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Implicit Time Integration Scheme for Real-Time Hybrid Test System

실시간 하이브리드 실험 시스템을 위한 Implicit 시간적분법

정래영[1)]
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국문 요약 >> 이 논문에서 소개되는 실시간 하이브리드 실험 시스템은 유사동적실험법을 근거로 하고 있으며, 실제 실험과 수학적 모델을 이용한 수치해석을 결합한 실험법이다. 이 시스템은 종래의 유사동적해석법에 비해 지진하중을 받는 구조물의 실시간 반응에 근접하는 현저히 높은 하중재하 속도를 고려할 수 있도록 설계되었다. 또한 다자유도 구조물에 대해 안정적인 해석환경을 제공하기위해 이 시스템은 implicit 시간적분법을 이용하여 수치해석을 수행한다. 이 논문은 연구를 통해 개발된 시스템의 전반적인 개요와 구성요소 그리고 이 시스템에서 사용하는 수치 해석법의 성능을 평가하기 위해 수행된 수치해석을 소개한다. 연구 결과 개발된 시스템에 적용된 수치해석법은 성능이 매우 우수하다는 것이 증명되었다.

주요어 지진공학, 하이브리드 실험, 유사동적실험법, 실시간 실험, implicit 시간적분법

ABSTRACT >> The Real-Time Hybrid Test system presented in this paper is based on the pseudodynamic test method, and it combines physical testing with model-based simulation. The system is designed to achieve a rate of loading that is significantly higher than that of a conventional pseudodynamic test approaching the real-time response of a structure subjected to earthquake loads. To provide robust computation environment for the analysis of many degree-of-freedom structures, the system adopts an implicit time integration scheme in the model-based simulation. This paper presents an overview of the developed system and numerical simulations that were conducted to evaluate the performance of the computation scheme adopted here. Results of these studies have demonstrated the good performance of the computation scheme for real-time multiple-degree-of-freedom tests.

Key words earthquake engineering, hybrid tests, pseudodynamic tests, real-time tests, implicit time integration method

1. Introduction

During the past few decades, considerable number of dynamic analysis and testing technologies have been developed to achieve more improved information on the dynamic behavior of structural systems. Even with vast research, however, the reliable prediction of inelastic structural performance during a severe seismic event is an extremely difficult task, due to the complex nonlinear behavior exhibited by members and connectors. Although nonlinear analytical models exist for such structural elements, the accuracy of the resulting response predictions is limited by assumptions in the mathematical description of the model. For this reason, experimental testing has been considered as the most reliable method to evaluate the inelastic behavior of structural systems and to devise structural details to improve seismic performance. As a series of this effort, substantial attention has been given to the development of the pseudodynamic test method for evaluation of the seismic performance of structure system.[(1-3)] The pseudodynamic test method consists of a numerical integration part to solve the equations of motion with assumed mass and damping and a physical test part to obtain the realistic load resisting characteristics of the structure. An explicit time integration method does not require an

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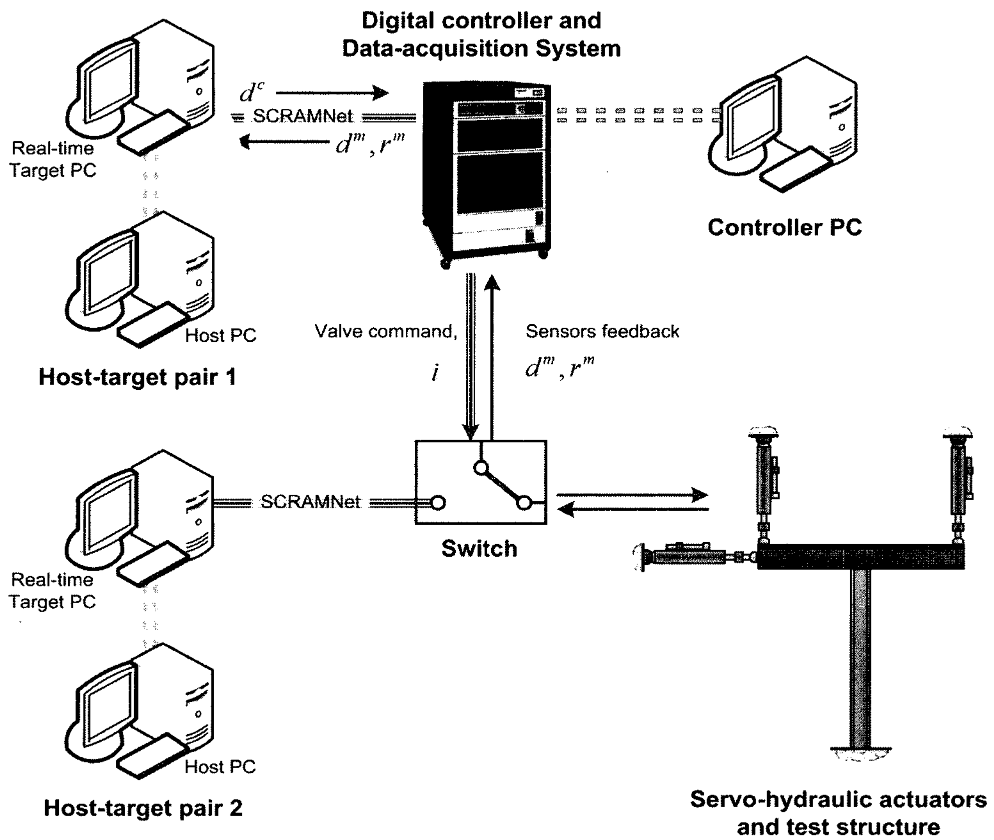
iterative technique to solve the equations of motion for structure. For this reason, in the early development of the pseudodynamic test method, an explicit method was adopted as the numerical analysis scheme.[4,5] However, with an increasing need for using the pseudodynamic test method for structures with many degrees of freedom or with very high frequency modes, especially in the context of substructure tests, implicit schemes have been developed and used. Recently, a Fast Hybrid Test (FHT) system has been developed[6,7] at the University of Colorado as a part of the George E. Brown, Jr. Network for Earthquake Engineering Simulation Program sponsored by the US National Science Foundation.

This paper presents a detailed description for the computation scheme and system architecture of the FHT system, which uses the α-method[8] with a Newton iterative procedure. Through this study, the performance and accuracy of the system are demonstrated. The performance of the system including the dynamics of a servo-hydraulic actuator will be presented in a future publication.

2. System Architecture

Figure 1 shows the basic architecture of the FHT system developed in this project for real-time testing. As shown in the figure, the FHT system consists of three main components; host-target computers, digital controller/data-acquisition system, and high-performance servo-hydraulic actuators.

The target PC is used to solve the equations of motion for the structure in real-time. The host PC is for program development and input preparation. Once the input information is prepared in the host PC, the execution file is downloaded to the target PC via an Ethernet. In this study, the real-time computation in the target PC uses Mathworks' xPC Target[9] which can run Simulink programs. The numerical algorithms for fast hybrid tests are programmed in Simulink for prototyping the testing methodology and for simple tests without substructuring. The communication between the digital controller/data-acquisition system and the target computer is through the Shared Common RAM Network (SCRAMNet), which transmits data among different processors at 150 *Mbits/s* via fiber optic cables.



〈Figure 1〉 Basic architecture of the FHT system.

3. Computation Scheme

3.1 Implicit Time Integration Scheme

In the α-method, the time-discretized equations of motion and the displacement and velocity approximations are expressed as follows.

$$Ma_{i+1}+(1+\alpha)C\nu_{i+1}-\alpha C\nu_i+(1+\alpha)r_{i+1}-\alpha r_i$$
$$=(1+\alpha)f_{i+1}-\alpha f_i \quad (1)$$

$$d_{i+1}=d_i+\Delta t\nu_i+\Delta t^2[(1/2-\beta)a_i+\beta a_{i+1}] \quad (2)$$

$$\nu_{i+1}=\nu_i+\Delta t[(1-\gamma)a_i+\gamma a_{i+1}] \quad (3)$$

in which M and C are the mass and damping matrices of the structural model; d_i, ν_i, a_i, and r_i are nodal displacement, velocity, acceleration, and restoring force vectors at time step i; f_i is the vector of applied forces; Δt is the integration time interval; and α, β, and γ are parameters that govern the numerical properties of the integration scheme. For unconditional stability and second-order convergence, it is required that $-1/3\le\alpha\le 0$, $\gamma=(1-2\alpha)/2$, and $\beta=(1-\alpha)^2/4$.[8]

3.2 Solution Scheme for Pseudodynamic Tests

For a nonlinear structure, the α-method requires a Newton-type iterative procedure to solve the equations of motion. Since the tangent stiffness of a test structure cannot be accurately estimated during a test, a modified Newton approach using the initial stiffness of the structure has to be used. With equations (1) through (3), the modified Newton solution strategy can be formulated as follows.

$$K^*\Delta d_{i+1}^{(k)}=R_{i+1}^{(k)} \quad (4)$$

$$d_{i+1}^{(k+1)}=d_{i+1}^{(k)}+\Delta d_{i+1}^{(k)} \quad (5)$$

where

$$K^*=\frac{\overline{M}}{\Delta t^2\beta}+(1+\alpha)K_{ini} \quad (6)$$

$$R_{i+1}^{(k)}=\frac{\overline{M}}{\Delta t^2\beta}\left[\hat{d}_{i+1}-d_{i+1}^{(k)}\right]-(1+\alpha)r_{i+1}^{(k)} \quad (7)$$

$$\overline{M}=M+(1+\alpha)\gamma\Delta tC \quad (8)$$

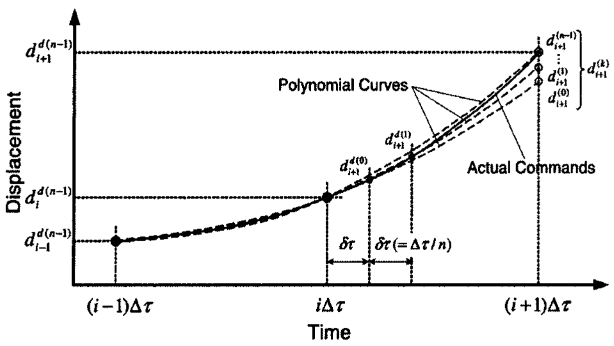
$$\hat{d}_{i+1}=d_i+\Delta t\nu_i+\Delta t^2(1/2-\beta)a_i+\Delta t^2\beta\overline{M}^{-1}[(1+\alpha)f_{i+1}$$
$$-\alpha f_i-C\nu_i-(1+\alpha)(1-\gamma)\Delta tCa_i+\alpha r_i] \quad (9)$$

in which $d_{i+1}^{(k)}$ and $r_{i+1}^{(k)}$ denote the displacements and the corresponding structural restoring forces in iteration k of time step $(i+1)$, and k_{ini} is the initial stiffness of the structure. In a non-real-time pseudodynamic test, the displacements $d_{i+1}^{(k+1)}$ updated in each iteration with equations (4) and (5) are imposed on the structural specimen through servo-hydraulic actuators or other loading apparatus, the resulting structural restoring forces $r_{i+1}^{(k+1)}$ are measured, and a new residual $R_{i+1}^{(k+1)}$ is then computed with equation (7) and used to update the response in the next iteration. This process is repeated until convergence is attained. After convergence, the acceleration and velocity response is updated with equations (2) and (3), and the solution is advanced to the time step. This method has been successfully used for non-real-time pseudodynamic tests where actuators move slowly following a linear ramp function, stop at the end of the ramp, and wait for the commands from the next iteration.[10]

3.3 Iterative Technique with Interpolation

In a real-time test for nonlinear structures, the trial displacements that obtained from the conventional Newton -type iterative method cannot be directly imposed on the structural specimen at the sampling points of the controller as this will lead to undesired velocity fluctuations during iteration. In addition, the number of iterative corrections required can vary from one time step to the next depending on the degree of nonlinearity developed by the structure. This uncertainty is not desirable for a real-time test where a converged solution has to be attained within a designated time interval. To solve this problem, a special iterative method that has a fixed number of iterations in each time step is used in the FHT system. This method relies on an interpolation technique to assure a smooth motion of the actuators during iteration. It is presented in detail below.

The iteration method proposed here can be viewed as a two-step procedure: (1) the trial displacement for each degree of freedom is evaluated as in a conventional Newton-type iteration process; (2) a small increment of the trial displacement is then computed by interpolation and imposed on the structural specimen. The interpolation is



〈Figure 2〉 Command generation scheme.

based on the trial displacement and the converged displacements in the previous time steps. The displacement subincrements are generated at the sampling frequency of the controller. The iteration procedure is illustrated in Figure 2.

In iteration step k, a quadratic polynomial is constructed to pass through three points. They are $d_{i-1}^{d(n-1)}$ and $d_{i}^{d(n-1)}$, which are the displacement commands generated at the last iteration step in the previous two time intervals and are considered as the converged solutions, and the recently updated trial displacement, $d_{i+1}^{(k)}$, in the current iteration step. Then, the desired displacement command, $d_{i+1}^{d(k)}$, in each iteration step is determined by interpolation using this polynomial function. However, in the very first time step, instead of using the two previous step displacements, the initial displacement and velocity are used. The mathematical expressions for the interpolated displacement commands are as follows.

$$d_{i+1}^{d(k)} = m^2 d_{i+1}^{(k)} + n(m-m^2)\nu_{ini} + (1-m^2)d_{ini} \quad (1^{\text{st}} \text{ step}) \tag{10}$$

$$d_{i+1}^{d(k)} = \frac{1}{2}(m^2-m)d_{i-1}^{d(n-1)} + (1-m^2)d_{i}^{d(n-1)} + \frac{1}{2}(m^2+m)d_{i+1}^{(k)} \quad \text{(other steps)} \tag{11}$$

in which k is iteration index (= 0,1,2 $\cdots$ $n-1$), n is number of iteration, $m=(k+1)/n$, $d_{i+1}^{d(k)}$ is the interpolated displacement vector which will be applied to the test structure at step k, d_{ini} and ν_{ini} are the initial displacement and velocity, $d_{i-1}^{d(n-1)}$ and $d_{i}^{d(n-1)}$ are the displacement commands generated in the last iteration at time steps i and $i-1$, and $d_{i+1}^{(k)}$ is the trial displacement vector at the current step k.

3.4 Response Update

It is inevitable that there will be a small residual error at the end because the number of iterations in each time step is fixed in the FHT system. Furthermore, since there are other possible delays in a test system, e.g., an actuator may not precisely track the command signal, there could be additional errors introduced at the end of iteration. To minimize the effect of these errors and enforce equilibrium at the end of iteration, a correction is introduced in the last iteration step. This correction can be introduced through the following displacement and force update in the last iteration step.

$$d_{i+1} = d_{i+1}^{d(n-1)} \tag{12}$$

$$r_{i+1} = r_{i+1}^{m(n-1)} + K_{ini}\left[d_{i+1}^{d(n-1)} - d_{i+1}^{m(n-1)}\right] \tag{13}$$

in which d_{i+1} and r_{i+1} are the updated displacements and forces that are treated as the converged solutions, $d_{i+1}^{d(n-1)}$ is the desired displacement vector computed in the $(n-1)$ -th iteration, which is the last iteration, and $d_{i+1}^{m(n-1)}$ and $r_{i+1}^{m(n-1)}$ are the displacements and restoring forces measured at the beginning of the $(n-1)$ -th iteration. The velocity and acceleration are then updated as follows.

$$a_{i+1} = \frac{1}{\Delta t^2 \beta}\left[d_{i+1} - d_i - \Delta t \nu_i - \Delta t^2\left(\frac{1}{2}-\beta\right)a_i\right] \tag{14}$$

$$\nu_{i+1} = \nu_i + \Delta t \gamma a_{i+1} + \Delta t(1-\gamma)a_i \tag{15}$$

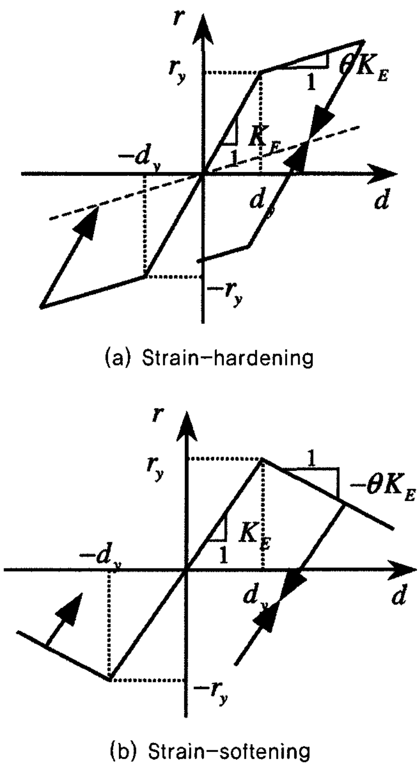
However, more accurate results can be obtained if the correction is introduced at the beginning of the next time step with one additional iteration using the more updated displacements and restoring forces measured. Nevertheless, this may significantly increase the computational effort in the first iteration step.

4. Performance Evaluation of the Computation Scheme

In order to evaluate the accuracy of the numerical integration scheme presented above, dynamic analyses are performed with single-, two-, and three-story shear frames having linearly elastic and inelastic load-displacement

⟨Table 1⟩ Elastic structural properties of the shear frames.

Model	Story	Mass (kg) $\times 10^3$	Stiffness (kN/m) $\times 10^3$	Modal Frequency (Hz)
Single-Story	1	35.054	3.678	f_1 = 1.63
Two-Story	1	35.054	3.678	f_1 = 0.95
	2	35.054	2.627	f_2 = 2.36
Three-Story	1	35.054	3.678	f_1 = 0.77
	2	35.054	2.627	f_2 = 1.86
	3	17.527	1.751	f_3 = 2.49



⟨Figure 3⟩ Inelastic constitutive models (Kinematic rule).

relations. The elastic structural properties of the shear frames are summarized in Table 1. To simulate the inelastic behavior of the structural system, bi-linear hardening and softening constitutive models as shown in Figure 3 are implemented.

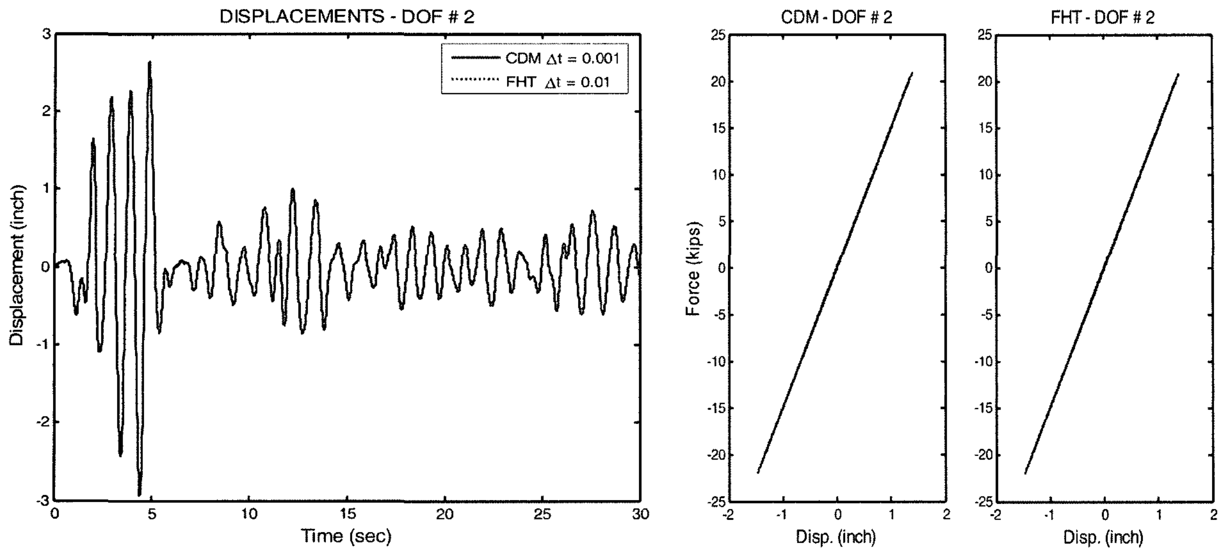
In order to reflect the time delay in the computation and the controller, two millisecond delay are introduced between $d^{d(k)}$ and $d^{m(k)}$. The values of the inelastic material parameters, d_y and θ, defined in Figure 3 are set to 0.5 and 0.3 for hardening and 1.0 and -0.3 for softening, respectively.

For all cases, the parameter α of the integration scheme is set to -0.25 and β and γ are set to $(1-\alpha)^2/4$ and $(1-2\alpha)/2$, respectively. The integration time step is 0.01 s with 10 iterations in each step. The NS component of the 1940 El Centro ground motion is used with the peak ground acceleration scaled to 0.2 g for the single- and three-story structures and 0.18 g for the two-story structure model. In addition, Rayleigh damping with 5% of the critical is used for all the natural modes of the structure. In the analysis of the inelastic shear frames, the initial stiffness is used for the iterative correction and the response update. The central difference method (CDM) with an integration time step Δt= 0.001 s is employed to obtain a benchmark solution, which has been shown to be close to the exact solution.

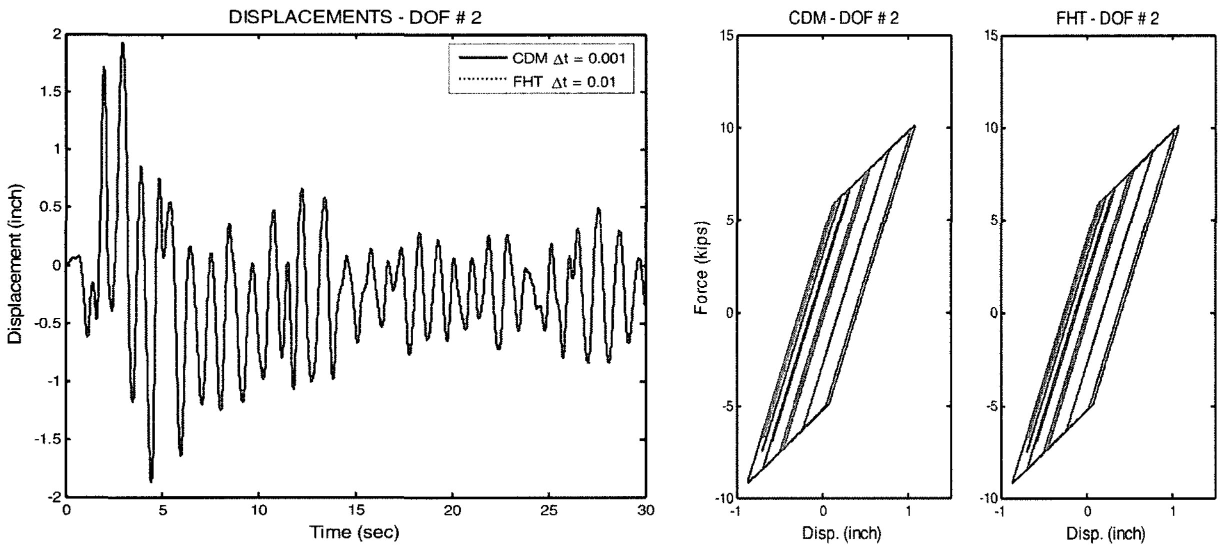
In Figure 4 and Figure 5, the displacement responses obtained with the proposed computation scheme are compared to the results of the central difference method. It has been observed that the computation scheme has compatible performance to evaluate the dynamic behavior of the single- and multi-degree-of-freedom structures. For conciseness, the responses of the single-degree-of-freedom structure are not shown here and only the top-story displacement responses are plotted for the multi-story shear frames. From these figures, it is observed that the responses obtained from the proposed scheme are very close to the results of the central difference method.

5. Conclusions

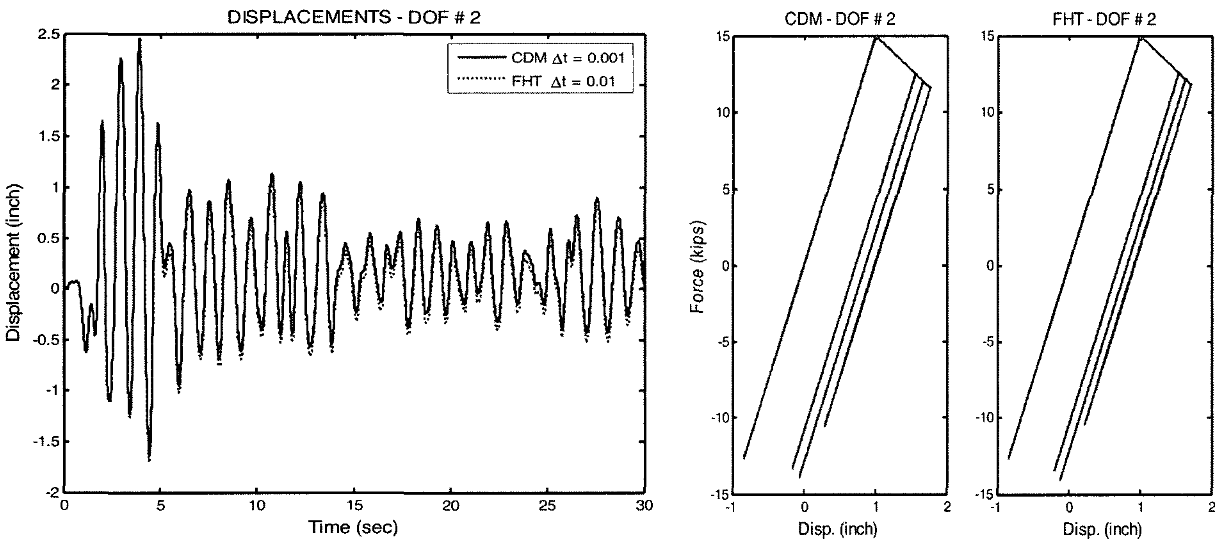
The key features and functionality of the Fast Hybrid Test system developed at the University of Colorado are presented in this paper. The FHT system has adopted an unconditionally stable implicit time integration scheme that provides a robust computational environmental for large-scale structural response simulations. The accuracy and a good performance of the computation scheme of the



(a) Linear elastic constitutive relation

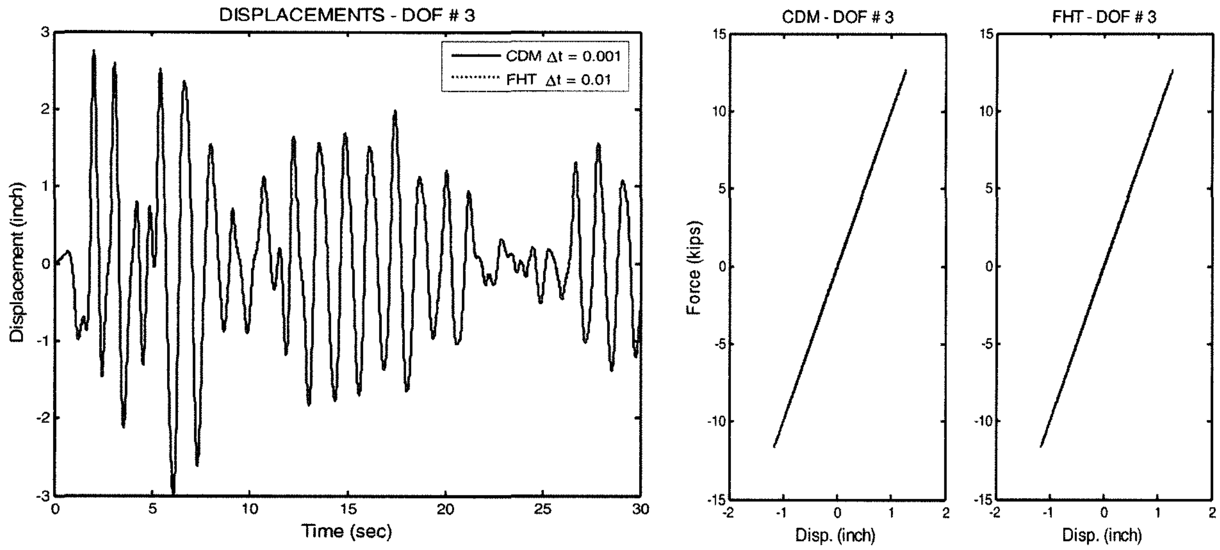


(b) Strain-hardening constitutive relation

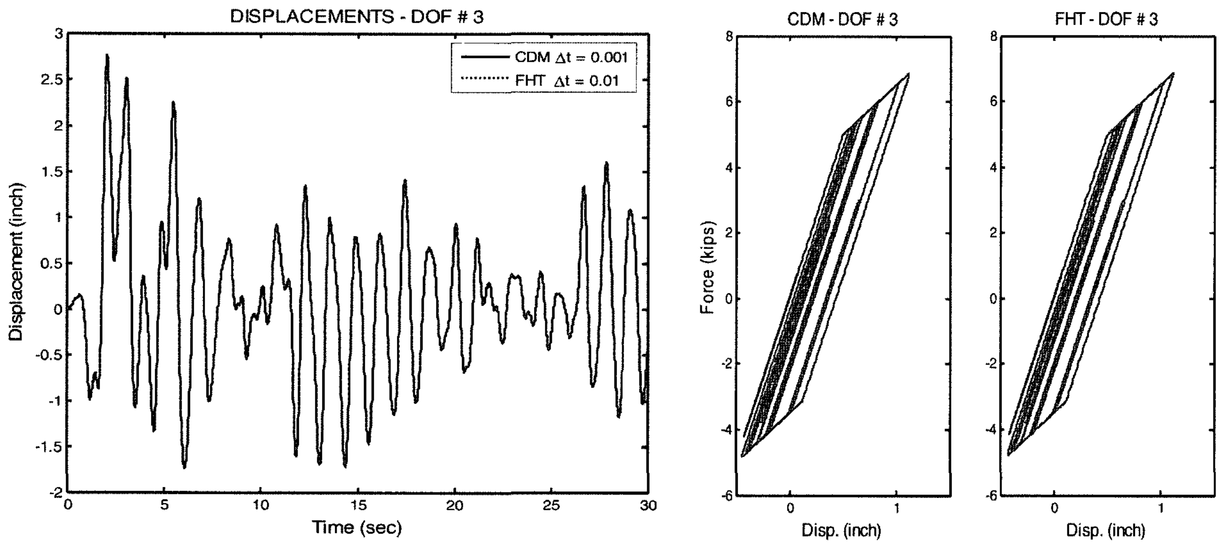


(c) Strain-softening constitutive relation

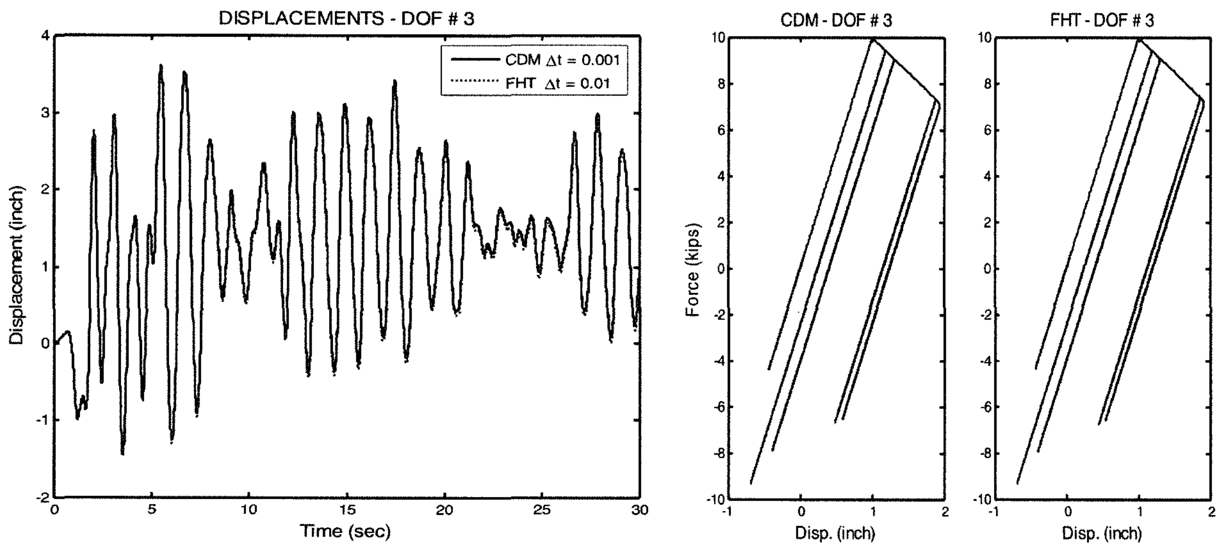
〈Figure 4〉 Displacement responses at top story of the two-story frame.



(a) Linear elastic constitutive relation



(b) Strain-hardening constitutive relation



(c) Strain-softening constitutive relation

⟨Figure 5⟩ Displacement responses at top story of the three-story frame.

system have been demonstrated with validation analyses performed with single- and multi-story shear frames having linearly elastic and inelastic load-displacement relations. Through the validation analyses, it has been observed that the responses obtained form the computation scheme proposed here for a real-time hybrid test are very close to the results of the central difference method. However, it has been also found that for a system with severe softening, increasing the number of iterative steps or time interval for integration is desirable to reduce the convergence error.

The system performance with an actual experiment system will be evaluated in a future publication.

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