Lever Arm Compensation for GPS/INS/Odometer Integrated System

Jaewon Seo, Hyung Keun Lee, Jang Gyu Lee, and Chan Gook Park*

Abstract: For more accurate navigation, lever arm compensation is considered. The compensation method for GPS and an odometer is introduced and new compensation methods are proposed for an odometer to consider the effect of coordinate transformation errors and the scale factor error. The methods are applied to a GPS/INS/odometer integrated system and the simulation and experimental results show its effectiveness.

Keywords: GPS, INS, integrated navigation system, Kalman filter, lever arm compensation.

1. INTRODUCTION

Navigation is defined as all the related theories and technologies for obtaining position, velocity and attitude of a vehicle. With the use of navigation technology, one can know his/her position and can plan trajectory for their destination. Nowadays, especially, the need for the navigation of land vehicles has rapidly increased. In the near future, ubiquitous personal navigation will be spread and navigation will become a more essential technology.

The Global Positioning System (GPS) is a satellite-based radio navigation system [1]. It allows a user with a receiver to obtain accurate position information anywhere on the globe. It provides position information whose errors would not increase with respect to time. However, a signal from the GPS satellite cannot often arrive at a receiver in an urban area. In those cases, it cannot provide any position information.

The Inertial Navigation System (INS) is a self-contained, hardly-disturbed, dead-reckoning navigation system [2-4]. It works according to the inertial principle and integrates the inertial measurement from inertial sensors. The accuracy of the INS depends on the ability of the inertial sensors. The biases and drifts of the sensors are accumulated in the navigation solution, and therefore, the navigation error of the INS increases unlimitedly with respect to time. On the other hand, it can provide a continuous navigation solution without disturbance.

These complementary properties of the GPS and the INS motivate their integration. The GPS/INS integrated system entails the Kalman filter for error estimation and compensation. However, as mentioned previously, the GPS is unavailable in some cases, and, in those cases, the GPS/INS integrated system becomes equivalent to an INS-only. In such instances, the GPS/INS/odometer integrated system is efficient and cost-effective. An odometer is a velocity sensor and it is also self-contained and hardly-disturbed. The odometer provides undisrupted velocity information continuously, and compensates the navigation error even in the case of unavailable GPS. Moreover, generally, it produces more accurate and more frequent velocity measurement than the GPS.

When integrating GPS, an INS, and an odometer, we can be confronted with some obstacles. Among them, the lever arm effect is addressed in this paper. Physically, the Inertial Measurement Unit (IMU) of an INS, the GPS antennas, and the odometers cannot be located at the same position. The lever arm is the distance between the sensing points of the sensors. The position that is calculated by the INS and the one calculated by the GPS are different by the vector of the lever arm. The GPS antenna is usually mounted on the outside of the vehicles, such as on the roof, while the IMU is located inside or at a different spot. In the same manner, the velocity of an IMU and that of an odometer are different due to the lever arm effect. The odometer is usually installed near the wheel of a vehicle. Therefore, the measurements by GPS, the

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IMU, and the odometer must be integrated in a systematic approach to compensate the lever arm effect. This systematic approach is called lever arm compensation. It makes the navigation solution more accurate. In [3], the lever arm effect and compensation are briefly explained. The lever arm effect between accelerometers and its application to gravimetry are described in [4]. Estimation of the measurement error of lever arm length and the misalignment errors of GPS antenna array is treated in [5]. In this paper, the lever arm compensation is explained and new methods are proposed. In the methods, the effect of coordinate transformation errors and the effect of an odometer scale factor error are considered. They are applied to the GPS/INS/odometer integrated system and their simulation and experimental results are presented.

In the next section, the general integration of the GPS, the INS, and an odometer is explained. Then the lever arm effect and compensation method are presented, and finally the simulation and experimental results are given.

2. GPS/INS/ODOMETER INTEGRATED SYSTEM

The GPS is a satellite-based radio navigation system. It provides position of a vehicle. For position calculation, generally at least four satellites must be visible. However, in an urban area, the number of visible satellites is often less than four due to tall buildings and trees. Therefore, with only a GPS receiver, continuous navigation is impossible. On the other hand, the INS is based on the inertial principle. In the system, the accelerations measured by accelerometers and the angular velocities by gyroscopes are integrated without external aids. Through the integration, linear velocity, position, and attitude can be obtained. Therefore, it can provide navigation solution continually. However, the measurement errors in accelerometers and gyroscopes must be integrated together with the true values, and then they accumulate in position solution without limit. The GPS/INS/odometer integrated system incorporates the characteristics of each navigation system. It integrates the output of the IMU at more frequent periods and if the external information by the GPS or an odometer is available at some instant, it combines those through some estimation method such as Kalman filtering.

2.1. Mechanization equations

The mechanization equations for the inertial navigation are

$$\dot{L} = \frac{v_N}{R_m + h}, \quad \dot{l} = \frac{v_E}{(R_t + h)\cos L}, \quad \dot{h} = -v_D,$$
$$\begin{aligned}\dot{V} &= \begin{bmatrix} \dot{v}_N & \dot{v}_E & \dot{v}_D \end{bmatrix} \\ &= C_b^n f^b - (2\omega_{ie}^n + \omega_{en}^n) \times V + g, \\ \dot{q} &= \frac{1}{2} q * (\omega_{ib}^b - C_n^b(\omega_{ie}^n + \omega_{en}^n)).\end{aligned} \tag{1}$$

$\begin{bmatrix} L & l & h \end{bmatrix}^T$ is a position vector in the earth-centered-earth-fixed (ECEF) geodetic frame, $\begin{bmatrix} v_N & v_E & v_D \end{bmatrix}^T$ is a velocity vector in the local level navigation frame, q is a quaternion used for attitude calculation, C_b^n is a direction cosine matrix from the body frame to the navigation frame, f^b is an acceleration vector measured by the IMU in the body frame, g is a gravity vector, ω_{ib}^b is an angular rate vector measured by the IMU in the body frame, $\omega_{ie}^n = \begin{bmatrix} \Omega_N & 0 & \Omega_D \end{bmatrix}$ is the earth rate resolved in the navigation frame, and $\omega_{en}^n = \begin{bmatrix} \rho_N & \rho_E & \rho_D \end{bmatrix}$ is the transport rate. R_m and R_t are meridian and traverse radii of curvature in the earth ellipsoid, and are given as follows:

$$R_m \equiv \frac{R_0(1-e^2)}{(1-e^2\sin^2 L)^{3/2}}, \quad R_t \equiv \frac{R_0}{(1-e^2\sin^2 L)^{1/2}}, \tag{2}$$

where R_0 is the equatorial earth radius and e is the eccentricity of the earth ellipsoid. The relationship between a quaternion and a direction cosine matrix is given as

$$C_b^n = \begin{bmatrix} q_0^2+q_1^2-q_2^2-q_3^2 & 2(q_1q_2-q_0q_3) & 2(q_1q_3+q_0q_2) \\ 2(q_1q_2+q_0q_3) & q_0^2-q_1^2+q_2^2-q_3^2 & 2(q_2q_3-q_0q_1) \\ 2(q_1q_3-q_0q_2) & 2(q_2q_3+q_0q_1) & q_0^2-q_1^2-q_2^2+q_3^2 \end{bmatrix}. \tag{3}$$

Detailed explanations of (1)~(3) are provided in [2-4,6].

2.2. Error model

With (1) and the linear perturbation method, the following error model is derived [2,6].

$$\begin{aligned}\dot{x}_I(t) &= F_I(t)x_I(t) + G(t)w(t) \\ &= \begin{bmatrix} F_{11} & F_{12} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ F_{21} & F_{22} & F_{23} & C_b^n & 0_{3\times3} \\ F_{31} & F_{32} & F_{33} & 0_{3\times3} & -C_b^n \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \end{bmatrix} x_I(t)\end{aligned}$$

$$+\begin{bmatrix} 0_{3\times3} & 0_{3\times3} \\ C_b^n & 0_{3\times3} \\ 0_{3\times3} & C_b^n \\ 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} \end{bmatrix} w(t), \tag{4}$$

$$\begin{aligned} x_I(t) &= \begin{bmatrix} x_f & x_a \end{bmatrix}^T, \\ x_f &= [\delta L \quad \delta l \quad \delta h \quad \delta V_N \quad \delta V_E \quad \delta V_D \\ &\qquad \phi_N \quad \phi_E \quad \phi_D], \\ x_a &= \begin{bmatrix} \nabla_x & \nabla_y & \nabla_z & \varepsilon_x & \varepsilon_y & \varepsilon_z \end{bmatrix}, \end{aligned} \tag{5}$$

where

$$F_{11} = \begin{bmatrix} \dfrac{R_{mm}\rho_E}{R_m+h} & 0 & \dfrac{\rho_E}{R_m+h} \\ \dfrac{\rho_N}{\cos L}\left(\tan L - \dfrac{R_{tt}}{R_t+h}\right) & 0 & -\dfrac{\rho_N \sec L}{R_t+h} \\ 0 & 0 & 0 \end{bmatrix},$$

$$F_{12} = \begin{bmatrix} \dfrac{1}{R_m+h} & 0 & 0 \\ 0 & \dfrac{\sec L}{R_t+h} & 0 \\ 0 & 0 & -1 \end{bmatrix},$$

$$F_{21} = \begin{bmatrix} \dfrac{\rho_E R_{mm} v_D}{R_m+h} - (\rho_N \sec^2 L + 2\Omega_N) v_E - \rho_N \rho_D R_{tt} & 0 & \dfrac{\rho_E}{R_m+h} v_D - \rho_N \rho_D \\ \left(2\Omega_N + \rho_N \sec^2 L + \dfrac{\rho_D R_{tt}}{R_t+h}\right) v_N - \left(\dfrac{\rho_N R_{tt}}{R_t+h} - 2\Omega_D\right) v_D & 0 & \dfrac{\rho_D}{R_t+h} v_N - \dfrac{\rho_N}{R_t+h} v_D \\ \rho_N^2 R_{tt} + \rho_E^2 R_{mm} - 2\Omega_D v_E & 0 & \rho_N^2 + \rho_E^2 \end{bmatrix},$$

$$F_{22} = \begin{bmatrix} \dfrac{v_D}{R_m+h} & 2\rho_D + 2\Omega_D & -\rho_E \\ -2\Omega_D - \rho_D & \dfrac{v_N \tan L + v_D}{R_t+h} & 2\Omega_N + \rho_N \\ 2\rho_E & -2\Omega_N - 2\rho_N & 0 \end{bmatrix},$$

$$F_{23} = \begin{bmatrix} 0 & -f_D & f_E \\ f_D & 0 & -f_N \\ -f_E & f_N & 0 \end{bmatrix},$$

$$F_{31} = \begin{bmatrix} \Omega_D & 0 & -\dfrac{\rho_N}{R_t+h} \\ 0 & 0 & -\dfrac{\rho_E}{R_m+h} \\ -\Omega_N - \rho_N \sec^2 L & 0 & -\dfrac{\rho_D}{R_t+h} \end{bmatrix},$$

$$F_{32} = \begin{bmatrix} 0 & \dfrac{1}{R_t+h} & 0 \\ -\dfrac{1}{R_m+h} & 0 & 0 \\ 0 & \dfrac{\tan L}{R_t+h} & 0 \end{bmatrix},$$

$$F_{33} = \begin{bmatrix} 0 & \Omega_D + \rho_D & -\rho_E \\ -\Omega_D - \rho_D & 0 & \Omega_N + \rho_N \\ \rho_E & -\Omega_N - \rho_N & 0 \end{bmatrix},$$

$$R_{mm} = \frac{\partial R_m}{\partial L}, \quad R_{tt} = \frac{\partial R_t}{\partial L}, \quad \begin{bmatrix} f_N \\ f_E \\ f_D \end{bmatrix} = C_b^n f^b.$$

Notation δ indicates that the variable is an error variable. For example, δL is the latitude error derived by the perturbation. ∇'s are accelerometer biases and ε's are gyroscope drifts. $[\phi_N \quad \phi_E \quad \phi_D]$ is an attitude error represented by the Euler angles. $w(t)$ is a measurement noise of inertial sensors and is assumed to be a white Gaussian process. $[\Omega_N \quad 0 \quad \Omega_D]$ is the earth rate resolved in the navigation frame and $[\rho_N \quad \rho_E \quad \rho_D]$ is the transport rate. For the GPS position measurement and the velocity measurement of an odometer, the following measurement models are given [2,6].

$$\begin{aligned} \delta z_{GPS} &= [I_{3\times3} \mid 0_{3\times12}] x_I(t_k) + v_{GPS}(t_k), \\ \delta z_{odo} &= [0_{3\times3} \mid I_{3\times3} \mid -V\times \mid 0_{3\times6}] x_I(t_k) + v_{odo}(t_k), \end{aligned} \tag{6}$$

where δz_{GPS} is position measurement residual of the GPS, δz_{odo} is velocity measurement residual by the odometer, v_{GPS} and v_{odo} are measurement noises of the GPS and the odometer, respectively. They are assumed to be white Gaussian noises. Subscript k of time t indicates the instant of acquisition of measurement.

In the derived error model, the estimated values are used for the system and measurement model matrix, instead of the true values. This is directly related to the compensation method of the navigation errors, namely, the indirect feedback method [9,10]. For the simple notation, it is not indicated that the variables are the estimated values. In fact, according to the definition of the perturbed errors, both the estimated values and the true values can be used. For the implementation, the estimated values are used in the model matrix.

2.3. Kalman filter

With (4)-(6), the Kalman filter can be designed as follows for navigation error estimation. Kalman filter is an optimal linear filter [7]. After discretization of (4), the discrete Kalman filter can be constructed. The equations of the discrete Kalman filter are given;

$$\begin{aligned} \hat{x}_{k+1}^- &= \phi_k \hat{x}_k, \\ P_{k+1}^- &= \phi_k P_k \phi_k^T + \bar{G}_k Q_k \bar{G}_k^T, \\ K_k &= P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1}, \\ \hat{x}_k &= \hat{x}_k^- + K_k (z_k - H_k \hat{x}_k^-), \\ P_k &= (I - K_k H_k) P_k^-, \end{aligned} \tag{7}$$

where ϕ_k is state transition matrix of (4), P_k is state covariance matrix, $\bar{G}_k$ is discretized input matrix, Q_k is covariance matrix of $w(t)$, H_k is observation matrix, and R_k is covariance matrix of v_{GPS} or v_{odo}. With (1)-(7), the error can be estimated and the GPS/INS/odometer integrated system can be constructed. In the next section, the lever arm compensation will be described.

3. LEVER ARM EFFECT & COMPENSATION

The lever arm effect can be compensated by considering it in the derivation of measurement models. The derivations that take the effect into account are considered in four cases.

3.1. GPS case

The position measurement of GPS can be expressed as

$$z_{GPS} = P_{IMU} + \Xi C_b^n r_b + v_{GPS},$$
$$\Xi = \begin{bmatrix} 1/(R_m + h) & 0 & 0 \\ 0 & 1/((R_t + h)\cos L) & 0 \\ 0 & 0 & -1 \end{bmatrix}. \tag{8}$$

The latitude, longitude, and height are abbreviated with P. The subscript IMU indicates that it is the position of the center of the IMU, as does GPS. r_b is the offset from the IMU to the GPS antenna resolved in the body frame. The calculated position of the same point will be given as

$$\hat{z}_{GPS} = \hat{P}_{IMU} + \Xi \hat{C}_b^n r_b. \tag{9}$$

The residual for the extended Kalman filter is

$$\begin{aligned} &\hat{z}_{GPS} - z_{GPS} \\ &= \hat{P}_{IMU} + \Xi \hat{C}_b^n r_b - P_{IMU} - \Xi C_b^n r_b - v_{GPS} \\ &= \delta P_{IMU} + \Xi (I - \phi\times) C_b^n r_b - \Xi C_b^n r_b - v_{GPS} \\ &= \delta P_{IMU} - \Xi \phi \times C_b^n r_b - v_{GPS} \\ &= \delta P_{IMU} + \Xi C_b^n r_b \times \phi - v_{GPS} \\ &= \begin{bmatrix} I_{3\times3} & 0_{3\times3} & \Xi C_b^n r_b \times & 0_{3\times6} \end{bmatrix} x_I - v_{GPS}, \end{aligned} \tag{10}$$

where $\delta P_{IMU} = [\delta L \quad \delta l \quad \delta h]^T$ and $\phi = [\phi_N \quad \phi_E \quad \phi_D]^T$.

3.2. Odometer case

The velocity measurement of the odometer is

$$z_{odo} = V_{IMU} + \Omega_{nb}^n r_n + v_{odo}. \tag{11}$$

Ω_{nb}^n is angular rate of the body frame, in which the velocity is measured by the odometer, with respect to the navigation frame, expressed in the navigation frame. r_n is the lever arm represented in the navigation frame. The subscript odo indicates that it is the value of the sensing point of the odometer. The calculated velocity is

$$\hat{z}_{odo} = \hat{V}_{IMU} + \hat{\Omega}_{nb}^n r_n. \tag{12}$$

The residual for the measurement is

$$\begin{aligned} &\hat{z}_{odo} - z_{odo} \\ &= \hat{V}_{IMU} + \hat{\Omega}_{nb}^n r_n - V_{IMU} - \Omega_{nb}^n r_n - v_{odo} \\ &= \delta V_{IMU} + \hat{C}_b^n \tilde{\Omega}_{nb}^b r_b - C_b^n \Omega_{nb}^b r_b - v_{odo} \\ &= \delta V_{IMU} + (I - \phi\times) C_b^n (\Omega_{nb}^b + \varepsilon\times) r_b - C_b^n \Omega_{nb}^b r_b - v_{odo} \\ &= \delta V_{IMU} + C_b^n \Omega_{nb}^b r_b \times \phi - C_b^n r_b \times \varepsilon - v_{odo} \\ &= \begin{bmatrix} 0_{3\times3} & I_{3\times3} & C_b^n \Omega_{nb}^b r_b \times & 0_{3\times3} & -C_b^n r_b \times \end{bmatrix} x_I - v_{odo}, \end{aligned} \tag{13}$$

where $\varepsilon = \begin{bmatrix} \varepsilon_x & \varepsilon_y & \varepsilon_z \end{bmatrix}^T$.

3.3. Odometer case considering coordinate transformation errors

In most cases, the odometer measures the velocity in the body frame because it is mounted on the vehicle directly. Therefore, it must be transformed into the navigation frame for the estimation process. In the previous subsection, the errors in coordinate transformation operation, which are corresponding to the attitude errors, were not considered. In this subsection, that effect is incorporated into the derivation of the residual. The velocity measurement from the body frame can be expressed as,

$$z_{odo} = \hat{C}_b^n V_{odo}^b + v_{odo}. \quad (14)$$

V_{odo}^b is the velocity of the odometer in the body frame. The calculated velocity of the odometer sensing point is the same as (12).

The residual can be derived:

$$\begin{aligned}
&\hat{z}_{odo} - z_{odo} \\
&= \hat{V}_{IMU} + \hat{\Omega}_{nb}^n r_n - \hat{C}_b^n V_{odo}^b - v_{odo} \quad (15) \\
&= \hat{V}_{IMU} + \hat{C}_b^n \tilde{\Omega}_{nb}^b r_b - \hat{C}_b^n V_{odo}^b - v_{odo} \\
&= V_{IMU} + \delta V_{IMU} + (I - \phi\times) C_b^n (\Omega_{nb}^b + \varepsilon\times) r_b \\
&\quad - (I - \phi\times) C_b^n V_{odo}^b - v_{odo} \\
&= \delta V_{IMU} - \phi \times C_b^n \Omega_{nb}^b r_b + C_b^n \varepsilon \times r_b + \phi \times C_b^n V_{odo}^b - v_{odo} \\
&= \delta V_{IMU} + C_b^n \Omega_{nb}^b r_b \times \phi - C_b^n r_b \times \varepsilon - C_b^n V_{odo}^b \times \phi - v_{odo} \\
&= \delta V_{IMU} - V_{IMU} \times \phi - C_b^n r_b \times \varepsilon - v_{odo} \\
&= \left[0_{3\times3} \mid I_{3\times3} \mid -V_{IMU}\times \mid 0_{3\times3} \mid -C_b^n r_b \times \right] x_I - v_{odo}.
\end{aligned}$$

3.4. Odometer case considering coordinate transformation errors and a scale factor error

The odometer produces only pulses that correspond to moving distance. The velocity can be calculated by multiplying these pulses by a scale factor. A scale factor converts the number of pulses to velocity, which is moving distance per time of a unit. In doing so, the scale factor error makes the measurement more inaccurate, and some sophisticated navigation systems must employ the efficient reduction method of the effect of the scale factor error. In that system, the error model (4), (5), and (6) becomes slightly different. (16) and (17) include the scale factor error in the state vector and the corresponding Kalman filter estimates that value. k_{xo} is a scale factor error of the odometer.

$$\begin{aligned}
\dot{x}_I(t) &= F_I(t) x_I(t) + G(t) w(t) \\
&= \begin{bmatrix} F_{11} & F_{12} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times1} \\ F_{21} & F_{22} & F_{23} & F_{24} & 0_{3\times3} & 0_{3\times1} \\ F_{31} & F_{32} & F_{33} & 0_{3\times3} & F_{35} & 0_{3\times1} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times1} \\ 0_{4\times3} & 0_{4\times3} & 0_{4\times3} & 0_{4\times3} & 0_{4\times3} & 0_{4\times1} \end{bmatrix} x_I(t) \quad (16) \\
&\quad + \begin{bmatrix} 0_{3\times3} & 0_{3\times3} \\ C_b^n & 0_{3\times3} \\ 0_{3\times3} & C_b^n \\ 0_{3\times3} & 0_{3\times3} \\ 0_{4\times3} & 0_{4\times3} \end{bmatrix} w(t),
\end{aligned}$$

$$\begin{aligned}
x_I(t) &= \left[x_f \quad x_a \right]^T, \\
x_f &= [\delta L \quad \delta l \quad \delta h \quad \delta V_N \quad \delta V_E \quad \delta V_D \\
&\qquad \phi_N \quad \phi_E \quad \phi_D], \quad (17) \\
x_a &= \left[\nabla_x \quad \nabla_y \quad \nabla_z \quad \varepsilon_x \quad \varepsilon_y \quad \varepsilon_z \quad k_{xo} \right].
\end{aligned}$$

The block matrices in (16) are the same as in (4). The scale factor error is assumed to be random constant bias in the above system. If the error is considered in the navigation system, the lever arm compensation method becomes different, too. The velocity measurement is given as

$$z_{odo} = \hat{C}_b^n (1 + k_{xo}) V_{odo}^b + v_{odo}. \quad (18)$$

The calculated velocity of the sensing point is the same as (12). The residual is derived as follows.

$$\begin{aligned}
&\hat{z}_{odo} - z_{odo} \\
&= \hat{V}_{IMU} + \hat{\Omega}_{nb}^n r_n - \hat{C}_b^n (1 + k_{xo}) V_{odo}^b - v_{odo} \\
&= \hat{V}_{IMU} + \hat{C}_b^n \tilde{\Omega}_{nb}^b r_b - \hat{C}_b^n (1 + k_{xo}) V_{odo}^b - v_{odo} \\
&= V_{IMU} + \delta V_{IMU} + (I - \phi\times) C_b^n (\Omega_{nb}^b + \varepsilon\times) r_b \\
&\quad - (I - \phi\times) C_b^n (1 + k_{xo}) V_{odo}^b - v_{odo} \\
&= \delta V_{IMU} + C_b^n \Omega_{nb}^b r_b \times \phi - C_b^n r_b \times \varepsilon - C_b^n V_{odo}^b \times \phi \\
&\quad - (V_{IMU} + \Omega_{nb}^n r_n) k_{xo} - v_{odo} \\
&= \delta V_{IMU} - V_{IMU} \times \phi - C_b^n r_b \times \varepsilon \\
&\quad - (V_{IMU} + \Omega_{nb}^n r_n) k_{xo} - v_{odo} \\
&= \left[0_{3\times3} \mid I_{3\times3} \mid -V_{IMU}\times \mid 0_{3\times3} \mid -C_b^n r_b \times \mid \right. \\
&\quad \left. -(V_{IMU} + \Omega_{nb}^n r_n) \right] x_I - v_{odo} \quad (19)
\end{aligned}$$

All the above cases made the linear relationships between the error states and the residuals. Therefore, they can be easily applied to the Kalman filter structure. When the lever arm vector is zero, all the models become well-known position or velocity measurement models. These are developed without considering the effect of the lever arm. Therefore, the proposed models are a more generalized version of the former.

4. SIMULATION & EXPERIMENT

To verify the improved performance of the proposed algorithm, a computer simulation was used. A trajectory of a vehicle is given in Fig. 1. The scenario for the vehicle movement is in Table 1. The Monte Carlo simulation iterates 100 times and the simulation parameters accord with the specification of the sensors to be used in the experiments. The lever arm vectors for the GPS receiver and an odometer are

Table 1. Scenario for the vehicle movement.

Time (second)	Movement	Velocity	Angular rate	Acceleration
0 ~ 100	Stop	0 m/s		
100 ~ 105	Acceleration	Increase		2 m/s^2
105 ~ 200	Uniform velocity for the east	10 m/s		
200 ~ 209	Left turn		10 deg/s	
209 ~ 300	Uniform velocity for the north	10 m/s		
300 ~ 309	Left turn		10 deg/s	
309 ~ 400	Uniform velocity for the west	10 m/s		
400 ~ 405	Deceleration	Decrease		-2 m/s^2
405 ~ 500	Stop	0 m/s		

Table 2. The lever arm vectors in the body frame.

Sensor	X	Y	Z
GPS antenna	0 m	-15 m	0 m
Odometer	-0.5 m	-20 m	20 m

Table 3. The time average of the RMS.

	Position error		
	N	E	height
No compensation	4.07 m	1.10 m	0.32 m
With compensation	0.88 m	0.44 m	0.26 m
	Velocity error		
	N	E	D
No compensation	0.19 m/s	0.058 m/s	0.011 m/s
With compensation	0.037 m/s	0.020 m/s	0.0054m/s
	Attitude error		
	Roll	Pitch	Heading
No compensation	0.042 deg	0.059 deg	1.58 deg
With compensation	0.025 deg	0.027 deg	0.54 deg

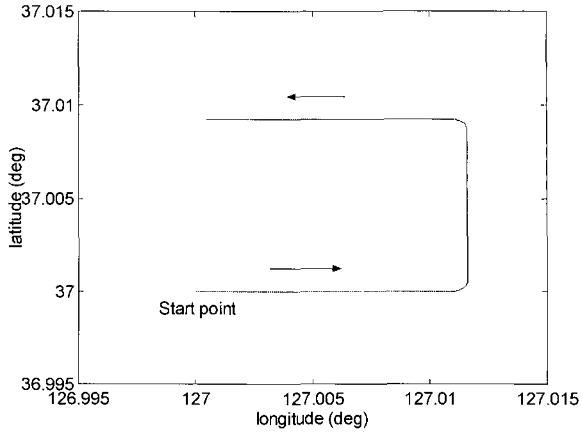

Fig. 1. Trajectory for simulation.

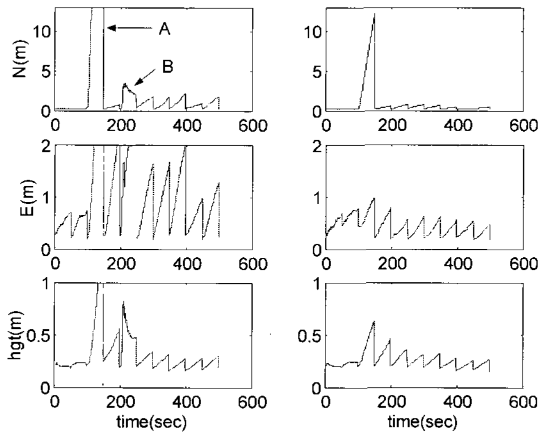

Fig. 2. RMS of the position errors.

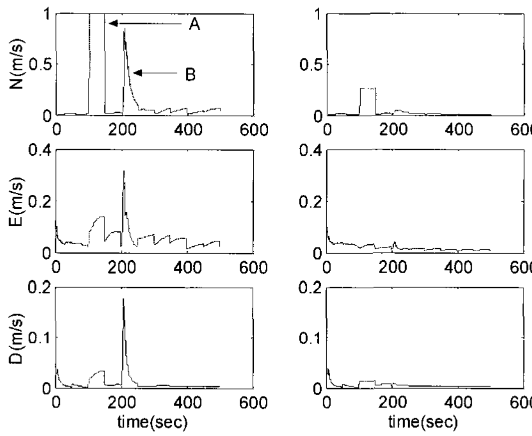

Fig. 3. RMS of the velocity errors.

indicated in Table 2. They are magnified intentionally to display the improvement clearly. The results are presented in Figs. 2-4. They denote the RMS of the errors. The left columns in the figures are the RMS of the errors without any compensation method. The other columns show the results with the lever arm compensation methods. It is evident that the proposed methods improve the performance. The 'A' and 'B' in the figures indicate the lever arm effects that correspond to the wrong measurements. The acceleration under the lever arm effect causes the 'A' and the turning under the effect causes the 'B'. With the compensation methods, they disappear. In Table 3, the time averages of the error RMS are given for the comparison.

The methods have been tested on a land vehicle. It

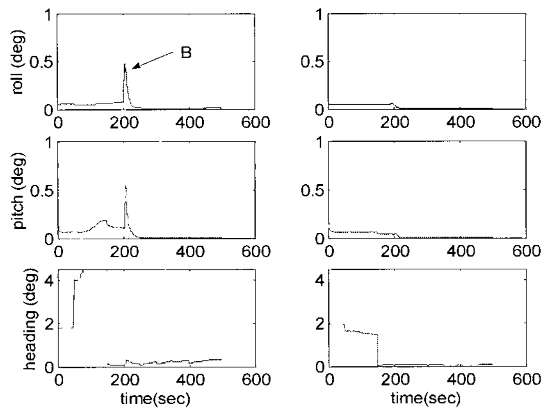

Fig. 4. RMS of the attitude errors.

is a van equipped with an IMU, a GPS antenna, and an odometer. The IMU is the LN-200 produced by Northrop Grumman Corporation. It uses fiber optic gyros (FOGs) and silicon accelerometers (SiAc's) to measure the vehicle angular rates and the linear accelerations. The detailed characteristics of the LN-200 are presented in [8]. It is installed at the center of the van's roof, which is not exactly the center of the gravity of the van. This can cause inaccurate navigation results, but, because the van does not suffer high dynamics, the effect can be small enough to disregard. The GPS antenna is also mounted on the roof, but is located at the left side of the LN-200. The odometer is installed at the left rear wheel, as can be seen in Fig. 5. The lever arm vectors from the origin of the body frame, which is the center of IMU, were measured and presented in Table 4.

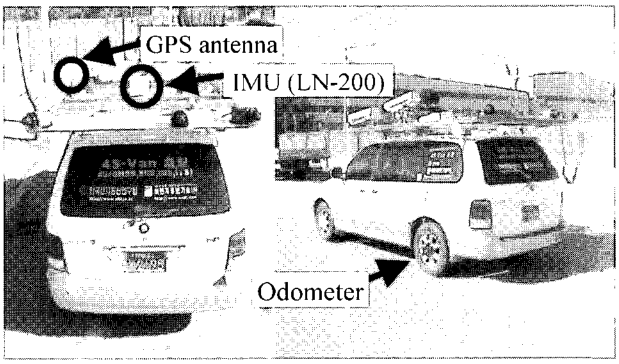

Fig. 5. The land vehicle for test.

Table 4. The lever arm vectors in the body frame.

Sensor	X	Y	Z
GPS antenna	-5 cm	-69 cm	-11 cm
Odometer	-48 cm	-71 cm	173 cm

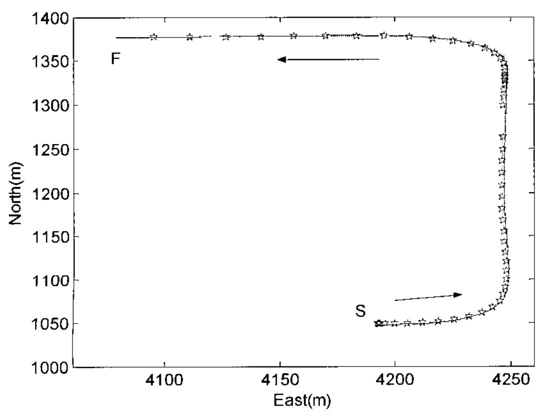

Fig. 6. The trajectory for test.

The trajectory is illustrated in Fig. 6. The van

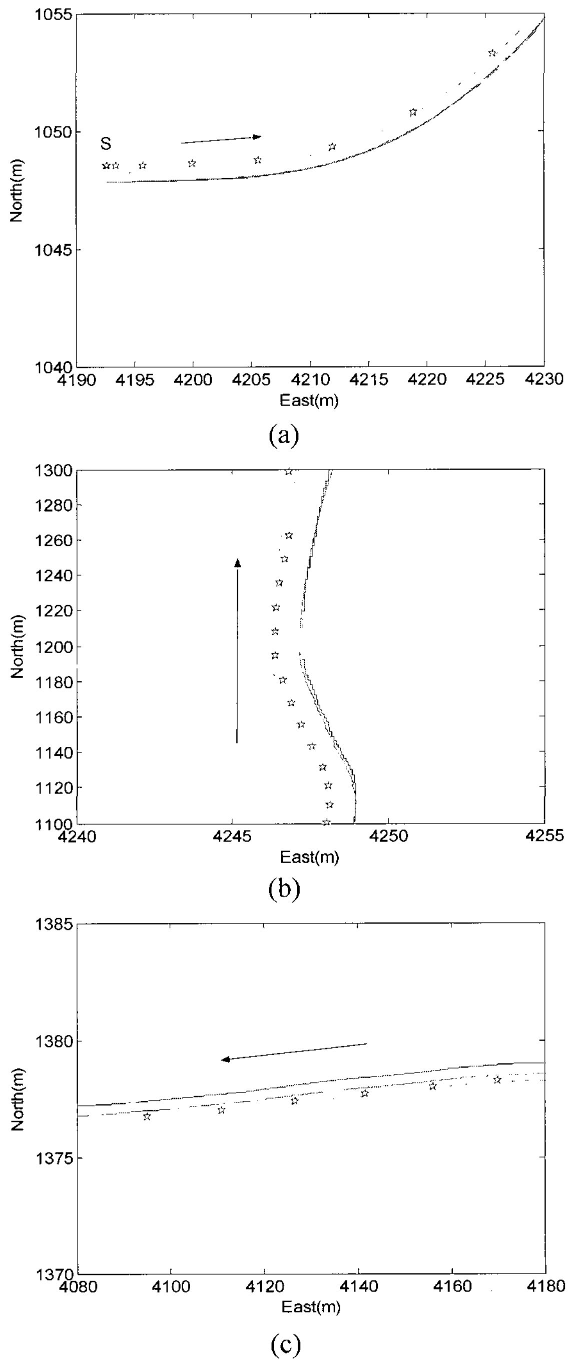

Fig. 7. The improved navigation result of the lever arm compensation.

started at 'S' and stopped at 'F'. The star markings indicate the position measurements of the GPS, which were used for the Kalman filter measurement. In Fig. 7(a), (b), and (c), the effect of the lever arm compensation is illustrated. The dotted line is the navigation result produced without any compensation method. The dashed line is the result of the lever arm compensation for the GPS and the odometer. The solid line is true trajectory of the center of the IMU. It can be obtained with the help of the additional dual-frequency GPS receiver and the CDGPS method. However, in the paper, it is assumed that only one receiver can be available to address the lever arm problem. The progress direction is described in the figures.

Since the antenna is mounted at the left side of the IMU, the measurement of the GPS is biased to the negative direction of the Y axis in the body frame with respect to the IMU. Compared to the result without the method, the navigation result with the lever arm compensation is shifted to the Y axis direction in the body frame, making good sense physically.

5. CONCLUSIONS

Lever arm compensation is addressed, and new compensation methods are proposed, one of which considers the effect of the coordinate transformation errors and the scale factor error of an odometer. They are applied to the GPS/INS/odometer integrated system and provide improved result.

The compensation method is simple, easy to apply, and useful for accurate navigation. It is not only for use in land vehicles, but also for any other transportation system, such as airplanes, ships if they are equipped with some position and velocity sensors as well as the GPS and an odometer.

In this paper, the measurement noises of the odometer are assumed to be white Gaussian, but they can be modeled with other expressions to consider the coordinate transformation errors and the scale factor error.

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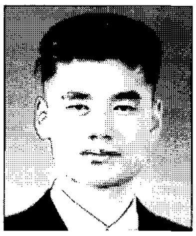

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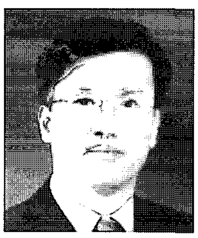

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