interpretations of numbers, but the (pragmatic) ordering is still involved. Finally I will show that bilateral ('exactly') interpretation is obtained even when a numeral is a member of a scale.

2. N entails $N-1$?

We have seen that from a sentence with a numeral we can get three possible interpretations, but few linguists believe that numerals are ambiguous in three ways. The question is which readings are basic and which readings are pragmatically inferred from the basic readings.

Since it has been observed that numerals behave differently from other scalar terms, some linguists assume that the 'exactly' reading is the basic meaning of a numeral. This is a plausible assumption considering the fact in (3a). When numerals have 'exactly' interpretations, a sentence with a number does not entail, or is not entailed by, a sentence with an alternative number. On the other hand, there are cases where numerals have 'at least' interpretations. And the Neo-Griceans assumed that numerals have 'at least' interpretations, based on the assumption that a sentence with a certain number entails, or is entailed by, a sentence with an alternative number. Geurts (2005) assumes that numerals are ambiguous, but he seems to assume that when numerals have 'at least' interpretations, they have entailment relationships. So before we take any position on the semantics of numerals, we have to check if numerals in 'at least' interpretation are in entailment relationship between themselves.

We can easily see an example which seems to show the entailment relationship. Suppose that I need 50 dollars. In this case, if I ask a friend of mine whether he has 50 dollars, it is supposed to mean '(at least)' 50 dollars, not 'exactly' 50 dollars.

(6) a. A: Do you have 50 dollars?
B: ??No, I have 300 dollars.
C: Yes. I have 300 dollars.

b. A: Do you have 50 dollars?
B: ??Neither of us have 50 dollars. I have 300 dollars and John has

only 40 dollars.

In these examples, speaker B's statements are odd because having 300 dollars is taken to entail having 50 dollars. If it is really an example of entailment, the examples in (3b) and (4b) are surprising.

One solution is that in (1) and (2) the numerals have 'exactly' interpretations and those in (6) 'at least' interpretation, ignoring which reading is derived from which. And the context determines the appropriate one. In ordinary contexts, when someone asks you how many children you have, it is normal to ask the exact number of your children. If the context requires the minimum number of your children, the whole story is different. This is shown below.

(7) a. A: You have to have three children to get this benefit. Do you have three children?
B: ?No, I have four children.
C: Yes, I have four children.

b. A: You have to have three children to get this benefit. Do you have three children?
B: ??Neither of us has three children. I have four and John has two.

If we assume that sentences with numerals have entailment relationships between themselves, 'exactly' readings of numerals should be obtained by pragmatic implications, as in the Neo-Gricean analysis. However, (4b) does not allow such a pragmatic solution because the reading is not obtained by applying some pragmatic principle to the whole statement. There is a possibility of metalinguistic negation. But this is not applicable here. It is generally observed that numerals do not show metalinguistic negation, as Breheny (2005) claims. To begin with, consider an example with an ordinary scalar term involved: Capitals in the examples below indicate special rising, contrastive intonation

(8) Mary: John got some$_F$ of the questions right
Bill: He didn't get SOME$_F$ of the questions right

Bill's statement can be understood as meaning that John got all of the questions right. He claims that this is the effect of interpreting the affirmative statement as saying that John didn't get some (but not all) of the questions right. On the other hand, this way of interpretation does not arise in the case of numerals.

(9) Mary: John got four$_F$ of the questions right
Bill: He didn't get FOUR$_F$ (of the questions) right

Here Bill's statement can be understood as meaning either 'John got three or less of the questions right' or 'John got more than four of the questions right', depending on the contexts.

In (4b), if having four kids entails having three kids, the discourse must be odd. If the 'exactly' reading is not obtained pragmatically, then we have to cast doubt on the idea that sentences with numbers show entailment relationships. Since 'at least' reading is obtained pragmatically and entailment relationships are assumed in those cases, we have to check if entailment relationships must hold when numerals have 'at least' interpretations. Suppose that John juggles with knives, and that it is all the more difficult to juggle with more knives. And for some reasons we do not know, it is impossible to juggle with an odd number of knives. In this context, the following sentence does not entail that John juggled with five knives.

(10) A: Anyone who can juggle with more than six knives can participate in the competition. Does John juggle with six knives?
B: Yes, he can. In fact, he can juggle with ten knives.

In this example, B's answer means that John can juggle with at least six knives, but it does not entail that John can juggle with five knives. This shows that even if numerals have 'at least' readings, entailment relationship does not hold between numerals.[2] This indicates that 'at least' reading is not based on entailment relationship.

From these observations, we could assume that numerals are ambiguous in as many ways as the ways subsets of numerals constitute scales, but this is extremely implausible. The only plausible assumption is that entailment does not hold between sentences with different numbers. So we have to give up the idea that numerals are ambiguous. Furthermore, whether a subset of numerals constitutes a scale also depends on the utterance context, not on the language system.

Even if a sentence with a larger number does not entail one with a smaller number, the numbers can constitute a scale, just as in the following example, if the context allows it.[3]

(11) John is at least an associate professor — perhaps even a full professor.

In this example, a relevant scale is like the following:

(12) <full professor, associate professor, assistant professor>.

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[2] Krifka (1999) discussed cumulative interpretations of numerals and pointed out that numerals do not show entailment relationships. When three boys ate at least two apples each, the following sentence does not entail 'Two boys ate six apples.'

(i) Three boys ate seven apples.

In this example, two numbers are involved, and cumulation can be taken to be a process of cumulation of events with original events intact. So John, Mary and Sue solved three problems, that is, Problem 1, Problem 2 and Problem 3 respectively. Then we can say the first sentence, but not the second in the following.

(ii) a. John, Mary and Sue solved Problem 1, Problem 2 and Problem 3.
b. John and Mary solved Problem 1 and Problem 3.

So the example of cumulative interpretation is not a semantic phenomenon.

[3] This has been pointed out by Hirschberg (1991) . Matsumoto (1997) suggests various tests for diagnosing scalarity showing that they are not equipollent: some tests work for a certain type of scalar terms, but not for others. Scalar terms which are not based on semantic strength pass the 'if not' or 'perhaps even' tests, but not the 'in fact' test.

In this scale, the alternatives are not ordered by semantic strength. Being a full professor does not entail being an associate professor or being an assistant professor. Numerals are like this. Then it does not depend on semantic strength, but on pragmatic strength, that numbers are often understood as meaning 'at least'. A scale of numerals is not based on semantic strength, but in many cases we feel as if it is so, unlike the scale in (12). We need to account for this difference.

Numerals are quantity words, and three apples physically include two apples. So if you ate three apples, it is possible that the event includes a (sub-)event of eating two apples. Seemingly entailment relationships are based on physical inclusion relationship, not meaning itself. But physical inclusion relationship can always be distorted depending on the context, as shown above. Another good example of distortion is a case where a NP with a numeral has group reading: when a piano is carried by six people, there is no event of moving a piano by five people. And even if physical relationship is not distorted, eating three apples is different from eating two apples, and there are contexts where the former is not taken to entail the latter. So apparent entailment relationships involving numerals are just pragmatic inference.

There are uses of numerals in which even such pragmatic inference is excluded, as pointed out by Partee (1986) and Geurts (2005). When numerals are used in predicate NPs, they just denote properties, not quantities of individuals. In this case, pragmatic inference cannot be involved.

(13) a. These are five cows.

b. These are four cows.

The first sentence cannot entail the second because the property of being five does not physically include that of being four. So there is no seemingly entailment relationship observed.

3. So-called 'at least' reading from pragmatic ordering

In the previous section, we have seen that numerals do not show entailment relationship between themselves. Then how is the 'at least' interpretation of a numeral possible? Traditionally numerals are assumed to entail smaller numerals. So if a sentence with a numeral is true, a corresponding sentence with a larger number is compatible with it because the former is entailed by the latter. Now that numerals are not in entailment relationship, we need to account for how 'at least' interpretation is possible. This has been attempted by Krifka (1999), Geurts (2005), etc. According to Krifka (1999), a numeral has the meaning $\lambda P \lambda x[\#(x) = n \wedge P(x)]$, where $\#(x)$ gives the number of atoms of the individual sum x. They basically assume that a numeral has 'exactly' reading, which is expressed by "$\#(x) = n$". And when there is no determiner in front of a numeral, it is assumed that there is an empty determiner which has the following interpretation.

(14) a. $[_{NP}\ \emptyset\ n\ \mathrm{CN}\ ]$

b. $[[\emptyset]] = \lambda P \lambda Q \exists x[P(x) \wedge Q(x)]$

Geurts (2005) instead assumes a type shifting from the original reading by Existential Closure. In either way, a numeral gets the reading of the existential quantifier. So the sentence "John ate five apples" is interpreted as follows.

(15) $[[\emptyset \text{ five apples}]_2 \text{ [John ate } t_2 \text{]]}$
$[[\text{five apples}]] = \lambda x[\#(x) = 5 \wedge \text{apples}(x)]$
$[[\emptyset \text{ five apples}]] = \lambda Q \exists x[\#(x) = 5 \wedge \text{apples}(x) \wedge Q(x)]$
$[[[[\emptyset \text{ five apples}]_2[\text{John ate } t_2 \text{]]]]} = \exists x[\#(x) = 5 \wedge \text{apples}(x) \wedge \text{ate}(j,x)]$

The NP with the numeral is quantifier-raised, and the meaning of the NP and the meaning of S is combined following Heim & Kratzer's (1998) Predicate Abstraction. This sentence is true, of course, when the number of apples John ate was "five". But the meaning says that there are five apples John ate. If John ate five apples and more, the more than five apples physically include a set of five apples John ate. If the context allows such a case to be taken to be a relevant one, the sentence with a larger number can be true. For this reason, the number is understood as meaning 'at least' five. The opposite direction of implication is also possible. If John ate five apples, it is also possibly true that John ate less than five apples. In interpreting a sentence, however, this second implication is ignored. This leads to the idea that the number mentioned is always the maximal number the speaker can say, because informativeness is the underlying principle in making an utterance.

There is one caveat on this generalization. Consider the examples in (6a-6b) again. Even if John has 300 dollars, the speaker says that he has 50 dollars because the question was about whether John has 50 dollars. That John has 300 dollars is not relevant in the utterance context. So the number mentioned is considered to be the maximal number *relevant in the context*. Informativeness is conditioned by relevance, which is actually reflected in Grice's (1975) first maxim of quantity: "make your contribution as informative as is required (*for the current purposes of the exchange*)".

The same idea can explain the difference between the following two examples. Van Kuppevelt (1996) claimed that one crucial factor determining whether a cardinal (or other scalar term) is given an "at least" or an "exactly" interpretation is whether the term is in the topic or comment part of the information structure of an utterance.

(16) a. How many children does John have?
   a. He has three children.
   b. *He has three children, in fact five.

b. Who has three children?
   a. John has three children.
   b. John has three children, in fact he has five.

In the first example, the relevant alternative numerals are not restricted in the range of numbers of children. So any number is relevant in the context. When the speaker has five children, he or she has to say so from the beginning. The answer

(b) is odd because the maxim of quantity or manner is violated. In the second example, the person who asks the question divides individuals into two groups: one is those who have at least three children, and the other those who do not have three children. So only the number *three* is relevant. So the speaker conforms to the requirement of the context first, then adds a more specific information.[4]

This could sound like the analysis by the Neo-Griceans. They claim that the semantics of a numeral n as 'at least n' and the pragmatic implicature 'at most n' from the maxim of quantity lead to the interpretation of a numeral as 'exactly n'. However, this is not the case. The maxim of quantity selects the most informative number the speaker can give, just as an underlying principle, and the actual reading is determined by some other factors, like the assumption that the speaker knows, for each number, whether John ate that number of apples, or whether a statement with a larger number is relevant in the context. In the example in question, the 'at least' meaning comes from the possibility that (the speaker does not know if) John ate more than five apples. It does not require the ambiguity of numerals.

Since the 'at least' reading comes from pragmatic inference, it can be canceled if some other pragmatic factors come in. In this case, we can say that numerals have 'exactly' readings. Note that even when a numeral n is interpreted as 'at least n', this is not part of the semantics of the numeral. This makes "n" different from "at least n", and the sentence can mean that B has only three children, together with the first submaxim of quantity. This can account for the differences between "three boys" and "at least three boys", which is accounted for differently by Krifka (1999).

(17) a. Three boys left, perhaps even four.

b. a. A: Three boys left.
B: No, four.
b. A: At least three boys left.
B: *No, four.

In the use of a simple numeral, the meaning of 'at least' is pragmatically derived, so it can be removed from the meaning of the sentence. This allows the possibility of 'exactly' reading. On the other hand, the meaning of 'at least' in "at least three" is really part of the semantics, so the possibility of 'more than three boys' is not excluded. The speaker cannot negate this when he or she knows that four boys left.

[4] On the basis of these examples, van Kuppevelt (1996) claims the possibility that numerals have 'exactly' interpretation. However, we find that the following example is fine, as Krifka (1999) pointed out.

(i) A: How many children does Nigel have?
B: Nigel has fóurteen$_F$ children, perhaps even fifteen$_F$.

It seems that, contrary to what van Kuppevelt claims, having focus does not necessarily lead to the 'exactly' reading. This shows that numerals do not have 'exactly' interpretations inherently. When they do have 'exactly' interpretations, it is just a pragmatic effect.

4. Context-dependency of numeral interpretations

Breheny (2005) claims that numerals have only 'exactly' reading, and the other readings are derived pragmatically. In this paper, I have shown that numerals do not entail other smaller ones in the strict sense of entailment, but if two alternative sentences are different only in corresponding numerals and there is a relation of physical inclusion, then one sentence may pragmatically implicate the other. This allows a possibility that a numeral has a unilateral reading. In this section, I will discuss effects of pragmatic factors, as Breheny did. In doing so, I will show that informativeness plays a role in determining the interpretation of numerals.

Consider an example in which a unilateral reading is observed. In the following example, which is an example Breheny (2005) discussed, the numeral can have 'at least' or 'at most' reading.

(18) No one who has three children is happy.

Suppose that more children means more prosperity. This leads to the following ordering of likelihood of being unhappy with respect to the number of children.

(19) a. $1 > 2 > ... > n-1 > n$

b. No one who has one child is happy &
No one who has two children is happy &
No one who has three children is happy &
NOT(No one who has four children is happy) &
...
NOT(No one who has n children is happy)

c. No one who has at most three children is happy.

When the number "three" is mentioned as the number of children which makes people unhappy, it is taken to be the limit. So the number *three* is taken to be the largest number of children which can make people unhappy. So the number is understood as meaning 'at most three'.

This can be contrasted with a context where the ordering is reversed. If more children means more stress on their parents, it is more likely that no one who has more than three children is happy. This makes the following ordering of likelihood of being unhappy with respect to the numbers of children.

(20) a. $n > n-1 > ... > 2 > 1$

b. No one who has n children is happy &
...
No one who has three children is happy &
NOT(No one who has two children is happy) &
NOT(No one who has one child is happy)

c. No one who at least three children is happy.

In this context, the number "three" is understood as the minimal number of children which makes people unhappy. So the number is understood as meaning 'at least three'. This example again shows that the basis lies in the ordering of likelihood with respect to the numbers, given as the background knowledge, and the mentioned number is taken to be the limit, either as 'at least' or 'at most'. Even if the ordering is involved in understanding the implication of the use of a specific number mentioned, it does not contribute to the semantics of the sentence itself. In this example, even if the numeral is interpreted as 'at least three', the domain of the quantification does not include people who have more than three children.[5] In this respect, as for any number n larger than three, the sentences must be admitted only as possibilities. Based on the discussion of examples like this, Breheny (2005) claims that the interpretation of a numeral depends on background knowledge about the world.

We have seen that the unilateral interpretation of a number is purely a pragmatic matter of likelihood, the actual interpretation depending on contexts. But then a question arises whether pragmatic strength in terms of informativeness does not play any role in determining interpretations of numerals. If a sentence "John ate five apples" is to be true, the actual state of affairs must be that John ate exactly five apples or that he ate more than five apples and his eating five apples is only pragmatically implicated. This leads to the 'at least' interpretation of the numeral. We cannot understand the sentence as meaning that John ate at most five apples.[6] And this is purely the effect of selecting the largest number possible plus the possibility of eating more apples. The latter comes from the possibility that the speaker is not in a position to exclude such a possibility.

If this is the case, it is surprising that numerals have 'at most' interpretations, since eating less than five apples cannot implicate eating five apples. It is necessary to find out when and how 'at most' interpretations arise. Consider the following examples.

(21) a. John may eat three apples.

b. John must eat three apples.

Obligation is imposing a restriction on possible actions or events and permission is considered to be lifting of a corresponding prohibition (a negative obligation). More permission means more concession of possible actions or events. These two sentences can be considered in two different contexts. First, consider cases where people want to eat more apples. In this case, it is likely that people are likely to eat more apples if there is no restriction on it. The ordering of likelihood of eating each number of apples is the following.

(22) a. $n > n{-}1 > ... > 2 > 1$

[5] According to the Neo-Gricean analysis, *no* introduces a downward entailing context, where no scalar implicature arises. And since the Neo-Gricean analysis assumes that a numeral has the semantics of 'at least'. So the domain of quantification is expected to include people who have more than three children. Intuitively, this does not seem to be the case in the example at hand.

[6] Breheny's example cannot be a test example because the numbers occur in downward-entailing context and we do not know for sure what happens in such a context.

b. John may eat n apples > ... > John may eat 3 apples > John may 2 apples > ...

c. NOT(John may eat n apples) &
...
NOT(John may eat 4 apples) &
John may eat 3 apples &
John may eat 2 apples &
John may eat 1 apple
= John may eat at most 3 apples

When a permission statement is uttered with a certain number, the larger the number, the more concession it means, and the specific number mentioned is taken to be the limit. So the sentence has the interpretation that John may eat at most three apples. The number 'three' is the one which conveys the strongest permission, and the possibility that John is allowed to eat more than three apples is excluded by the fact that 'three' is the maximal number of apples John is permitted to eat.

When an obligation statement is uttered, the smaller the number, the larger restriction the obligation sentence imposes on the behavior of the subject. So the pragmatic ordering of obligation sentences is like the following.

(23) John must eat 1 apple > John must eat 2 apples > John must eat three apples > ... > John must eat n apples

When the sentence with "three" is mentioned, the number must be the limit. Since that number is the smallest, it should be understood as 'at least three'. However, this is not an intuitively correct interpretation of the sentence. Even if the numbers 1 and 2 are excluded from possible obligations for pragmatic reasons, the negation of the two sentences with those two numbers does not exclude the possibility of eating less than three apples, as shown below.

(24) a. NOT(John must eat one apple) &
NOT(John must eat two apples) &
John must eat three apples &
...
John must eat n apples
= John must eat n apples

b. NOT(John must eat one apple) = John is allowed not to eat one apple

In the given context, the obligation of eating four apples is a weaker restriction than that of eating three apples, if the relative ordering from the comparison of informativeness between the two numbers is ignored. So the number *three* should be the one that leads to the strongest restriction on the subject's behavior. The problem is that the smallest number has proper effects only when the possibility of eating more than three apples is excluded, because if stronger alternatives were not excluded, the resulting meaning would be that John must eat n apples, which is the pragmatically weakest restriction. So in order for the sentence not to be trivial, the more informative alternatives must be excluded. So the resulting interpretation is not that John must eat at least three apples, even though "three" must be the minimal number, but that John must eat at most three apples. This shows that an ordering from informativeness works here too.

When a more informative alternative is excluded, the reading must be (25b) rather than (25a).

(25) a. NOT(John must eat n apples)

b. John must NOT(eat n apples)

The way that the reading (25b) is obtained is proposed in Chierchia (2004) and Yeom (2006): when a scalar implicature is calculated, the negation operator should be incorporated with some narrow scope than some operator in the sentence uttered. One example is given below.

(26) a. Some students who watched TV or played games last night failed maths.

b. NOT[Some students who watched TV and played games failed maths]

c. Some students who NOT[watched TV and played games] failed maths

In calculating a scalar implicature, normally the negation operator has the widest scope, as in (26a), but it is not plausible. A more plausible interpretation is (26b), where the negation operator has narrower scope than the existential operator. The thing happens in the example we are concerned with. I will not go into details about the way that this reading is obtained.

Breheny (2005) claims that numerals have basically 'exactly' meaning, and that pragmatic background knowledge determines the unilateral reading of a numeral, but this is only partially true. The number mentioned in a sentence is always the most informative one among possible alternatives, that is, the largest one the speaker can give. This is based on the idea that numerals do not have 'exactly' meaning, but 'at least' meaning, in their traditional meaning. In this paper, 'at least' is a derived meaning, but it is assumed to exist, separately from pragmatic likelihood. This opens the possibility that a sentence with a larger number holds which may allow the sentence with the smaller one to hold. Pragmatic background knowledge affects this possibility.

Now consider cases where the ordering of likelihood is reversed. Consider the following examples.

(27) a. John may solve three problems.

b. John must solve three problems.

People want to solve fewer problems. In this case the ordering of likelihood with respect to numbers is the following:

(28) $1 > 2 > ... > n-1 > n$

In this case a permission statement means the more concession, the smaller the number. If a number is mentioned in a permission sentence, it is taken to be the limit, that is, the minimal number. Then the pragmatically stronger alternatives are negated, and the resulting interpretation would be that John may solve at least three problems. But solving more problems are still allowed and the resulting meaning is that John may solve n problems, again.

(29) NOT(John may solve one problem) &
NOT(John may solve two problem) &
John may solve three problems &
John may solve four problems &
...
John may solve n problems
= John may solve n problems.

If the number mentioned is to be non-trivial, the more informative alternatives also should be excluded. So the actual reading of the sentence is not that John may solve at least three problems, even though "three" should be the least number pragmatically. This leads to the interpretation of the number as 'at least three'. Here again ordering of informativeness plays a role.

Obligation sentences are different. In the given context, an obligation sentence becomes pragmatically the stronger, the larger the number.

(30) John must solve n problems > ... > John must solve three problems > John must solve two problems > John must solve one problem

Mentioning "three" in the obligation sentence negates the stronger alternatives.

(31) NOT(John must solve n problems) &
...
NOT(John must solve four problems) &
John must solve three problems &
John must solve two problems &
John must solve one problem
= John must solve at least three problems

This leads to the interpretation of the sentence as meaning that John must solve *at least* three problems, and possibly more. It allows the possibility that John must solve one or two problems. The ordering of informativeness coincides with that of likelihood provided by the context, so no further adjustment is necessary.

So far I have shown that a numeral can be interpreted as 'at least' or 'at most', depending on what is the ordering of likelihood in the set of alternative numbers given in a context. When there is no such ordering between them, there is no such interpretation. Likelihood is not a semantic relation but a pragmatic one. For this reason, the ordering can be reversed depending on contexts. However, regardless of the ordering of pragmatic likelihood, the ordering in terms of informativeness plays a role in determining the interpretation. Informativeness is like the Law of Gravity which is omnipresent in our world, whether we feel it or not. In this respect, interpretation of numerals here is similar to that in the Neo-Gricean analysis. But we do not assume that a scale of numerals is based on semantic strength. A scale is also pragmatically determined. The next question is what happens if there is no ordering between numerals at all. This is a case where the number has so-called 'exactly' interpretation, but I do not suppose that ordering and bilateral interpretation are incompatible. I will discuss this issue below.

5. Numerals in cumulative sentences

In the previous section, we have seen how 'at most' readings are possible, along with 'at least' interpretations. What is common in the examples is that numerals occur within the scope of quantifiers, but what is more important is that the background knowledge involves modality, something like likelihood. If such modality is not involved, we have only seen that numerals have 'at least' interpretations, as in (15). A question is whether 'at most' interpretation is possible in cases where there is no modality involved. One such case is discussed in Krifka (1999).

(32) a. In Guatemala, three percent of the population owns seventy percent of the land.

b. In Guatemala, two percent of the population owns eighty percent of the land.

These sentences are about the distortion of statistical distribution of the land. In this respect, (32b) is more informative than (32a), but the former does not entail the latter. If two percent of the population owns eighty percent of the land, it is also true that it owns seventy percent of the land. But the speaker has to be informative enough, and (s)he should say the maximal percentage of the land that is possessed by two percent of the population. On the other hand, the opposite can be said of the percentage of the population who possess some portion of the land. When two percent of the population owns eighty percent of the land, it is the case that three percent of the population possesses eighty percent of the land. This shows that smaller numbers lead to stronger readings, and two percentage is the lower limit. The number ultimately given in the most informative reading is the smallest. The 'upper bounding' reading of the numeral comes from the fact that at the speaker's current information state the numeral is the smallest one. There is still a possibility that a stronger sentence can hold, where the corresponding number is the smaller.

This example is compared with the following.

(33) Three boys ate seven apples.

This sentence means 'three boys ate apples and seven apples were eaten by boys'. One difference from the previous example is that it is not known how many boys and how many apples there are. When (33) is uttered, the numbers *three* and *seven* are the maximal numbers the speaker knows at the moment.[7] The numbers can be larger if he/she gets more information. The two numbers are taken as the minimal maximal numbers that can be given. They are maximal because they are the largest that can be given at the current knowledge state, and they are minimal because if information changes, it will be bigger. In the example of the distribution of land over the population, on the other hand, even if some percent of the population owns a significant part of the land, we know that there is more land and more people who owns it since we know the sizes of the land and the population. So they are not informative at all. As for the distribution of the land, the portion of the population is the maximal minimal number and the portion of the land is the minimal maximal number. The interpretation of numbers is determined by the range of the most informative statement, as the information increases.

Comparing the two examples, when a numeral is taken to be the upper limit in the current information state, it is given 'at least' interpretation, as in most cases. But when a numeral is the lower limit in the current information state, it is given 'at most' interpretation. Here again informativeness plays a significant role, and the upper or lower limit comes from the consideration of ordering with respect to informativeness. A upper limit could possibly get higher and a lower limit lower, if more information is available. So in some sense, the interpretation of numerals involves epistemic modality.

This indicates that unilateral readings arise when a certain kind of modality is involved. Each statement is taken to be the most informative statement, and the interpretation of the numerals in the statement is determined by the modality provided by the context. When a numeral in a sentence ϕ has a unilateral interpretation without any additional modality, possible information increase provides the necessary modality. In this case a statement ϕ can be understood as meaning 'the speaker S can say at least in the current knowledge state k that ϕ.

(34) If a speaker S utters ϕ in a knowledge state k, it means 'SAY_AT_ LEAST(S, k, ϕ)'.

ϕ is the most informative statement at the current knowledge state, and the numbers in ϕ can be the lower or upper limit, depending on the context. Possible changes in the knowledge state determines the interpretation of numbers. This accounts for sentences like (15) and (32). In the latter example, the more distorted the distribution, the more informative the statement is.

6. Exactly?

In the previous sections, we have seen examples which include numerals with unilateral interpretations. But those sentences also have so-called 'exactly' readings.

(35) a. No one who has (exactly) three children is happy.

b. John may have (exactly) three apples.

[7] This does not mean that the sentence entails that two boys ate six apples. If each boy ate two or more apples, two boys only ate five apples.

c. John must solve (exactly) three problems.

When numerals do not constitute a scale, each member of numerals other than the number mentioned is negated.

(36) a. No one who has three children is happy &
for any n $\neq$ 3, NOT(No one who has n children is happy)

b. John may have three apples &
for any n $\neq$ 3, NOT(John may have n apples)

c. John must solve three problems &
for any n $\neq$ 3, NOT(John must solve n problems)

The reason that numerals here do not form a scale is pragmatic, as shown above. For this reason, we might think that numerals which are interpreted as having so-called 'exactly' meaning do not form a scale, but this is not correct. Suppose that if you catch one fish, you get one dollar. So it is important to know exactly how many fish you catch. In this situation, if you say you caught three fish, it means that you caught exactly three fish. However, the number is a scalar term. This is clear from the meaning of the following sentence.

(37) John caught only three fish.

This does not mean that the number of the fish John caught is three, not any other, but that he caught three fish, but not more. This indicates that the numeral is interpreted as a scalar term.

In contrast, even if numerals do not have 'exactly' reading, they do not form scales.

(38) a. Neither of us spent 400 dollars this month. John spent 300 dollars and I spent 500 dollars.

b. ??Neither of us spend 400 dollars this month. John spend 399 dollars and I spend 401 dollars.

Here in normal contexts, 400 does not mean 'exactly 400'. This is why the second example is a little odd in ordinary contexts. Even though the numbers do not have 'exactly' reading, they do not form a scale. The degree of being preciseness is a different matter from scalarity, which is not directly related to bilateral interpretation.

What is crucial in bilateral readings is not whether numerals form a scale, but whether a sentence with a numeral opens the possibility that a sentence with an alternative number also holds and that number is relevant at the current context. When the speaker knows for every alternative number whether a sentence with that number holds or not, the numbers simply get bilateral readings.

7. Conclusion

In this paper I have tried to specify the semantics of numerals and derive various readings of them by pragmatic reasoning. Semantics of numerals is hard to specify, because various readings are interwound with various assumptions and pragmatic reasons. But I have decided several points to determine the semantics and pragmatics of numerals. First, I have shown that numerals do not have entailment relationship, even when they have unilateral interpretations. This is an important starting point, because it can eliminate the necessity of the assumption that numerals are ambiguous. Second, I have shown that despite the lack of entailment relationship between numerals, we need an ordering of pragmatic strength between numbers so that we can decide which alternative is more informative. This plays a role in determining actual unilateral interpretations, regardless of whether numerals have 'at least' or 'at most' interpretations. Finally, I have shown that even if numerals have so-called 'exactly' interpretations, they can form a scale. And whether numerals have unilateral or bilateral interpretations depends on whether the truth of each alternative sentence with an alternative number is known and the number is relevant at the current context.

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