Double-Frequency Jitter in Chain Master-Slave Clock Distribution Networks: Comparing Topologies

Jose Roberto Castilho Piqueira and Andrea Zaneti Caligares

Abstract: **Master-slave (M-S) strategies implemented with chain circuits are the main option in order to distribute clock signals along synchronous networks in several telecommunication and control applications. Here, we study the two types of master-slave chains: Without clock feedback, i.e., one-way master-slave (OWMS) and with clock feedback, i.e., two-way master-slave (TWMS) considering the slave nodes as second-order phase-locked loops (PLL) for several types of loop low-pass filters.**

Index Terms: **Clock, double-frequency jitter, filter, master-slave (M-S), network, phase-locked loop (PLL).**

I. INTRODUCTION

Network synchronization deals with the distribution of phase and frequency signals along several nodes located in a certain geographic area. There are three different implementation strategies: Plesiochronous, synchronous master-slave (M-S), and synchronous full-connected [1], [2].

In the plesiochronous case, all nodes have their own precise free oscillator. Supposedly, they have the same frequency and, from time to time, manual adjustment is performed. Synchronous networks have their wholex time basis automatically adjusted during system operation by using the clock signals from individual nodes.

Synchronous full-connected networks give the same weight for all nodes in order to determine the whole synchronous state. Master-slave networks present a priority scheme with an atomic pattern oscillator, working as master, giving the time basis to the tributary nodes.

M-S networks can be implemented by two kinds of topology: One way master-slave (OWMS) and two ways master-slave (TWMS). The master clock is an atomic oscillator that sends its time basis to the slave nodes, represented by phase-locked loops (PLLs). PLLs work like band-pass filters tracking angle-modulated signals and tuning over the range of frequency variation.

In the single chain topology (OWMS), the clock signal is sent from the master to the first slave that synchronizes its local oscillator and sends the time basis to the second one. The process is repeated up to the synchronization of the last node of the chain.

In the double chain (TWMS), the process is similar, but the control signal sent by the master to the node 1 considers a feedback clock signal from node 2. Analogously, the control signal sent from node $j-1$ to node j considers a feedback control signal from node $j+1$.

In this work, we study the single (OWMS) and double (TWMS) chain topologies, which are widely employed in public telecommunications networks, robotics, and many other engineers' applications. This great variety of applications is due to their reliable behavior, construction facility, and low cost.

Considering that commercial versions of PLLs include a phase detector modeled by a signal multiplier [3] double-frequency terms are unavoidable. Oscillations around the synchronous state appear in spite of the low-pass filters attenuate these terms [4]. These oscillations are called double-frequency jitter [5].

The effect of this kind of jitter for M-S chains is studied and its accumulation along the nodes is modeled, considering phase step perturbations in the master signal.

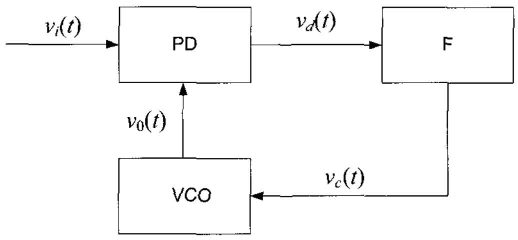

Fig. 1. PLL block diagram.

II. ANALYTIC NETWORK MODEL

The master clock of chain networks is stable and accurate represented by

$$\Phi_M = \omega_M t + \Psi(t) \tag{1}$$

where ω_M is the free running frequency and $\Psi(t)$ is the perturbation signal.

In this work, the perturbation is a step and, according to the (1), it causes a discontinuity on the master's phase and, consequently, on its frequency. Therefore, the role of the slave nodes is re-acquiring the synchronous state.

The basic slave node of the chain can be modeled by a PLL, i.e., a closed loop composed of a phase-detector (PD), a low-pass filter (F), and a voltage controlled oscillator (VCO), as shown in Fig. 1.

Oscillation generated by the VCO belonging to the PLL located in the j-slave node, $v_0^{(j)}(t)$, must be synchronized with its input signal $v_i^{(j)}(t)$, i.e., the phase error, $\theta_i^{(j)} - \theta_0^{(j)}$, must be constant and, consequently, the frequency error, $\dot{\theta}_i^{(j)} - \dot{\theta}_0^{(j)}$, must be zero.

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Table 1. Linear stable low-pass filters.

Filter type	Transfer function
Passive lag-lead	$H(s) = \frac{s\tau_2+1}{s(\tau_2+\tau_1)+1}$
Active lag-lead	$H(s) = \frac{s\tau_2+1}{s\tau_1+1}$
Active PI	$H(s) = \frac{s\tau_2+1}{s\tau_1}$

As the phase detector considered is a signal multiplier, its output $v_d^{(j)}$ is given by

$$v_d^{(j)}(t) = v_i^{(j)}(t)v_o^{(j)}(t). \tag{2}$$

Three types of linear stable filters with transfer functions given in Table 1 are considered: Passive lag-lead, active lag-lead, and active proportional and integral (PI) [3]. These filters are used in the main commercial integrated PLLs, for instance, in CD 4046.

We suppose that $v_i^{(j)}(t)$ has a periodic form with central angular frequency ω_M and time-varying phase $\theta_i^{(j)}(t)$. Signal $v_o^{(j)}(t)$ has also a periodic form with central angular frequency ω_M and adjustable phase $\theta_o^{(j)}(t)$.

In order to allow the phase detection, input and output signals have a 90° phase difference and for each j-node can be written as

$$v_i^{(j)}(t) = V_i \sin[\omega_M t + \theta_i^{(j)}(t)] \tag{3}$$

$$v_o^{(j)}(t) = V_o \cos[\omega_M t + \theta_o^{(j)}(t)] \tag{4}$$

where V_i and V_o are the amplitudes of the signals.

As a consequence of (2)–(4), the expression for $v_d^{(j)}$ has a second-harmonic term added to the sine of the phase error. In fact, $v_d^{(j)}$ is given by [4], [5]

$$v_d^{(j)}(t) = \frac{k_d V_i V_o}{2}\sin[2\omega_M t + \theta_i^{(j)}(t) + \theta_o^{(j)}(t)] + \frac{k_d V_i V_o}{2}\sin[\theta_i^{(j)}(t) - \theta_o^{(j)}(t)] \tag{5}$$

where k_d is the PD gain.

The output phase $\theta_o^{(j)}$ is controlled by the filter output signal $v_c^{(j)}(t)$ according to

$$\dot\theta_o^{(j)}(t) = k_o v_c^{(j)}(t). \tag{6}$$

Combining (2)–(6) with the filter transfer functions, one can show that the dynamics of the phase $\theta_0^{(j)}(t)$ of each slave node is described by a second-order non linear differential equation, depending on the filter type. These equations have the whole PLL gain $G = \frac{k_d k_0 V_i V_0}{2}$ as a parameter [3].

According to [3], we assume that the dynamics of the local phase error $\phi^{(j)} = \theta_i^{(j)}(t) - \theta_0^{(j)}(t)$ of each slave node can be described by

$$\ddot\phi^{(j)} + \mu\dot\phi^{(j)} + \mu G^{(j)}\sin\phi^{(j)} = \ddot\theta_i^{(j)} + \mu\dot\theta_i^{(j)} + A^{(j)}\sin(2\omega_M t + \psi_0^{(j)}) \tag{7}$$

where μ is the time constant of the low-pass filter, and $A^{(j)}$ and $\psi_0^{(j)}$ are the amplitude and phase of the double-frequency term remaining after filtering. Consequently, $A^{(j)}$ is supposed to be small.

In our previous works [4], [5], by using analytical and numerical tools, it was shown that this kind of PLL, described by (7) presents double-frequency jitter in a single node analysis.

Here, we study the behavior of this jitter in OWMS and TWMS chains with the master presenting a phase step perturbation.

For this type of network, the double-frequency jitter does not depend on transmission delays [6]. Consequently, we will analyze the behavior of single and double-chain networks submitted to a step perturbation in the master.

If we consider the OWMS architecture, the input of the first slave PLL (node 2) is given by

$$\theta_i^{(2)}(t) = \Phi_M(t - t_{1,2}) \tag{8}$$

with $t_{1,2}$ representing the propagation time from node 1 to node 2.

For each j-slave node, $j = 3, 4, \cdots, N$, phase of the input signal is written as

$$\theta_i^{(j)}(t) = \theta_0^{(j-1)}(t - t_{j-1,j}) \tag{9}$$

with $t_{j-1,j}$ representing the propagation time from node $j-1$ to node j.

Therefore, if the delays are neglected and a step perturbation appears in the master, phase error dynamic equation for node 2, the first slave of a OWMS network, becomes

$$\ddot\phi^{(2)} + \mu\dot\phi^{(2)} + \mu G^{(2)}\sin\phi^{(2)} = \dot\theta_0^{(1)} + A^{(2)}\sin(2\omega_M t + \psi_0^{(2)}). \tag{10}$$

For the other $j = 3, \cdots, N$ nodes of the OWMS network, phase error dynamics equation is

$$\ddot\phi^{(j)} + \mu\dot\phi^{(j)} + \mu G^{(j)}\sin\phi^{(j)} = \dot\theta_0^{(j-1)} + A^{(j)}\sin(2\omega_M t + \psi_0^{(j)}). \tag{11}$$

Observing (10), we can see that it is similar to the simple pendulum with a small periodic perturbation. Its permanent solution is a double-frequency oscillation around a synchronous equilibrium point [7].

Equation (11) presents the same behavior as (10), but with a smaller double-frequency jitter as j increases because the number of low-pass filters in the way of the signal is increased.

For high values of $G^{(j)}$, the terms with derivatives become much smaller than the containing $\sin\phi^{(j)}$ in the left side of (10) and (11). Besides, the right side of (11), after the transient, contains a synchronous solution added to the double-frequency jitter.

Consequently, for high $G^{(j)}$, all nodes present almost the same double-frequency jitter in the OWMS architecture.

Considering the TWMS architecture [8], for the first slave PLL (node 2), phase of input signal is given by

$$\theta_i^{(2)}(t) = \theta_M(t - t_{12}) - \frac{1}{2}\theta_0^{(2)}(t - t_{12} - t_{21}) + \frac{1}{2}\theta_0^{(3)}(t - t_{32}). \tag{12}$$

For each j-slave node, $j = 3, 4, \cdots, N-1$, phase of the input signal is written as

$$\theta_i^{(j)}(t) = \frac{1}{2}\theta_0^{(j-1)}(t - t_{i-1,i}) + \frac{1}{2}\theta_0^{(j+1)}(t - t_{j+1,j}). \quad (13)$$

For slave node N, phase of the input signal is

$$\theta_i^{(N)}(t) = \theta_0^{(N-1)}(t - t_{N-1,N}). \quad (14)$$

Therefore, if the delays are neglected and a step perturbation appears in the master, phase error dynamic equation for node 2, the first slave of a TWMS network, becomes

$$\ddot{\phi}^{(2)} + \mu\dot{\phi}^{(2)} + \mu G^{(2)} \sin\phi^{(2)} = -\frac{1}{2}\dot{\theta}_0^{(2)}(t) + \frac{1}{2}\dot{\theta}_0^{(3)}(t) + A^{(2)}\sin(2\omega_M t + \psi_0^{(2)}). \quad (15)$$

For $j = 3, \cdots, N-1$, the phase error dynamics equation for TWMS nodes is

$$\ddot{\phi}^{(j)} + \mu\dot{\phi}^{(j)} + \mu G^{(j)} \sin\phi^{(j)} = -\frac{1}{2}\dot{\theta}_0^{(j-1)}(t) + \frac{1}{2}\dot{\theta}_0^{(j+1)}(t) + A^{(j)}\sin(2\omega_M t + \psi_0^{(j)}). \quad (16)$$

For $j = N$, the phase error dynamics equation in TWMS architecture is

$$\ddot{\phi}^{(N)} + \mu\dot{\phi}^{(N)} + \mu G^{(N)} \sin\phi^{(N)} = \frac{1}{2}\dot{\theta}_0^{(N-1)}(t) + A^{(N)}\sin(2\omega_M t + \psi_0^{(N)}). \quad (17)$$

Observing (15) and (10), we conclude that the jitter performance must be the same because, in the permanent regime, for (15) the terms related to $\dot{\theta}^{(3)}(t)$ and to $\dot{\theta}^{(2)}(t)$ will be cancelled. Therefore, the first slave node of OWMS and TMWS will present the same double-frequency jitter behavior.

Comparing (15) with (16) and (17), we can see that the double-frequency jitter will be smaller as j increases due to the increasing number of filters in the signal way.

Long-term behavior of (16) and (17), representing TWMS networks, when compared to (11), seems to present better attenuation for the oscillation around the equilibrium state. This fact is a consequence of the two derivatives terms presented by (16), instead of one term, presented by (11), producing a more effective filtering process.

For high values of $G^{(j)}$, because the same facts explained for OWMS, (15)–(17) have lower differences for the double-frequency jitter behavior.

III. NUMERICAL SIMULATIONS

In order to complement our view about the double-frequency jitter in OWMS and TWMS topologies, we simulated them, neglecting propagation delays. We used built in MATLAB-Simulink blocks [9] and we compared the performance of different kinds of filters for OWMS and TWMS systems.

Adjusting the gain (G), we measured the peak-to-peak permanent jitter (J) after a step phase perturbation in the master.

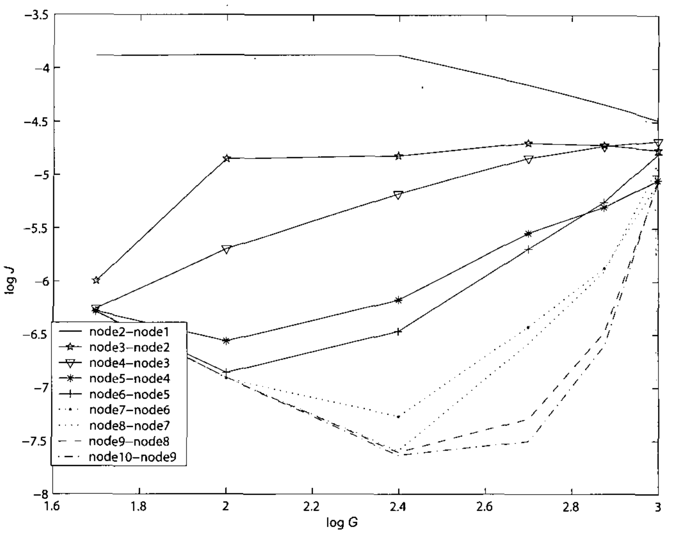

Fig. 2. OWMS—permanent double-frequency jitter versus gain—active lag filter.

J is measured comparing the output of two consecutive filters after a step perturbation in the master. When the transient vanishes, we measure the peak-to-peak of the resulting oscillation around the synchronous state.

Chains with a master and ten slaves are simulated choosing the "ode-23 Bogacki-Shampine" [9] as integration method with variable step and 10^{-12} of relative tolerance. The chosen number of slave nodes is ten, according to UTI-T Recommendation G.812 [10].

The free running angular frequency ω_M is set in 1 rad/s to normalize the results. The unitary step $\Psi(t)$ is added to the master phase $\Phi_M(t)$.

The filter parameters are $\tau_1 = 6280$ and $\tau_2 = 62.80$, providing a good compromise between transient and permanent responses.

We started with the OWMS single chain and, to determine the network behavior after the step phase signal, we adjusted the gain and measured the peak to peak double-frequency jitter in permanent regime.

Figs. 2–4 show the peak-to-peak jitter J_M varying the gain G, in logarithm scales, for active lag, passive lag, and active PI filters, respectively.

Simulations were repeated for TWMS double chain with results shown is Figs 5–7, for active lag, passive lag, and active PI filters, respectively.

A quantitative idea of the measured values can be given considering that the maximum jitter, according to the ITU-T Recommendation G.811 [11], is 0.015 unitary intervals-peak-to-peak (UIpp), in order to maintain acceptable the network performance for 2048 kbps links.

This value was adapted to our normalized 1 rad/s free-running angular frequency corresponds to $\log J = -1.8$.

IV. CONCLUSION

Considering the maximum ITU recommended jitter, numerical simulation results show a large difference between the

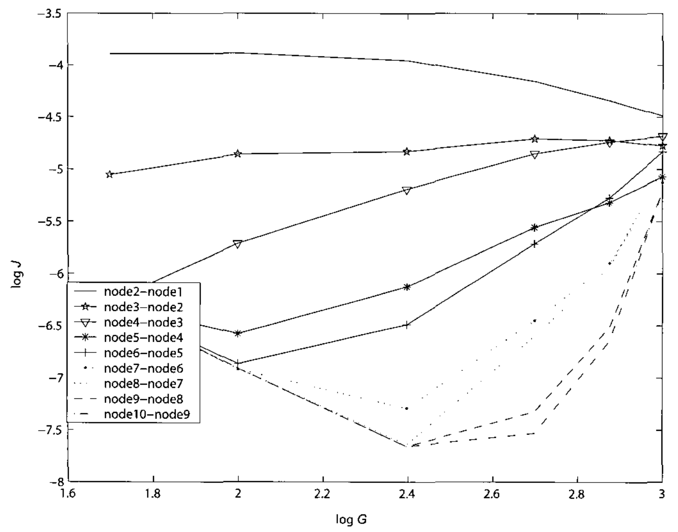

Fig. 3. OWMS—permanent double-frequency jitter versus gain—passive lag filter.

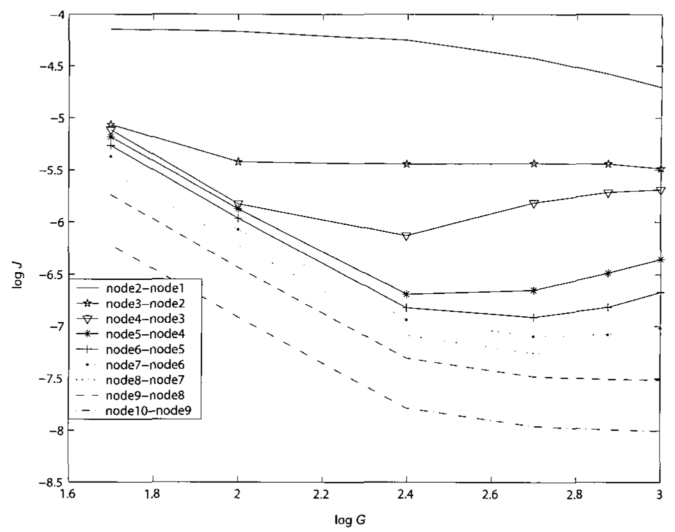

Fig. 5. TWMS—permanent double-frequency jitter versus gain—active lag filter.

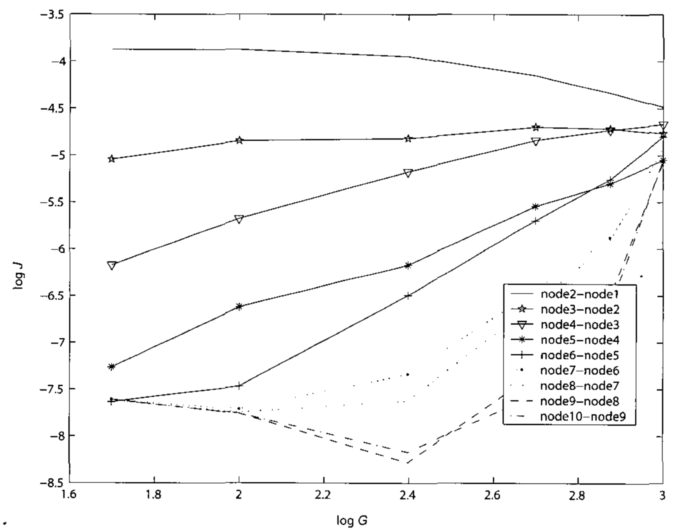

Fig. 4. OWMS—permanent double-frequency jitter versus gain—active PI filter.

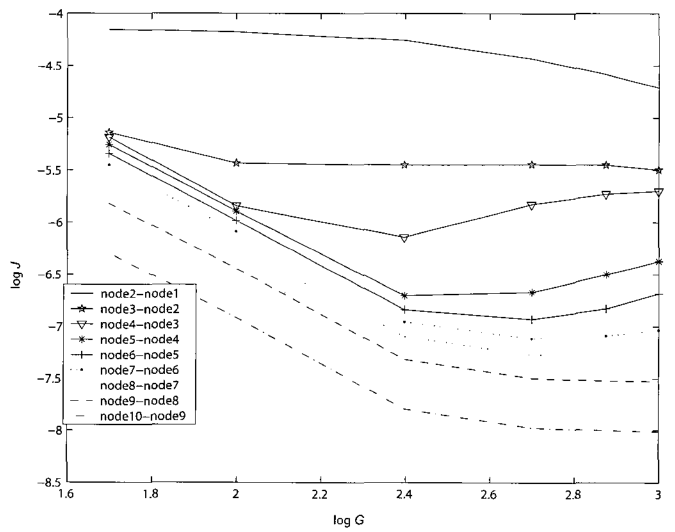

Fig. 6. TWMS—permanent double-frequency jitter versus gain—passive lag filter.

double-frequency jitter and the ITU specified limit, even for low gains of the PLL slave nodes.

Consequently, one could argue that the results are indicating that the double-frequency jitter is too low, being not important for designing networks.

Even though, mainly in wireless networks, for the accumulated jitter in the network, double-frequency jitter must be considered together with cross-talk, noise, distortion, inter-symbol interference, and quantization noise if a good performance for the time distribution network is required [1], [4], [5], [10], [11].

In order to give some support to the design of synchronous master-slave chain networks we summarize some conclusions about the double-frequency jitter, obtained analytically (Section II) and using simulations (Section III).

- Except for node 1, the measured values are better for TWMS than for the corresponding OWMS case. Therefore, this topology could present better performance, in spite of not being used frequently;
- for the both topologies, the double-frequency jitter does not accumulate and decreases as the nodes become more distant from the master, along the chain;
- active PI filters present a better performance with a low level of double-frequency jitter in the both cases. Increasing the gain, corresponding to a wider lock-in range for the network, the three types of filter seems to be equivalent;
- for OWMS and TWMS, as gain increases the double-frequency jitter performance of all nodes seems to become closer but, as simulations show, this phenomenon is stronger in the OWMS case.

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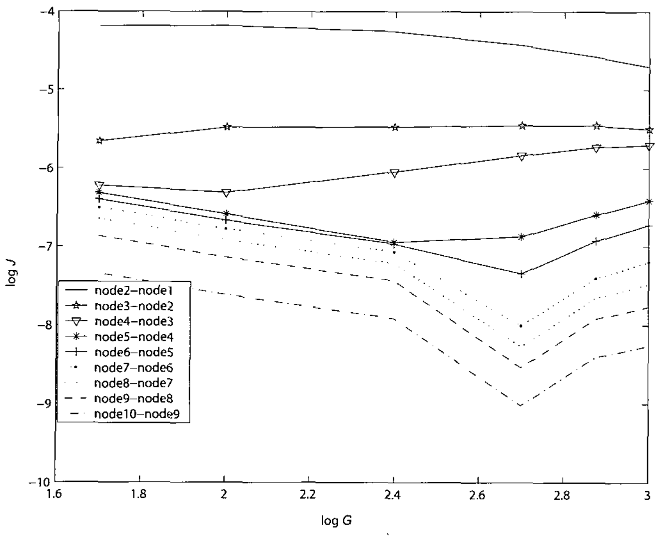

Fig. 7. TWMS—permanent double-frequency jitter versus gain—active PI filter

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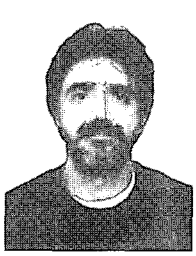

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