Robust Stability Analysis of an Uncertain Nonlinear Networked Control System Category

Minrui Fei, Jun Yi, and Huosheng Hu

Abstract: In the networked control system (NCS), the uncertain network-induced delay and nonlinear controlled object are the main problems, because they can degrade the performance of the control system and even destabilize it. In this paper, a class of uncertain and nonlinear networked control systems is discussed and its sufficient condition for the robust asymptotic stability is presented. Further, the maximum network-induced delay that insures the system stability is obtained. The Lyapunov and LMI theorems are employed to investigate the problem. The result of an illustrative example shows that the robust stability analysis is sufficient.

Keywords: Networked control system, nonlinear controlled object, robust asymptotic stability, uncertain network-induced delay.

1. INTRODUCTION

Walsh [1], Zhang [2,3], Branicky [4,5], etc. contribute to the research of networked control system stability. Zhang[2] analyzed the networked control system stability of network-induced delay in a discontinuous system. In his paper, delay is constant, sensor is time-driven, of which the sample cycle is h, and controller and actuator are event-driven. The stability of delay $\tau<h$ and $\tau>=h$ is analyzed and a stable area graph for each special object is drawn. Another aspect of stability research is that search maximum network-induced delay for guarantee networked control system stability, based on Riccati equation and Lyapunov stability theorem. Walsh [1] defined the maximum allowable time interval to guarantee networked control system stability in the meaning of Lyapunov stability theorem. Separately, Kim [6,7] and Park [8] classified maximum allowable delay bound to guarantee networked control system stability for continuous and discontinuous time models. However, all of the above researches have a problem in that their objects are linear, non time-varying and accurate. Lin [9] analyzed networked control system robust stability in the condition of uncertain delay, losing package and disturbance, but the object is always linear and accurate, which is quite different in the case of the actual controlled object.

In real projects, many uncertain, nonlinear objects come into existence. A certain linear networked control system model provides just an approximate description of the real facts. When requesting precise control, the control effect is greatly weakened by use of a certain linear networked control system model. Network-induced delay falls in the category of uncertain delay. It is very difficult for a control system to obtain precise design when uncertain predicted errors exist, despite the fact that the excellent compensation algorithm is applied. Therefore, the robust analysis of the uncertain nonlinear networked control system seems to be emphasized. Up to now, almost no research concerning this aspect has been reported. As such, the work regarding the uncertain nonlinear networked control system is a more open and challenging research. Obviously, the first academic problem that needs to be solved is the robust stability analysis of the uncertain nonlinear networked control system.

2. NCS MODEL FOR UNCERTAIN NONLINEAR CONTROLLED OBJECT

The typical networked control system model structure for an uncertain controlled object is shown in Fig. 1.

Manuscript received October 1, 2005; accepted January 12, 2006. Recommended by Editor Zengqi Sun. This work was supported by the National Natural Science Foundation of China under grant 60274031, Program for New Century Excellent Talents in University under grant NCET-04-0433, Research Project for Doctoral Disciplines in University under grant 20040280017. This work was jointly supported by the Keystone Foundation Project of the Shanghai Municipal Commission of Science and Technology under grant 04JC14038, Shanghai Leading Academic Disciplines under grant T0103, and the Shanghai Key Laboratory of Power Station Automation Technology.

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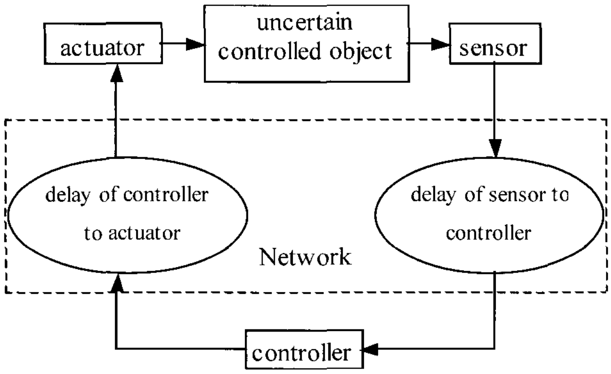

Fig. 1. Structure graph of the uncertain nonlinear networked control system.

Suppose τ_{sc} is delay of controller to actuator and τ_{ca} is delay of sensor to controller. As reference [10] is concerned, control results reflect samples before network delay $\tau=\tau_{sc}+\tau_{ca}$ by event-driven computing strategy.

By analysis of the network-induced delay mechanism in reference [11], network-induced delay is either unbounded or bounded. In this paper, network-induced delay is a random delay in which maximum delay is bounded, namely

$$0 \le \tau \le \tau_{\max} = \max(\tau_{sc} + \tau_{ca}).$$

A class of networked control system model for an uncertain nonlinear controlled object is given:

$$\begin{cases} \dot{x}(t) = f(x(t)) + \Delta f(x(t)) + g(x(t))u(t-\tau) \\ y = h(x), \end{cases} \tag{1}$$

where state vector $x \in R^n$, control input $u \in R$, control output $y \in R$, $f(\cdot)$, $g(\cdot)$ and $h(\cdot)$ are smooth vector field, $f(\cdot) = 0$, $\Delta f(x)$ is system uncertainty, τ is network delay. $x(t)=\varphi(t)$, $\forall t \in [-\tau, 0]$ is initial condition. Four Hypotheses are supposed as follows:

Hypothesis 1: Span

$$\left\{ g(x), ad_f g(x), \cdots, ad_f^{n-2} g(x) \right\}$$

is nonsingular involuntary distribution.

Hypothesis 2: Rank meets

$$r\left[g(x), ad_f g(x), \cdots, ad_f^{n-1} g(x) \right] = n.$$

Hypothesis 3:

$$\left\| \begin{bmatrix} L_{\Delta f} h(x) & L_{\Delta f} L_f h(x) & \cdots & L_{\Delta f} L_f^{n-1} h(x) \end{bmatrix}^T \right\|$$
$$\le \eta(x) \left\| \begin{bmatrix} h(x) & L_f h(x) & \cdots & L_f^{n-1} h(x) \end{bmatrix} \right\|$$

and $\lim_{x \to 0} \eta(x) = 0$, $\eta(x)$ is positive continuous function.

Hypothesis 4: System (1) is zero state detectable.

3. MAIN PRINCIPLE FOR THE ROBUST ASYMPTOTIC STABILITY

Lemma 1: For arbitrary real $\beta > 0$ and arbitrary positive definite symmetric matrix S, the following inequation comes into existence [12]:

$$-2u^T v \le \beta u^T S^{-1} u + \beta^{-1} v^T S v.$$

Specially, when $\beta=1$, that

$$-2u^T v \le u^T S^{-1} u + v^T S v.$$

Theorem 1: For system (1), if there exists positive definite symmetric matrixes P, P_1, P_2, and matrix $K_{1\times n}$, positive real ε, $q > 1$, the following matrix inequation comes into existence:

$$\left[-Q + \varepsilon I \right] + \tau_{\max} (M_1 + M_2 + \varepsilon q \delta \| A_\tau \| I) < 0,$$

where

$$M_1 = \inf(P A_\tau P_1 A_\tau^T P + A^T P_1^{-1} A),$$
$$M_2 = \inf(P A_\tau P_2 A_\tau^T P + A_\tau^T P_2^{-1} A_\tau),$$
$$A_\tau = BK, \quad \delta = \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(Q)}}.$$

Maximum network delay meets

$$\tau_{\max} < \inf(\lambda(-(-Q + \varepsilon I)(M_1 + M_2 + \varepsilon q \delta \| A_\tau \| I)^{-1}))$$

where

$$-Q = (A + A_\tau)^T P + P(A + A_\tau),$$

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix},$$

$$\phi: \begin{array}{l} z_1 = h(x), \\ z_2 = L_f h(x), \\ \quad \vdots \\ z_n = L_f^{n-1} h(x). \end{array}$$

The system shows robust asymptotic stability for arbitrariness uncertain network delay $0 \le \tau \le \tau_{\max}$ and the control law is

$$u(t-\tau) = \left[L_g L_f^{n-1} h(x(t)) \right]^{-1} \left[K\phi(x(t-\tau)) - L_f^n h(x) \right].$$

Proof: For state variable x of system (1), consider new coordinate

$$z=\phi(x)=\begin{bmatrix}\phi_1(x)\\ \phi_2(x)\\ \vdots\\ \phi_n(x)\end{bmatrix}=\begin{bmatrix}\phi_1(x_1,x_2,\cdots,x_n)\\ \phi_2(x_1,x_2,\cdots,x_n)\\ \vdots\\ \phi_n(x_1,x_2,\cdots,x_n)\end{bmatrix}, \quad (2)$$

where $\phi(x)$ is a smooth differentiable function in which the contradictory map comes into existence, that is to say, coordinate transform (2) is a diffeomorphism. As reference [12] is concerned, two systems in which diffeomorphism coordinate transforms come into existence are feedback equivalence. If one is unflappability, the other is also unflappability.

By equations (1) and (2), we get:

$$\begin{cases}\dot{z}=\dfrac{\partial\phi}{\partial x}\dot{x}=\dfrac{\partial\phi}{\partial x}f(x)+\dfrac{\partial\phi}{\partial x}\Delta f(x)+\dfrac{\partial\phi}{\partial x}g(x)u(t-\tau)\\ \dot{y}=\dfrac{\partial h}{\partial x}\dot{x}=\dfrac{\partial h}{\partial x}f(x)+\dfrac{\partial h}{\partial x}\Delta f(x)+\dfrac{\partial h}{\partial x}g(x)u(t-\tau).\end{cases} \quad (3)$$

According to Hypotheses 1 and 2, the comparative rank of system (1) is n. So Li differential satisfies:

$$\begin{aligned}&L_g h(x)=L_g L_f h(x)=L_g L_f^2 h(x)=\\ &\cdots=L_g L_f^{n-2}h(x)=0\ \ L_g L_f^{n-1}h(x)\neq 0.\end{aligned} \quad (4)$$

In the condition, choose the following diffeomorphism transform:

$$\phi:\ \begin{aligned}&z_1=h(x),\\ &z_2=L_f h(x),\\ &\ \vdots\\ &z_n=L_f^{n-1}h(x).\end{aligned} \quad (5)$$

So system (1) can transform the following standard model:

$$\dot{z}(t)=Az(t)+Bv(t-\tau)+\Delta H(z(t)). \quad (6)$$

The control law is

$$u(t-\tau)=\left[L_g L_f^{n-1}h(x(t))\right]^{-1}\left[v(t-\tau)-L_f^n h(x)\right].$$

(A,B) is Brounovsky standard model, in which

$$A=\begin{bmatrix}0&1&0&\cdots&0\\ 0&0&1&\cdots&0\\ \vdots&\vdots&\vdots&\vdots&\vdots\\ 0&0&0&\cdots&1\\ 0&0&0&\cdots&0\end{bmatrix},\qquad B=\begin{bmatrix}0\\0\\ \vdots\\0\\1\end{bmatrix}.$$

The uncertain item is

$$\Delta H(z)=\left[L_{\Delta f}h(\phi^{-1}(z))\ \ L_{\Delta f}L_f h(\phi^{-1}(z))\ \cdots\ L_{\Delta f}L_f^{n-1}h(\phi^{-1}(z))\right]^T. \quad (7)$$

According to Hypothesis 3,

$$\lim_{z\to 0}\frac{\|\Delta H(z)\|}{\|z\|}=0. \quad (8)$$

The initial condition is:

$$z(t)=\phi(\varphi(t))=\Psi(t),\quad \forall t\in[-2\tau,0].$$

Suppose the control law of (6)

$$v(t)=Kz(t). \quad (9)$$

So (1) can transform:

$$\dot{z}(t)=Az(t)+BKz(t-\tau)+\Delta H(z(t)).$$

Suppose $A_\tau=BK$, then

$$\dot{z}(t)=Az(t)+A_\tau z(t-\tau)+\Delta H(z(t)) \quad (10)$$

$$\begin{aligned}z(t-\tau)&=z(t)-\int_{-\tau}^{0}\dot{z}(t+\theta)d\theta\\ &=z(t)-\int_{-\tau}^{0}\begin{bmatrix}Az(t+\theta)+A_\tau z(t-\tau+\theta)\\ +\Delta H(z(t+\theta))\end{bmatrix}d\theta.\end{aligned}$$

Then

$$\begin{aligned}\dot{z}(t)=&(A+A_\tau)z(t)+\Delta H(z(t))\\ &-A_\tau\int_{-\tau}^{0}\left[Az(t+\theta)+A_\tau z(t-\tau+\theta)+\Delta H(z(t+\theta))\right]d\theta.\end{aligned} \quad (11)$$

Define the following Lyapunov function

$$V(z,t)=z(t)^T Pz(t)+W(z,t), \quad (12)$$

where P is positive symmetric matrix, and

$$\begin{aligned}W(z,t)=&\int_{-\tau}^{0}\int_{t+\theta}^{t}z(s)^T A^T P_1^{-1}Az(s)dsd\theta\\ &+\int_{-\tau}^{0}\int_{t+\theta-\tau}^{t}z(s)^T A_\tau^T P_2^{-1}A_\tau z(s)dsd\theta,\end{aligned} \quad (13)$$

where P_1, P_2 is positive symmetric matrix. Obviously, real $q>1$ exists when $t-\tau<\gamma<t$, then

$$\lambda_{\min}(Q)\|z(\gamma)\|^2\le V(z,t)\le q\lambda_{\max}(P)\|z(t)\|^2. \quad (14)$$

The above inequation can become:

$$\|z(\gamma)\|\le q\delta\|z(t)\|,\quad \delta=\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(Q)}}.$$

$V(z,t)$ derivation along system equation (6) is:

$$\begin{aligned}\dot{V}(z,t)&=\dot{z}(t)^T Pz(t)+z(t)^T P\dot{z}(t)+\dot{W}(z,t)\\ &=\dot{W}(z,t)-z(t)^T Qz(t)-\{[A_\tau\int_{-\tau}^{0}Az(t+\theta)d\theta]^T Pz(t)\end{aligned}$$

$$+z(t)^T P[A_\tau \int_{-\tau}^{0} Az(t+\theta)d\theta]\}$$
$$-\{[A_\tau \int_{-\tau}^{0} A_\tau z(t-\tau+\theta)d\theta]^T Pz(t)$$
$$+z(t)^T P[A_\tau \int_{-\tau}^{0} A_\tau z(t-\tau+\theta)d\theta]\}$$
$$-\{[A_\tau \int_{-\tau}^{0} \Delta H(z(t+\theta))d\theta]^T Pz(t) \qquad (15)$$
$$+z(t)^T P[A_\tau \int_{-\tau}^{0} \Delta H(z(t+\theta))d\theta]\}$$
$$+\{z(t)^T P\Delta H(z(t))+\Delta H(z(t))^T Pz(t)\}$$
$$=\dot{W}(z,t)-z(t)^T Qz(t)$$
$$+\eta_1(z,t)+\eta_2(z,t)+\eta_3(z,t)+\eta_4(z,t),$$

where

$$\eta_1(z,t)=-\{[A_\tau \int_{-\tau}^{0} Az(t+\theta)d\theta]^T Pz(t) + z(t)^T P[A_\tau \int_{-\tau}^{0} Az(t+\theta)d\theta]\},$$
$$\eta_2(z,t)=-\{[A_\tau \int_{-\tau}^{0} A_\tau z(t-\tau+\theta)d\theta]^T Pz(t) + z(t)^T P[A_\tau \int_{-\tau}^{0} A_\tau z(t-\tau+\theta)d\theta]\},$$
$$\eta_3(z,t)=-\{[A_\tau \int_{-\tau}^{0} \Delta H(z(t+\theta))d\theta]^T Pz(t) + z(t)^T P[A_\tau \int_{-\tau}^{0} \Delta H(z(t+\theta))d\theta]\},$$
$$\eta_4(z,t)=z(t)^T P\Delta H(z(t))+\Delta H(z(t))^T Pz(t).$$

As Lemma 1 is concerned, for arbitrariness positive symmetric matrixes P_1 and P_2 we have

$$\eta_1(z,t)\le \tau z(t)^T PA_\tau P_1 A_\tau^T Pz(t) + \int_{-\tau}^{0} z(t+\theta)^T A^T P_1^{-1} Az(t+\theta)d\theta,$$
$$\eta_2(z,t)\le \tau z(t)^T PA_\tau P_2 A_\tau^T Pz(t) + \int_{-\tau}^{0} z(t-\tau+\theta)^T A_\tau^T P_2^{-1} A_\tau z(t-\tau+\theta)d\theta.$$

As formula (8) is concerned, for arbitrariness small positive real $\varepsilon>0$, positive α always exists, allowing the following inequation to come into existence:

$$\frac{\|\Delta H(z(t))\|}{\|z(t)\|}<\frac{\varepsilon}{2\|P\|}, \quad \forall\|z(t)\|<\alpha,$$
$$\frac{\|\Delta H(z(t+\theta))\|}{\|z(t+\theta)\|}<\frac{\varepsilon}{2\|P\|}, \quad \forall\|z(t+\theta)\|<\alpha.$$

So we can get:

$$\eta_3(z,t)\le 2\|z(t)\|\|P\|\|A_\tau\|\left\|\int_{-\tau}^{0} \Delta H(z(t+\theta))d\theta\right\|<d\varepsilon q\delta\|z(t)\|^2,$$
$$\eta_4(z,t)\le 2\|z(t)\|\|P\|\|\Delta H(z(t))\|<\varepsilon\|z(t)\|^2.$$

$W(z,t)$ derivation for t is

$$\dot{W}(z,t)=\int_{-\tau}^{0}\left[z(t)^T A^T P_1^{-1}Az(t)-z(t+\theta)^T A^T P_1^{-1}Az(t+\theta)\right]d\theta$$
$$+\int_{-\tau}^{0}\left[z(t)^T A_\tau^T P_2^{-1}A_\tau z(t)-z(t-\tau+\theta)^T A_\tau^T P_2^{-1}A_\tau z(t-\tau+\theta)\right]d\theta$$
$$=z(t)^T\left(\tau A^T P_1^{-1}A+\tau A_\tau^T P_2^{-1}A_\tau\right)z(t)-\int_{-\tau}^{0} z(t+\theta)^T A^T P_1^{-1}Az(t+\theta)d\theta$$
$$-\int_{-\tau}^{0} z(t-\tau+\theta)^T A_\tau^T P_2^{-1}A_\tau z(t-\tau+\theta)d\theta. \qquad (16)$$

Let (16) be put into (15):

$$\dot{V}(z,t)<-z(t)^T Qz(t)+\varepsilon\|z(t)\|^2+\tau\varepsilon q\delta\|A_\tau\|\|z(t)\|^2$$
$$\tau z(t)^T PA_\tau P_1 A_\tau^T Pz(t)+\tau z(t)^T PA_\tau P_2 A_\tau^T Pz(t)+$$
$$z(t)^T\left(\tau A^T P_1^{-1}A+\tau A_\tau^T P_2^{-1}A_\tau\right)z(t)$$
$$=z(t)^T[(-Q+\varepsilon I)+\tau\varepsilon q\delta\|A_\tau\|I+\tau(PA_\tau P_1 A_\tau^T P+A^T P_1^{-1}A)$$
$$+\tau(PA_\tau P_2 A_\tau^T P+A_\tau^T P_2^{-1}A_\tau)]z(t). \qquad (17)$$

By choosing the appropriate positive symmetric matrixes P_1 and P_2 satisfy:

$$PA_\tau P_1 A_\tau^T P+A^T P_1^{-1}A=\inf(PA_\tau P_1 A_\tau^T P+A^T P_1^{-1}A)=M_1$$
$$PA_\tau P_2 A_\tau^T P+A_\tau^T P_2^{-1}A_\tau=\inf(PA_\tau P_2 A_\tau^T P+A_\tau^T P_2^{-1}A_\tau)=M_2$$

So inequation (17) can become:

$$\dot{V}(z,t)<z(t)^T\{[-Q+\varepsilon I]+\tau(M_1+M_2+\varepsilon q\delta\|A_\tau\|I)\}z(t)$$
$$\le z(t)^T\{[-Q+\varepsilon I]+\tau_{\max}(M_1+M_2+\varepsilon q\delta\|A_\tau\|I)\}z(t).$$

Suppose

$$Q_1=-\{[-Q+\varepsilon I]+\tau_{\max}(M_1+M_2+\varepsilon q\delta\|A_\tau\|I)\}>0. \qquad (18)$$

The corresponding maximum network delay is:

$$\tau_{\max}<\inf(\lambda(-(-Q+\varepsilon I)(M_1+M_2+\varepsilon q\delta\|A_\tau\|I)^{-1})).$$

The feedback control law is:

$$u(t-\tau)=\left[L_g L_f^{n-1}h(x(t))\right]^{-1}\left[K\phi(x(t-\tau))-L_f^n h(x)\right].$$

So $\dot{V}(x,t)<-Q_1<0$. Then as Hypothesis 4 is concerned, system (1) is robust asymptotic stability. This concludes the Proof. □

4. EXAMPLE SIMULATIONS

Consider the following networked control system for an uncertain nonlinear controlled object:

$$\begin{cases}\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}x_1^2+x_2\\ x_1^2-x_2\end{bmatrix}+\begin{bmatrix}0\\ 1+x_2^2\end{bmatrix}u(t-\tau)+\Delta f\\ y=x_1,\end{cases} \qquad (19)$$

where $\Delta f = \begin{bmatrix} x_1^2 \sin\theta & 0 \end{bmatrix}^T$, the uncertain parameter satisfies $-2 \le \theta \le 2$. We can get:

$$L_f h(x) = x_1^2 + x_2, \quad L_g L_f h(x) = 1 + x_2^2,$$
$$L_f^2 h(x) = 2x_1^3 + 2x_1 x_2 + x_1^2 - x_2,$$
$$ad_f g(x) = \begin{bmatrix} -1 - x_2^2 & 1 + 2x_1^2 - x_2^2 \end{bmatrix}^T \ne 0.$$

Therefore, Hypotheses 1 and 2 are true. Then diffeomorphism coordinate transforms correspond to equation (5) such that:

$$\phi: \; z_1 = x_1, \quad z_2 = x_1^2 + x_2.$$

The uncertain item that corresponds to system (6) is:

$$\Delta H = \begin{bmatrix} z_1^2 \sin\theta & 2z_1^3 \sin\theta \end{bmatrix}^T.$$

Obviously, $\lim\limits_{z \to 0} \dfrac{\|\Delta H\|}{\|z\|} = 0$ satisfies Hypothesis 3.

By computation:

$$F = \begin{bmatrix} -1 & -1 \end{bmatrix}, P = \begin{bmatrix} 10 & 4 \\ 4 & 10 \end{bmatrix}, \; P1 = \begin{bmatrix} 0.093 & 0 \\ 0 & 0.001 \end{bmatrix},$$
$$P_2 = \begin{bmatrix} 0.001 & 0 \\ 0 & 0.131 \end{bmatrix} \delta = 1.02, \; q = 2, \; \varepsilon = 0.1,$$
$$\tau_{\max} < \inf(\lambda(-\left[(A + A_\tau)^T P + P(A + A_\tau) + \varepsilon I\right] *$$
$$\left[M_1 + M_2 + \varepsilon q \delta \|A_\tau\| I\right]^{-1})) = 0.2856.$$

So we can choose $\tau_{\max} = 285ms$, here

$$\left[(A + A_\tau)^T P + P(A + A_\tau) + \varepsilon I\right] +$$
$$\tau_{\max}\left[M_1 + M_2 + \varepsilon q \delta \|A_\tau\| I\right] = \begin{bmatrix} -4.6412 & 0.7487 \\ 0.7487 & -0.1576 \end{bmatrix} < 0.$$

The control law of system (1) is:

$$u(t - \tau) = \left[L_g L_f^{n-1} h(x(t))\right]^{-1} \left[v(t - \tau) - L_f^n h(x)\right]$$
$$= -\frac{x_1(t-\tau) + x_1^2(t-\tau) + x_2(t-\tau)}{1 + x_2^2(t)}$$
$$- \frac{2x_1^3(t) + 2x_1(t)x_2(t) + x_1^2(t) - x_2(t)}{1 + x_2^2(t)}.$$

Consider the furthest uncertain case, choosing $\theta = -\pi/2$, the initial value is $x(0) = \begin{bmatrix} -1.5 & 4 \end{bmatrix}$. In these simulations, considering the furthest severity network, network-induced delay is the maximum delay. Of course, when network-induced delay $\tau < \tau_{\max}$, control capability is improved. By simulations, it is found that not only when network maximum delay $\tau_{\max} = 285ms$, the system has stability (see Fig. 2), but also when $\tau_{\max} = 450ms$, the system also has stability, however system state convergent rate becomes slower (see Fig. 3). When network maximum delay $\tau_{\max} = 500ms$, the system begins to become instable (see Fig. 4).

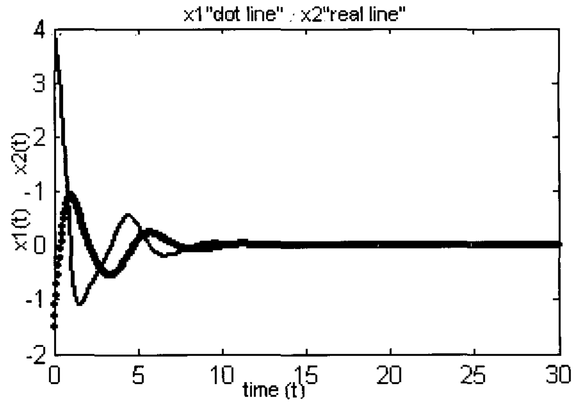

Fig. 2. The state graph of $\tau_{\max} = 285ms$.

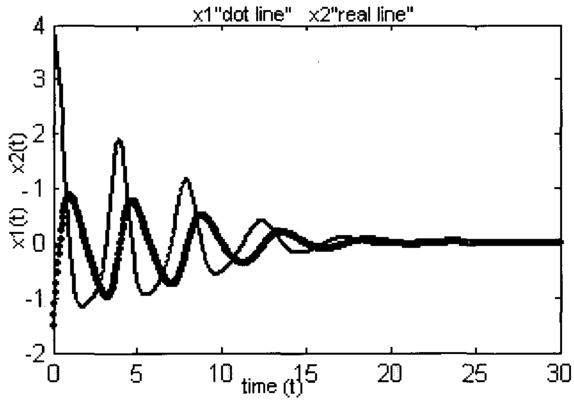

Fig. 3. The state graph of $\tau_{\max} = 450ms$.

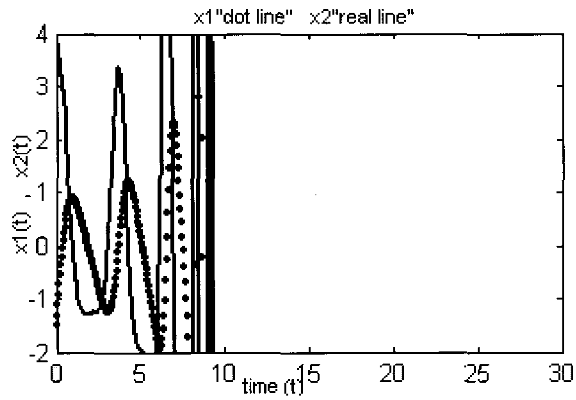

Fig. 4. The state graph of $\tau_{\max} = 500ms$.

5. CONCLUSIONS

In this paper, for an uncertain nonlinear networked control system model, networked control system robust stability is analyzed and the sufficient condition for the robust asymptotic stability is presented. Simulation results indicate that the sufficient condition for the robust asymptotic stability is correct. However, the results are conservative. How to reduce conservation is one of the most important issues needing to be addressed. Furthermore, the sufficient condition is based on neglect of network losing package and bandwidth limitation. If bandwidth limitation is concerned, the problem will be more complicated. That will be studied in the future.

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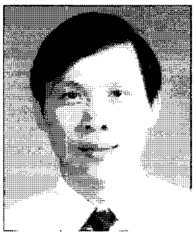

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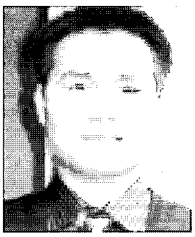

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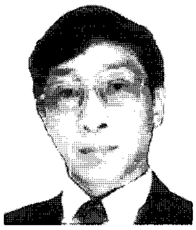

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