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Supply Chain Coordination Under a Trade Credit Contract and a Quantity Discount Contract

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■ Abstract ■

Consider a supply chain in which a vendor supplies a product to a buyer. We assume that the buyer's and vendor's inventory cost structures are different, resulting in differences in inventory order/delivery cycle times. Here, if one party insists on its individually optimal order/delivery quantity, the other party will suffer from mismatches in cycle times. Under this scenario, coordination contracts that make use of either a Net Term/Two parts Term Trade Credit or a Quantity Discount are designed to align individually optimal order quantities. We compare and analyze the performances of these contracts. The focus of the comparison is the ability of contracts to generate a lower cost for the supply chain. We show that a Trade Credit policy can be effectively used to coordinate a supply chain. In many cases it will result in a lower supply chain cost compared to that achieved by using a Quantitative Discount policy.

Keyword : Supply Chain Coordination, Trade Credit, Quantity Discount, Incentive and Contracting

1. Introduction

In the last decade, the notion of supply chain integration has drawn a great deal of attention from both the business community and academics. Advocates argue that all of the sub-

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systems of a supply chain are connected; thus, trying to find the best set of trade offs for any one subsystem is not sufficient. An integration of the complete scope of the supply chain needs to be considered so that collective strategies can be designed to optimize the joint objectives of the supply chain. While the importance of achieving integration in the supply chain is generally well recognized, designing a sophisticated integrated system for real world applications is an arduous task. Few firms are so powerful that they can manage the entire provision of the supply chain so as to drive individual members to a superimposed integrated objective. Rather, a more realistic approach is to design a coordination contract with incentives to induce supply chain members to cooperate with others under a voluntary compliance base. A great body of research focuses on designing supply chain coordinating contracts. Examples include coordination via buyback contracts [10], markdown allowances contracts [14], price only contracts [5], quantity flexibility contracts [15], and price protection contracts [13]. Among these studies, one stream of research has focused on using a *Quantity Discount* contract in a deterministic EOQ setting to synchronize mismatches in the supply chain order/delivery cycle times [1-4, 7, 9, 16-18]. Our work falls into this category of study; however, we contribute by addressing an alternative supply chain contract using *Trade Credit* policies as means for coordination. We propose that a Trade Credit policy, if properly designed, could be used as an incentive for coordinating a supply chain. That is, if a supplier has an advantage over a buyer in financing ability, then the supplier could continuously draw on his/her own sources of credit, and provide the buyer a cheaper financing opportunity as a channel incentive for supply chain coordination. A similar model was studied in Lee [6], in which a supply chain coordinating trade credit contract is analyzed under a stochastic Newsboy model. In this work, we study the coordination contract under a deterministic Economic Order Quantity model.

This paper is structured as follows. We present a description of the problem, assumptions, and notations. Then we develop the inventory cost functions of the buyer and the vendor. We analyze the individually optimal policies as well as the jointly optimal policy. Next, supply chain contracts that make use of either a *Net Term/Two parts Term Trade Credit* policy, or a *Quantity Discount* policy are designed to align the individually optimal order/delivery quantities. We compare and analyze performances of these contracts. The focus of the comparison is the ability of contracts to generate a lower cost for the supply chain. We show that a Trade Credit policy can be effectively used to coordinate a supply chain. In many cases it will result in a lower supply chain joint cost compared to that achieved by using a Quantitative Discount policy. We conclude with a discussion of the results.

2. Model Description

We begin our analysis by introducing the notations. The buyer and vendor will be referred to as party 1 and party 2 respectively. Generally, then, we will use subscripts "1," "2," and "J" to designate the buyer's, the vendor's, and the joint set of parameters.

D = Buyer's demand rate
Q = order quantity
W = Wholesale price per unit paid by the buyer to the vendor
S_i = Setup cost per order, $i=1,2$
h_1 = Inventory opportunity cost for the buyer
h_2 = Inventory opportunity cost for the vendor
n_1 = buyer's non-opportunity inventory cost (such as storage space cost, property taxes)

Our model is restricted to a relatively simple EOQ transaction scenario. An additional assumption unique to our model is as follows:

The vendor finances order process/production cost from a specialized financial institution. The buyer can either borrow from a specialized financial institution, and pay the vendor directly, or finance its inventories from the vendor's Trade Credit policy.

Consider a base case scenario in which the buyer periodically orders a certain quantity from a vendor. Upon receiving the order quantity, the buyer borrows money from a financial institution and pays the wholesale price to the vendor. The buyer's average inventory cost for a wholesale price $W=W^0$ is $C_1(Q|W^0)=DS_1/Q+Q(n_1+h_1)/2+W^0D$. The optimal order quantity is $Q_1=\sqrt{2DS_1/(n_1+h_1)}$. The vendor's average inventory cost is $C_2(W^0|Q_1)=S_2D/Q_1-W^0D$ where the order quantity Q_1 is determined by the buyer. Note that inventory carrying cost is not included in the vendor's cost function. This model is valid under some circumstances. For example, (1) the vendor is a manufacturer who has a very high production rate (production lead- time is negligible), and operates under an order-for-order production principle, or (2) the vendor is a distributor/wholesaler who operates under an order-for-order delivery principle. A similar vendor's inventory cost function has been formulated in Monahan[9] where inventory carrying cost is not included. Clearly, from the vendor's perspective, the ideal order quantity minimizing $C_2(W^0|Q)$ is greater than Q_1.

3. The Coordination Contracts

Assume now that the vendor proposes a contract to ask the buyer to increase its order size from the current level of Q_1 to a new level $Q\in[Q_1,\in\infty)$. Should the buyer accept this proposal, an incentive capable of compensating any cost penalty will be given as a reward. Now, with no additional compensation, the buyer's cost penalty is $\triangle C(Q)=C_1(Q|W^0)-C_1(Q_1|W^0)$; thus, the incentive must be at least as great as $\triangle C(Q)$ (Monahan [9] referred to this as a "break-even discount"). Let $C_J(Q)=D(S_1+S_2)/Q+Q(n_1+h_1)/2$ denote the joint supply chain inventory cost of the vendor and the buyer. Including an incentive $\triangle C(Q)+\epsilon$ with an arbitrarily small $\epsilon\geq 0$, the vendor's cost function now changes to $C_2(W^0|Q)+C_2(W^0|Q)+\triangle C(Q)+\epsilon=C_J(Q)-C_1(Q_1|W^0)+\epsilon$. The first order condition shows that the optimal order quantity $\min_Q C_2(W^0|Q)+\triangle C(Q)+\epsilon\Rightarrow\min_Q C_J(Q)$ is $Q_J=\sqrt{2D(S_1+S_2)/(n_1+h_1)}$. This tells us that the vendor can minimize its individual inventory cost by inducing the buyer to adjust the order size from the currently optimal Q_1 to Q_J, and by giving the buyer a compensation $\triangle C(Q_J)+\epsilon$ with an arbitrarily small $\epsilon\geq 0$ that is just enough to draw the buyer's indifference. In what follows, we will consider three types of incentive transfer schemes for coordination. These are the (1) Quantity Discount (QD) contract (see, for example, [9]), (2) Net Tem (NT) Trade Credit contract, and (3) Two-Parts Term Trade Credit contract.

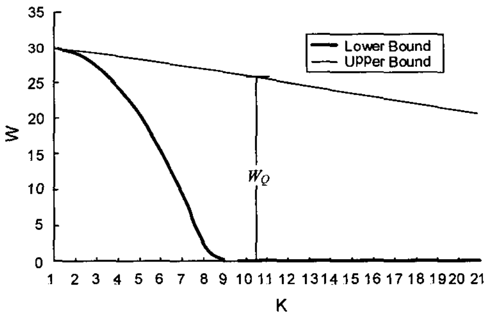
3.1 The Quantity Discount (QD) Contract

Consider a coordination contract in which the vendor proposes a *Quantity Discount* with a discounted wholesale price $W_Q \le W^0$ as an incentive to induce the buyer to increase its order size from Q_1 to Q_J. Here, to reach a win win result, the discount in the wholesale price must at least offset the buyer's cost penalty incurred from adjusting the currently optimal order size, i.e., $C_1(Q_1|W^0) \ge C_1(Q_J|W_Q)$. Note also that the discount should not incur a cost penalty to the vendor ; thus, one must also make sure that $C_2(W_Q|Q_J) \le C_2(W^0|Q_1)$.

Proposition 1. *(The proof for Proposition 1 appears in the Appendix.)*
Denote $K: Q_J/Q_1 = \sqrt{1+S_2/S_1}$. *To accomplish win win results, the vendor designs a discounted wholesale price* $\mathbf{W}_Q \in [W_{LB}, W_{UB}]$ *where :*
1.1 $C_1(Q_1|W^0) \ge C_1(Q_J|W_Q)$ *leads to an upper bound for the wholesale price* $W_{UB} = W^0 - S_1(k-1)^2/KQ_1$. *Any* $W \le W_{UB}$ *will be more beneficial for the buyer.*
1.2 $C_2(W_Q|Q_J) \le C_2(W^0|Q_1)$ *leads to a lower bound for the wholesale price* $W_{LB} = W^0 - (1+K)S_1(K-1)^2/KQ_1$. *Any* $W > W_{LB}$ *will be more beneficial for the vendor.*
1.3 Both W_{LB} *and* W_{UB} *decrease as* K *increases, and* $W^0 > W_{UB} > W_{LB}$ *when* $K > 1$. □

[Figure 1] provides a numerical experiment illustrating W_{UB} and W_{LB}. The parameters are : Q_1=100, S_1=50, and W_0=30. The numerical example verifies Proposition 1.3. W_{UB} can be rearranged to $W^0 - W_{UB} = \sqrt{2S_1(n_1+h_1)/D}\ (K-1)^2/2K$. Monahan [9] referred to this as a "break-even discount". We see that designing $W = W_{UB}$ will be sufficient to make the buyer indifferent. Any wholesale price lower than the "break-even" wholesale price W_{UB} will be more beneficial for the buyer. On the other hand any wholesale price higher than W_{LB} will be more beneficial for the vendor. Thus, the vendor and the buyer will negotiate through a mutually agreeable discounted wholesale price $W_{LB} \le \mathbf{W_Q} \le W_{UB}$. Upon coordination, the supply chain average inventory cost is :

$$C_J(Q_J, W_Q) = D(S_1+S_2)/Q_J + Q_J(n_1+h_1)/2. \quad (1)$$



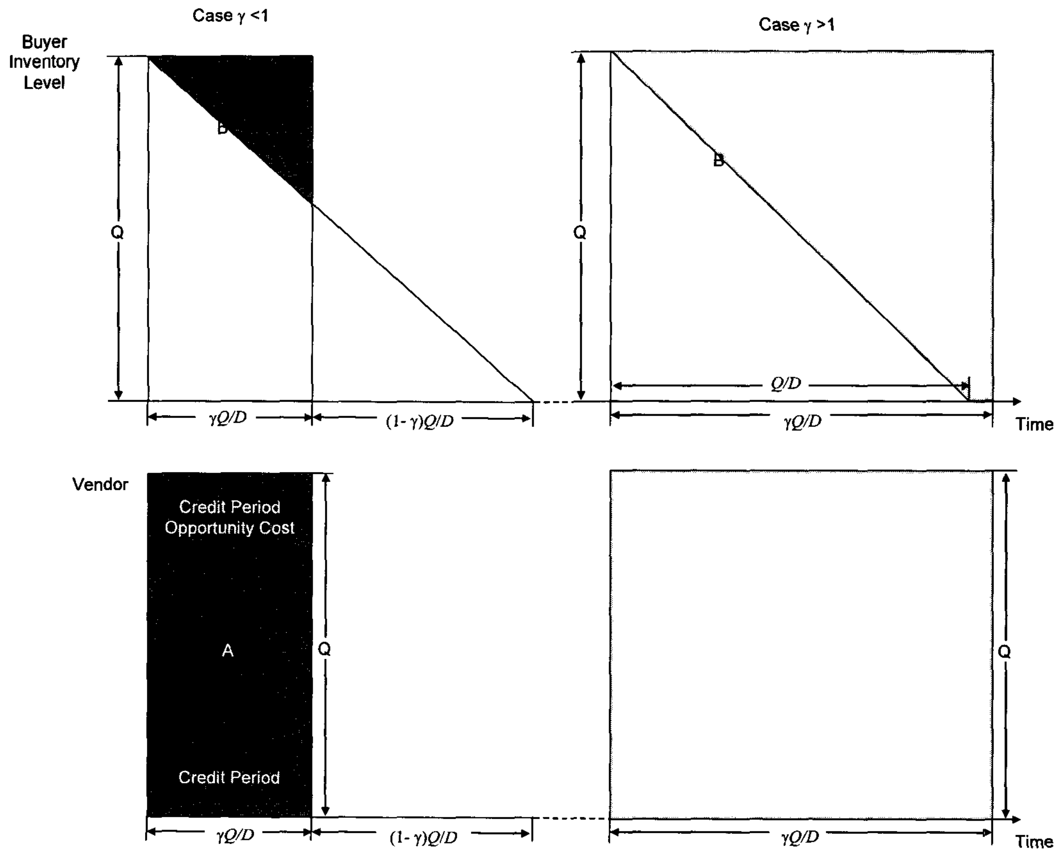
[Figure 1] Upper and Lower Bounds as a Function of K

3.2 The Net Term (NT) Trade Credit Contract

Let us now consider a second contract. Here, a *Trade Credit* is offered as an incentive for supply chain coordination. In general, there are two basic forms of Trade Credit policies — a *Net Term* policy and a *Two-Parts Term* policy. In a Net Term policy, a vendor offers a trade credit that allows the buyer's payment to be delayed until the *Net Date* (e.g., net 30 days, net 60 days). In a Two-Parts Term policy, a vendor offers a contract consisting of three basic clauses : (1) *Discount Percentage*, (2) *Discount Period*, and (3) *Net Date* (e.g., 2% Discount/Discount Period 10days/Net 30 days). Here, a buyer receives a discount for paying within the *Discount Period*, but then must pay the full price at the *Net Date* if the buyer misses the *Discount Period*.

Let us first consider a Net Term (NT) trade credit contract. Here, payment for the shipment will be delayed until after the *Net Date* $\gamma Q/D$ with a $\gamma > 0$ (see [Figure 2]). We assume that the trade credit loans will be settled by the buyer according to a one-time payment schedule at the end of the net date. We also assume that the buyer recoups its capital immediately upon using the inventory. In this case, a "negative" opportunity cost could be generated during the net date. That is, the buyer is recapturing capital before it is spent. Then, each time unit in the net date could generate a "negative opportunity cost".

In [Figure 2], the shaded region in B is the "negative opportunity cost". The buyer's inventory carrying cost can be computed by subtracting the inventory opportunity cost saving of region $B = Qh_1\gamma$ from the inventory carrying cost $Q(n_1 + h_1)/2$. By assumption, the vendor finances



[Figure 2] Inventory Policy

order production costs through loans from a financial institution; thus, during the credit period, the vendor incurs an opportunity cost for the account receivable (see [Figure 2], region A). As in Schwartz [12], we assume that the vendor's opportunity cost shows a strictly increasing trend with respect to γ to reflect the trade credit risk. We let $h_2(\gamma)=a+b\gamma$ as a linearly increasing function of γ. As in the QD contract, the vendor proposes a *Net Term* contract with a credit period $\gamma_N Q_J/D>0$ as an incentive to induce the buyer to increase its order size from Q_1 to Q_J. Upon coordination, the vendor's average inventory cost is $C_2(W^0,\gamma|Q_J)=S_2D/Q_J+Q_J(a+b\gamma)\gamma-W^0D$, the buyer's average inventory cost is $C_1(Q_J|W^0,\gamma)=S_1D/Q_J+Q_J[(n_1+h_1)/2-\gamma h_1]+W^0D$, and the supply chain average inventory cost is :

$$C_J(Q_J,\gamma_N)=D(S_1+S_2)/Q_J+Q_J[(n_1+h_1)/2 -\gamma_N(h_1-a-b\gamma_N)]. \quad (2)$$

As in the QD contract, win win results can be reached by simultaneously satisfying $C_1(Q_1|W^0,\gamma=0)\geq C_1(Q_J|W^0,\gamma_N)$ and $C_2(W^0,\gamma_N|Q_J)\leq C_2(W^0,\gamma=0|Q_1)$.

Proposition 2. *(The proof for Proposition 2 appears in the Appendix.)*

To accomplish win win results, the vendor designs a Net Date $\gamma_N\in[\gamma_{LB},\gamma_{UB}]$ *where :*

2.1 $C_1(Q_1|W^0,0)\geq C_1(Q_J|W^0,\gamma_N)$ *leads to a lower bound* $\gamma_{LB}=(K-1)^2(n_1+h_1)/2h_1K^2$. *Any* $\gamma\geq\gamma_{LB}$ *will be more beneficial for the buyer.*

2.2 $C_2(W^0,\gamma_N|Q_J)\leq C_2(W^0,0|Q_1)$ *leads to an upper bound*

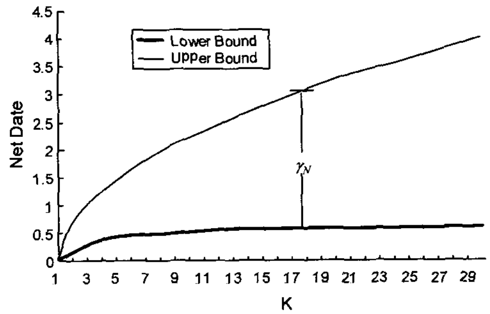
$$\gamma_{UB}=[\sqrt{a^2+4b(1+K)(K-1)^2(n_1+h_1)/2K^2} -a]/2b$$

Any $\gamma<\gamma_{UB}$ *will be more beneficial for the vendor.*

2.3 $\gamma_{UB}>\gamma_{LB}>0$ *when* $K>\max[1,(a+b\gamma_{UB})/h_1-1]$. *Both* γ_{LB} *and* γ_{UB} *increase as* K *increases.*

2.4 Supply chain inventory cost functions (1) and (2) show that $C_J(Q_J,\gamma_N)<[>]C_J(Q_J,W_Q)$ *when* $h_2(\gamma_N)<[>]h_1\Rightarrow\gamma_N<[>](h_1-a)/b$. □

As in Propositions 1.1 and 1.2, Propositions 2.1 and 2.2 provide a "break-even" lower bound γ_{LB} and an upper bound γ_{UB} to assure win-win results for both the vendor and the buyer. Proposition 2.4 shows that if the vendor has an advantage over the buyer in financing ability $h_1\geq h_2(\gamma_N)$, then the vendor should continuously draw on its own sources of credit, and provide the buyer a reliable and cheaper financing opportunity as an incentive for supply chain coordination. Note that if $h_2(\gamma_N)=h_2$ (if the vendor's inventory opportunity cost is a constant), then the decision is a binary type policy. $C_J(Q_J,\gamma_N)>(\leq)C_J(Q_J,W_Q)$ if $h_2>(\leq)h_1$, and the vendor will use a quantity discount contract (net term trade credit contract) to coordinate a supply chain.



[Figure 3] Upper and Lower Bounds as a Funnction of K

[Figure 3] provides a numerical experiment illustrating γ_{UB} and γ_{LB}. The parameters are : $Q_1=100$, $n_1=2$, $h_1=8$, $a=0$, and $b=9$. The numerical example verifies Proposition 2.3.

3.3 The Two-Parts Term Trade Credit Contract

Alternatively, the vendor could consider offering a Two-Parts Term trade credit contract with payment terms that include cash due in a specified period and a prompt payment discount option. To the buyer, the benefit of a Two-Parts Term policy can be described by the following example. Consider that a buyer receives a discount for paying within a prescribed Discount Period, but then must pay the full price if the buyer misses the Discount Period. Assume that a cash discount of 2% is offered if the payment is made within the Discount Period of 10 days, and the gross amount is due in 30 days. If the buyer takes the discount opportunity and pays within 10 days, the buyer's effective interest cost savings are equivalent to an annual interest rate of 36%. This can be computed when one knows that the buyer's cost of using that amount of money for an additional 20 more days is 2%. There are approximately 18 20-day periods in a year; thus, the interest saving amounts to 36% (see, for example, [8]).

The vendor will design two sub-contracts in a Two-Parts Term trade credit contract. The first applies to the prompt payment case. We will refer to it as a Cash Discount (CD) contract. The second applies to the case in which the buyer forgoes the discount opportunity and pays the gross amount at the net date. The second contract can be formulated exactly like that in the NT contract. As in QD and NT contracts, the vendor proposes a Two Parts Term contract, with a *discount period* $\gamma_D < \gamma_N$ and a discounted wholesale price $W_D < W^0$ as incentives to induce the buyer to increase its order size from Q_1 to Q_J. Upon coordination, the supply chain average inventory cost is :

$$C_J(Q_J,\gamma_D) = D(S_1+S_2)/Q_J + Q_J[(n_1+h_1)/2 - \gamma_D(h_1-a-b\gamma_D)]. \quad (3)$$

The following two properties are observed :

Property 1 : $C_J(Q_J,\gamma)$ is a convex function with respect to γ. The first order condition shows that the $\gamma^* = (h_1-a)/2b$ minimizes $C_J(Q_J,\gamma)$.

Property 2 : For any given $\gamma_N \in [\gamma_{LB},\gamma_{UB}]$, solving $C_1(Q_J|W_D,\gamma_D) = C_1(Q_J|W^0,\gamma_N)$ yields $W_D(\gamma_D) = W^0 - (\gamma_N-\gamma_D)h_1Q_J/D$. Let $\delta := \gamma_N - W^0D/h_1Q_J$. $\delta \le \gamma_D \Rightarrow W_D(\gamma_D) \ge 0$.

Property 3 : For any given $\gamma_N \in [\gamma_{LB},\gamma_{UB}]$, solving $C_1(Q_J|W_Q,0) = C_1(Q_J|W_D,\gamma_D) = C_1(Q_J|W^0,\gamma_N)$ yields $W_D(\gamma_D) = W^0 - (\gamma_N-\gamma_D)h_1Q_J/D$ and $W_Q(\gamma_N) = W^0 - \gamma_N h_1 Q_J/D$. $\delta \le 0 \Rightarrow W_Q(\gamma_N) \ge 0$.

Property 2 shows that the vendor may manipulate two parameters (γ_D, W_D) to achieve $C_1(Q_J|W^0,\gamma_N = C_1(Q_J|W_D,\gamma_D)$ so that the buyer is indifferent to choose between the two contracts. Thus, when $\gamma_N > \gamma^* > 0$, the vendor may design $\gamma_D = \gamma^*$ and $W_D(\gamma_D = \gamma^*)$ to minimize the supply chain (vendor's) inventory cost. Proposition 3 verifies this observation.

Proposition 3. *Supply chain cost functions (2) and (3) show that $C_J(Q_J,\gamma_D) < [>] C_J(Q_J,\gamma_N)$ when $\gamma_D[h_1-h_2(\gamma_D)] > [<] \gamma_N[h_1-h_2(\gamma_N)] \Rightarrow \gamma_D+\gamma_N > [<](h_1-a)/b$. Similarly, cost functions (1) and (3) show that $C_J(Q_J,\gamma_D) < [>] C_J(Q_J, W_Q)$ when $h_2(\gamma_D) < [>] h_1 \Rightarrow \gamma_d < [>](h_1-a)/b$. Thus, when $\gamma_N > \gamma^* > 0$, the vendor minimizes the supply chain (vendor's)* *cost by designing a CD contract with* $\gamma_D = \gamma^*$ *and* $W_D(\gamma_D = \gamma^*)$. □

Proposition 3 reveals that the vendor will prefer using a CD contract when $\gamma_D[h_1 - h_2(\gamma_D) > [>] \gamma_N[h_1 - h_2(\gamma_N)] \Rightarrow \gamma_D + \gamma_N > [<](h_1 - a)/b$. This condition compares "opportunity cost savings" from the two trade credit contracts. If the opportunity cost savings $\gamma_D[h_1 - h_2(\gamma_D)]$ from the CD contract is greater than that of the NT contract $\gamma_N[h_1 - h_2(\gamma_N)]$, then the vendor will use a CD contract to coordinate the supply chain. Property 1 reveals that the supply chain joint cost under the CD contract is convex, and the optimal $\gamma_D = (h_1 - a)/2b$; thus, the vendor will prefer using a CD contract when $\gamma_N > (h_1 - a)/2b$. Based on Properties 1, 2, 3, Propositions 2.4 and 3, the vendor designs the following procedure for formulating a supply chain contract. Assume that the contractual agreement between the buyer and the vendor requires that the buyer's cost to be no more than C_B. That is, C_B is a maximum cost that is acceptable to the buyer. Assume also that (i) a $\gamma_N \in [\gamma_{LB}, \gamma_{UB}]$ satisfies $C_1(Q_J | W^0, \gamma = \gamma_N) = C_B$, and (ii) $\delta \leq \Rightarrow W_Q(\gamma_N) \geq 0$ (see Property 3).

- Supply Chain Coordination Contracts

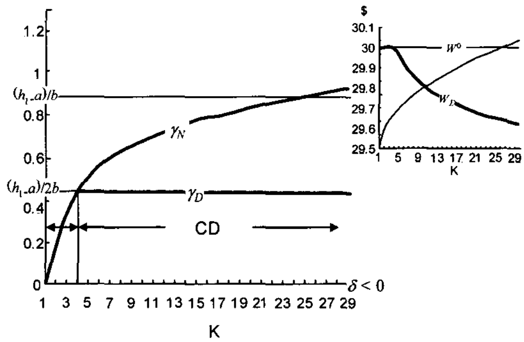
(i) QD contract (When $h_1 < a \Rightarrow \gamma^*(h_1 - a)/2 < 0$) : The vendor will not offer a NT contract (Proposition 2.4). Since $C_J(Q_J, \gamma)$ is a convex function with respect to γ (Property 1), the optimal CD contract = QD contract. The vendor offers a QD contract with a wholesale price = $W_Q(\gamma_N)$ (Property 3).

(ii) NT contract (When $h_1 > a$ and $\gamma_N \leq (h_1 - a)/2b$) : The vendor will not offer a QD contract (Proposition 2.4). Since $C_J(Q_J, \gamma)$ is a convex function with respect to γ, the optimal CD contract = NT contract. The vendor offers a NT contract with a wholesale price= W^0 and a net date = $\gamma_N \in [\gamma_{LB}, \gamma_{UB}]$.

(iii) CD contract (When $h_1 > a$ and $\gamma_N > (h_1 - a)/2b$) : The vendor offers a two-parts term contract with a discount period $\gamma_D = \gamma^*$ (Proposition 3). The vendor charges a wholesale price = W^0 for a late payment case, and charges a wholesale price $W_D(\gamma_D) - \epsilon$ (Property 2) for a prompt payment case. The Two-Parts Term contract can be summarized as follows:

(Discount = $100 \times (W^0 - W_D + \epsilon)/W^0\%$ / Discount Period = $(h_1 - a)/2b < 0$/Net Date=γ_N).

Since $C_1(Q_J | W_D - \epsilon, \gamma_D) < C_B$, the buyer prefers the CD contract. Let N:= number of $(\gamma_N - \gamma_D)Q_J/D$ period in a year. By taking the cash discount opportunity, the buyer's implicit opportunity cost saving amounts to effectively $100 \times (W^0 - W_D + \epsilon)/W^0\%$ annum.



[Figure 4] Supply Chain Coordination Contracts

[Figure 4] furnishes a numerical experiment illustrating the supply chain coordination contracts. The parameters are : $Q_1 = 100$, $n_1 = 2$, $h_1 = 8$, $W^0 = 30$, $L = 1000$, $a = 0$, and $b = 9$. We assume that $\gamma_N = 0.9\gamma_{LB} + 0.1\gamma_{UB}$. The numerical example shows that the vendor will use a NT contract when $K \leq 4$, and a CD contract when $K > 4$.

4. Conclusion

We presented a deterministic EOQ inventory system consists of a vendor and a buyer. Mismatches occur in individually optimal cycle times due to the difference in inventory cost structures. Means of settlement that focus on synchronizing mismatches in individually optimal cycle times are analyzed to minimize the joint inventory cost. The logic behind this approach is intuitively clear : The joint inventory cost generated from adopting an individually optimal lot size can never be smaller than that generated by adopting a lot size that minimizes the joint inventory cost. Therefore, a Pareto efficient solution can be obtained, and the two parties can design a win-win based fair arrangement that divides the cost savings generated from adopting the joint optimal lot size. We have here designed three supply chain coordinating contracts to facilitate the fair sharing of joint inventory cost savings. These are the: Quantity Discount contract, Net Term trade credit contract, and Two-Parts Term trade credit contract. The analysis has been carried out under the managerial scenario in which we assume that the vendor's opportunity cost during the credit period shows a strictly increasing trend with respect to the length of the credit period to reflect the trade credit risk. Two bounds for assuring win-win conditions are developed. These bounds are formulated to guarantee that the buyer's cost sacrifice can at least be compensated, and that the inducement should not result in the vendor's inventory cost being greater than the original cost. Our research reveals that trade credit contracts (Net Term and Two-Parts Term trade credit contracts) can be effectively used to coordinate a supply chain and that in many cases it will result in a lower supply chain joint cost compared to that achieved by using a Quantitative Discount contract.

In the present study, we set out to analyze the possibility of implementing a trade credit contract to coordinate a supply chain that achieves system-wide improvement. In our view, the analysis has some limitations. First, our research is done in a simplified setting where we consider a vendor with a sole/major buyer case. In discussing the topic, including multiple heterogeneous customers might provide more meaningful results. Second, we assume that the demand is independent of retail price. An extension of the work to include retail price sensitivity might be more appropriate. These limitations indicate possible extensions in future studies.

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Appendix

Proof of Proposition 1 :

Proposition 1.1 : It is seen that $\partial C_1(Q_1|W)/\partial W = D \geq 0$; thus, $C_1(Q_1|W)$ is an increasing function of W. Solving W_{UB} satisfies:

$$C_1(Q_1|W^0) = C_1(Q_J|W_{UB}) \Rightarrow S_1 D(1-1/K)/Q_1 + Q_1(n_1+h_1)(1-K)/2 + D(W^0 - W_{UB}) = 0$$
$$\Rightarrow DS_1[Q_1(W^0 - W_{UB})/S_1 - (K-1)^2/k]/Q_1 = 0$$
$$\Rightarrow W_{UB} = W^0 - S_1(K-1)^2/Q_1 K.$$

Thus, any wholesale price $W \leq W_{UB} \Rightarrow C_1(Q_1|W^0 \geq C_1(Q_J|W)$.

Proposition 1.2 : It is seen that $\partial C_2(W|Q_1)/\partial W = -D \leq 0$; thus, the vendor's inventory cost decreases as the wholesale price increases. Solving W_{LB} satisfies :

$$C_1(W^0|Q_J) = C_2(W^0|Q_1) \Rightarrow S_2 D(1-1/K)/Q_1 + D(W_{LB} - W^0) = 0$$
$$\Rightarrow W_{LB} = W^0 - S_1(K+1)(K-1)^2/Q_1 K. \qquad \text{since } S_2 = S_1(K^2-1).$$

Thus, any wholesale price $W \geq W_{UB} \Rightarrow C_2(W|Q_J) \leq C_2(W^0|Q_1)$.

Proposition 1.3 : $W_{UB} - W_{LB} = S_1(k-1)^2/Q_1 > 0$ and $W^0 - W_{UB} = S_1(k-1)^2/KQ_1 > 0$ if $K > 1$.

Proof of Proposition 2 :

Proposition 2.1 : $\partial C_1(Q_1|\gamma)/\partial\gamma = -Q_J h_1 \leq 0$; thus, the buyer's inventory cost decreases as γ increases. Solving γ_{LB} satisfies :

$$C_1(Q_J|\gamma_{LB}) = C_1(Q_1|\gamma = 0) \Rightarrow S_1 D(1-1/K)/Q_1 + Q_1(n_1+h_1)(1-K)/2 + KQ_1\gamma h_1 = 0$$
$$\Rightarrow S_1 D[2\gamma K h_1/(n_1+h_1) - (K-1)^2/K]/Q_1 = 0$$
$$\Rightarrow \gamma_{LB} = (K-1)^2(n_1+h_1)/2h_1 K^2.$$

Thus, any net date $\gamma \geq \gamma_{LB} \Rightarrow C_1(Q_1|\gamma = 0) \geq C_1(Q_J|\gamma_{LB})$.

Proposition 2.2 : $\partial C_2(\gamma|Q_1)\partial\gamma = Q_J h_2(\gamma) \geq 0$; thus, the vendor's inventory cost increases as γ increases. Solving γ_{UB} satisfies :

$$C_2(\gamma_{UB}|Q_J) = C_2(\gamma = 0|Q_1) \Rightarrow \gamma_{UB} h_2(\gamma_{UB}) = (1+K)(K-1)^2(n_1+h_1)2K^2.$$

Substituting $h_2(\gamma_{UB})a + b\gamma_{UB} \Rightarrow \gamma_{UB} = \left[\sqrt{a^2 + 4b(1+K)(K-1)^2(n_1+h_1)/2K^2} - a\right]/2b$

Thus, any wholesale price $\gamma \leq \gamma_{UB} \Rightarrow C_2(\gamma | Q_J) \geq C_2(\gamma = 0 | Q_J)$.

Proposition 2.3 : It is seen that

$$(\gamma_{UB} - \gamma_{LB})h_2(\gamma_{UB}) = (n_1 + h_1)(K-1)^2\{(1+K)h_1 - h_2(\gamma_{UB})\}/2h_1K^2 \ ;$$

thus, $\gamma_{UB} - \gamma_{LB} > 0$ if $K > h_2(\gamma_{UB})/h_1 - 1$ and $K > 1$. Substituting $h_2(\gamma_{UB})a + b\gamma_{UB}$ leads to $K > \max[1, (a + b\gamma_{UB})/h_1 - 1]$.