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Controllability for the Nonlinear Fuzzy Control System with Nonlocal Initial Condition in E_N^n

Bu Young Lee[1], Dong Gun Park[1], Gyu Tak Choi[2]
and
Young Chel Kwun[1]

[1] DEPARTMENT OF MATHEMATICS, DONG-A UNIVERSITY, BUSAN 604-714, KOREA
[2] DEPARTMENT OF INDUSTRIAL ENGINEERING, KYUNGNAM COLLEGE OF INFORMATION & TECHNOLOGY, BUSAN 617-701, KOREA

Abstract

In this paper we study the exact controllability for the nonlinear fuzzy control system with nonlocal initial condition in E_N^n by using the concept of fuzzy number of dimension n whose values are normal, convex, upper semicontinuous and compactly supported surface in R^n. E_N^n be the set of all fuzzy numbers in R^n with edges having bases parallel to axis $X_1, X_2, \cdots, X_n$.

Key words : fuzzy number of dimension n, nonlinear fuzzy control system, nonlocal controllability

1. Introduction

Many authors have studied several concepts of fuzzy systems.

Kaleva[2] studied the existence and uniqueness of solution for the fuzzy differential equation on E^n where E^n is normal, convex, upper semicontinuous and compactly supported surface in R^n.

Seikkala[10] proved the existence and uniqueness of fuzzy solution for the initial value problem on E^1.

Subrahmanyam and Sudarsanam [11] studied fuzzy volterra - integral equation.

Park etal.[9] are proved the existence and uniqueness of fuzzy solution for the nonlinear fuzzy differential equation on E_N^n with nonlocal initial condition.

Kwun etal.[6] are studied controllability for the nonlinear fuzzy control system on E_N^n, where E_N^n be the set of all fuzzy numbers in with edges having bases parallel to axis $X_1, X_2, \cdots, X_n$. For example E_N^2 be the set of all fuzzy pyramidal numbers in R^2 with edges having rectangular bases parallel to the axis X_1 and X_2.

The purpose of this paper is to investigate the exact controllability of the nonlinear fuzzy control system in E_N^n.

Let E_N^n be the set of all fuzzy numbers in R^n with edges having bases parallel to axis $X_1, \cdots, X_n$.

For example, E_N^2 be the set of all fuzzy pyramidal numbers in R^2 with edges having rectangular bases parallel to the axis X_1 and X_2 ([6]).

We consider the exact controllability for the following nonlinear fuzzy control system with nonlocal initial condition:

$$(F.C.S). \qquad \begin{cases} \dot{x}(t) = a(t)x(t) + f(t, x(t)) + u(t)\,, \\ x(0) + g(t_1, t_2, \cdots, t_p, x(\cdot)) = x_0, \\ \qquad \cdot \in \{t_1, t_2, \cdots, t_p\} \end{cases}$$

where $a : [0,T] \to E_N$ is fuzzy coefficient, initial value $x_0 \in E_N^n$. $f : [0,T] \times E_N^n \to E_N^n$ and $g : [0,T]^p \times E_N^n \to E_N^n$ are nonlinear function and $u(t) \in E_N^n$ is control function.

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Corresponding Author : Young Chel Kwun
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2. Properties of fuzzy numbers

We consider a fuzzy graph $G \subset R^n$ that is a functional fuzzy relation in R^n such that its membership function $\mu_G(x_1, \cdots, x_n) \in [0,1]$, $(x_1, \cdots, x_2) \in R^n$ has the following properties:

1. For all $x_i \in R$, $(i = 1, \cdots, k-1, k+1, \cdots, n)$,

$$\mu_G(x_1, \cdots, x_k, \cdots, x_n) \in [0,1]$$

is a convex membership function.

2. For all $\alpha \in [0,1]$,

$$\{(x_1, \cdots, x_n) \in R^n : \mu_G(x_1, \cdots, x_n) = \alpha\}$$

is a convex set.

3. There exists $(x_1, \cdots, x_n) \in R^n$,

$$\mu_G(x_1, \cdots, x_n) = 1.$$

If the above conditions are satisfied, the fuzzy subset G is called a fuzzy number of dimension n.

The first projection of G is

$$\vee_{\{x_2, \cdots, x_n\}} \mu_G(x_1, \cdots, x_n) = \mu_{A_1}(x_1),$$

the second projection of G is

$$\vee_{\{x_1, x_3, \cdots, x_n\}} \mu_G(x_1, \cdots, x_n) = \mu_{A_2}(x_2)$$

and the i-th projection of G is

$$\vee_{\{x_1, \cdots, x_{i-1}, \cdots, x_{i+1}, \cdots, x_n\}} \mu_G(x_1, \cdots, x_n) = \mu_{A_i}(x_i),$$

where $i = 3, \cdots, n$.

We denote by fuzzy number in E_N^n

$$A = (a_1, a_2, \cdots, a_n),$$

where a_i is projection of A to axis X_i, $(i = 1, \cdots, n)$. And a_i, $(i = 1, \cdots, n)$ is fuzzy number in R.

The α-level set of fuzzy number in E_N^n is defined by

$$[A]^\alpha = \{(x_1, \cdots, x_n) \in R^n : (x_1, \cdots, x_n) \in \prod_{i=1}^n [a_i]^\alpha\},$$

where notation $\prod$ is the Cartesian product of sets.

Let A and B in E_N^n, for all $\alpha \in (0,1]$,

$$(2.1) \qquad A = B \iff [A]^\alpha = [B]^\alpha.$$

$$(2.2) \qquad [A *_n B]^\alpha = \prod_{i=1}^n [a_i * b_i]^\alpha,$$

where $*_n$ is operator in E_N^n and $*$ is operator in E_N.

Let $\prod_{i=1}^n [a_i]^\alpha$, $0 < \alpha \leq 1$, be a given family of nonempty areas.

If

$$(2.3) \qquad \prod_{i=1}^n [a_i]^\beta \subset \prod_{i=1}^n [a_i]^\alpha \text{ for } 0 < \alpha < \beta < 1$$

and

$$(2.4) \qquad \prod_{i=1}^n \lim_{k \to \infty} [a_i]^{\alpha_k} = \prod_{i=1}^n [a_i]^\alpha$$

whenever (α_k) is a nondecreasing sequence converging to $\alpha \in (0,1]$, then the family $\prod_{i=1}^n [a_i]^\alpha$, $0 < \alpha \leq 1$, represents the α-level sets of a fuzzy number $A \in E_N^n$.

Conversely, if $\prod_{i=1}^n [a_i]^\alpha$, $0 < \alpha \leq 1$, are the α-level sets of a fuzzy number in R^n, then the conditions (2.3) and (2.4) hold true.

We denote the metric d_∞ on E_N^n and the suprimum metric H_1 on $C([0,T] : E_N^n)$.

Definition 2.1. Let $A, B \in E_N^n$.

$$\begin{aligned} d_\infty(A,B) &= \sup\{d_H([A]^\alpha, [B]^\alpha) : \alpha \in (0,1]\} \\ &= \sup\{d_H(\Pi_{i=1}^n [A_i]^\alpha, \Pi_{i=1}^n [B_i]^\alpha) : \alpha \in (0,1]\} \\ &= \sup\{\sqrt{\sum_{i=1}^n (d_H([A_i]^\alpha, [B_i]^\alpha))^2} : \alpha \in (0,1]\}, \end{aligned}$$

where d_H is the Hausdorff distance.

Definition 2.2. The supremum metric H_1 on $C([0,T] : E_N^n)$ is defined by

$$H_1(x,y) = \sup\{d_\infty(x(t), y(t)); t \in [0,T]\}$$

for all $x, y \in C([0,T]; E_N^n)$

Definition 2.3. Nonlinear regular fuzzy function $f : [0,T] \times E_N^n \times E_N^n \to E_N^n$ is satisfied, $x, y \in E_N^n$,

$$\begin{aligned} f(t, [x]^\alpha) &= f(t, \Pi_{m=1}^n [x_m]^\alpha) \\ &= \Pi_{m=1}^n f_m(t, [x_m]^\alpha) \\ &= \Pi_{m=1}^n f_m^\alpha(t, x) \\ &= f^\alpha(t,x) = [f(t,x)]^\alpha. \end{aligned}$$

3. Nonlocal controllability

In this section, we show the exact controllability for the following nonlinear fuzzy control system:

$$(F.C.S.)\qquad \begin{cases} \dot{x}(t) = a(t)x(t) + f(t,x(t)) + u(t)\,, \\ x(0) + g(t_1,t_2,\cdots,t_p,x(\cdot)) = x_0, \\ \qquad \cdot \in \{t_1,t_2,\cdots,t_p\} \end{cases}$$

with fuzzy coefficient $a : [0,T] \to E_N^n$, initial value $x_0 \in E_N^n$ and control $u : [0,T] \to E_N^n$ and given nonlinear regular fuzzy function $g : [0,T]^p \times E_N^n \to E_N^n$, $f : [0,T] \times E_N^n \to E_N^n$ are satisfy global Lipschitz condition.

The (F.C.S.) is related to the following fuzzy integral system:

$$(F.I.E.)\qquad \begin{cases} x(t) =S(t)(x_0 - g(t_1,t_2,\cdots,t_p,x(\cdot))) \\ \qquad + \int_0^t S(t-s)f(s,x(s))ds \\ \qquad + \int_0^t S(t-s)u(s)ds, \\ x(0) =x_0 + g(t_1,t_2,\cdots,t_p,x(\cdot)) \in E_N, \\ \qquad \cdot \in \{t_1,t_2,\cdots,t_p\}. \end{cases}$$

where $S(t)$ is fuzzy number of dimension n and

$$[S(t)]^\alpha = \prod_{i=1}^n [S_i(t)]^\alpha = \prod_{i=1}^n [S_{il}^\alpha(t),\ S_{ir}^\alpha(t)]$$

where $S_{il}^\alpha(t)$ is $\exp\{\int_0^t a_l^\alpha(s)ds\}$ and $S_{ir}^\alpha(t)$ is $\exp\{\int_0^t a_r^\alpha(s)ds\}$. And $S_{ij}^\alpha(t)$ $(j = l,r)$ is continuous. That is, there exists a constant $c > 0$ such that $|S_{ij}^\alpha(t)| \le c$ for all $t \in [0,T]$.

Definition 3.1. The (F.I.S.) is nonlocal exact controllable if, there exists $u(t)$ such that the fuzzy solution $x(t)$ of (F.I.S.) satisfies

$$x(T) = x^1 - g(t_1,t_2,\cdots,t_p,x(\cdot))$$

$$\text{(i.e., } [x(T)]^\alpha = \prod_{i=1}^n [x_i(T)]^\alpha$$
$$= \prod_{i=1}^n [(x^1)_i - g_i(t_1,t_2,\cdots,t_p,x(\cdot))]^\alpha$$
$$= [x^1 - g(t_1,t_2,\cdots,t_p,x(\cdot))]^\alpha \text{)}$$

where x^1 is target set.

We assume that the following linear fuzzy control system with respect to nonlinear fuzzy control system (F.C.S.):

$$(F.C.S.1)\qquad \begin{cases} \dot{x}(t) = a(t)x(t) + u(t), \\ x(0) + g(t_1,t_2,\cdots,t_p,x(\cdot)) = x_0 \in E_N^n \end{cases}$$

is exact controllable. Then

$$\begin{aligned} & x(T) \\ =& S(T)(x_0 - g(t_1,t_2,\cdots,t_p,x(\cdot))) \\ & + \int_0^T S(T-s)u(s)ds \\ =& x^1 \end{aligned}$$

and

$$\begin{aligned} & [x(T)]^\alpha \\ =& \big[S(T)(x_0 - g(t_1,t_2,\cdots,t_p,x(\cdot))) \\ & + \int_0^T S(T-s)u(s)ds\big]^\alpha \\ =& \prod_{i=1}^n [S_i(T)(x_0 - g(t_1,t_2,\cdots,t_p,x(\cdot)))_i \\ & + \int_0^T S_i(T-s)u_i(s)ds]^\alpha \\ =& \prod_{i=1}^n [S_{il}^\alpha(T)(x_0 - g(t_1,t_2,\cdots,t_p,x(\cdot)))_{il}^\alpha \\ & + \int_0^T S_{il}^\alpha(T-s)u_{il}^\alpha(s)ds, \\ & S_{ir}^\alpha(T)(x_0 - g(t_1,t_2,\cdots,t_p,x(\cdot)))_{ir}^\alpha \\ & + \int_0^T S_{ir}^\alpha(T-s)u_{ir}^\alpha(s)ds] \\ =& \prod_{i=1}^n [(x^1)_{il}^\alpha,\ (x^1)_{ir}^\alpha] \\ =& [x^1]^\alpha\,. \end{aligned}$$

Defined the fuzzy mapping $\widetilde{g} : \widetilde{\mathrm{P}}(R^n) \to E_N^n$ by

$$\widetilde{g}^\alpha(v) = \begin{cases} \int_0^T S^\alpha(T-s)v(s)ds\,, & v \subset \overline{\Gamma_u}\,, \\ 0\,, & \text{otherwise}\,. \end{cases}$$

Then there exists $\widetilde{g}_i : \widetilde{\mathrm{P}}(R) \to E_N$ $(i = 1,2,\cdots,n)$ such that

$$\widetilde{g}_i^\alpha(v_i) = \begin{cases} \int_0^T S_i^\alpha(T-s)v_i(s)ds\,, & v_i(s) \subset \overline{\Gamma_{u_i}}, \\ 0\,, & \text{otherwise} \end{cases}$$

where u_i is projection of u to axis X_i, $(i = 1,\cdots,n)$ respectively and there exists $\widetilde{g}_{ij}^\alpha$ $(j = l,r)$

$$\widetilde{g}_{il}^\alpha(v_{il}) = \int_0^T S_{il}^\alpha(T-s)v_{il}(s)ds\,,$$
$$v_{il}(s) \in [u_{il}^\alpha(s),\ u_i^1(s)]\,,$$
$$\widetilde{g}_{ir}^\alpha(v_{ir}) = \int_0^T S_{ir}^\alpha(T-s)v_{ir}(s)ds\,,$$
$$v_{ir}(s) \in [u_i^1(s),\ u_{ir}^\alpha(s)]\,.$$

We assume that $\widetilde{g}_{il}^{\alpha}, \widetilde{g}_{ir}^{\alpha}$ are bijective mappings.

Hence α-level of $u(s)$ are

$$\begin{aligned}
&[u(s)]^{\alpha} \\
&= \prod_{i=1}^{n}[u_i(s)]^{\alpha} = \prod_{i=1}^{n}[u_{il}^{\alpha}(s),\ u_{ir}^{\alpha}(s)] \\
&= \prod_{i=1}^{n} \big[(\widetilde{g}_{il}^{\alpha})^{-1}\big((x^1 - g(t_1, t_2, \cdots, t_p, x(\cdot)))_{il}^{\alpha} \\
&\qquad - S_{il}^{\alpha}(T)(x_0)_{il}^{\alpha}\big), \\
&\qquad (\widetilde{g}_{ir}^{\alpha})^{-1}\big((x^1 - g(t_1, t_2, \cdots, t_p, x(\cdot)))_{ir}^{\alpha} \\
&\qquad - S_{ir}^{\alpha}(T)(x_0)_{ir}^{\alpha}\big)\big].
\end{aligned}$$

Thus we can be introduced $u(s)$ of nonlinear system

$$\begin{aligned}
&[u(s)]^{\alpha} \\
&= \prod_{i=1}^{n}[u_i(s)]^{\alpha} = \prod_{i=1}^{n}[u_{il}^{\alpha}(s),\ u_{ir}^{\alpha}(s)] \\
&= \prod_{i=1}^{n}\ (\widetilde{g}_{il}^{\alpha})^{-1}\ (x^1 - g(t_1, t_2, \cdots, t_p, x(\cdot)))_{il}^{\alpha} \\
&\qquad - S_{il}^{\alpha}(T)(x_0)_{il}^{\alpha} - \int_0^T S_{il}^{\alpha}(T-s) f_{il}^{\alpha}(s, x_{il}^{\alpha}(s))ds\ , \\
&\qquad (\widetilde{g}_{ir}^{\alpha})^{-1}\ (x^1 - g(t_1, t_2, \cdots, t_p, x(\cdot)))_{ir}^{\alpha} \\
&\qquad - S_{ir}^{\alpha}(T)(x_0)_{ir}^{\alpha} - \int_0^T S_{ir}^{\alpha}(T-s) f_{ir}^{\alpha}(s, x_{ir}^{\alpha}(s))ds\ .
\end{aligned}$$

Then substituting this expression into the (F.I.S.) yields $\alpha-$level of $x(T)$. For each $i = 1, \cdots, n$,

$$\begin{aligned}
&[x_i(T)]^{\alpha} \\
&= S_{il}^{\alpha}(T)(x_0 - g(t_1, t_2, \cdots, t_p, x(\cdot)))_{il}^{\alpha} \\
&\qquad + \int_0^T S_{il}^{\alpha}(T-s) f_{il}^{\alpha}(s, x_{il}^{\alpha}(s))ds \\
&\quad + \int_0^T S_{il}^{\alpha}(T-s)(\widetilde{g}_{il}^{\alpha})^{-1}\ (x^1 - g(t_1, t_2, \cdots, t_p, x(\cdot)))_{il}^{\alpha} \\
&\quad - S_{il}^{\alpha}(T)(x_0 - g(t_1, t_2, \cdots, t_p, x(\cdot)))_{il}^{\alpha} \\
&\qquad - \int_0^T S_{il}^{\alpha}(T-s) f_{il}^{\alpha}(s, x_{il}^{\alpha}(s))ds\ ds, \\
&S_{ir}^{\alpha}(T)(x_0 - g(t_1, t_2, \cdots, t_p, x(\cdot)))_{ir}^{\alpha} \\
&\qquad + \int_0^T S_{ir}^{\alpha}(T-s) f_{ir}^{\alpha}(s, x_{ir}^{\alpha}(s))ds \\
&\quad + \int_0^T S_{ir}^{\alpha}(T-s)(\widetilde{g}_{ir}^{\alpha})^{-1}\ (x^1 - g(t_1, t_2, \cdots, t_p, x(\cdot)))_{ir}^{\alpha} \\
&\quad - S_{ir}^{\alpha}(T)(x_0 - g(t_1, t_2, \cdots, t_p, x(\cdot)))_{ir}^{\alpha} \\
&\qquad - \int_0^T S_{ir}^{\alpha}(T-s) f_{ir}^{\alpha}(s, x_{ir}^{\alpha}(s))ds\ ds \\
&= S_{il}^{\alpha}(T)(x_0 - g(t_1, t_2, \cdots, t_p, x(\cdot)))_{il}^{\alpha} \\
&\qquad + \int_0^T S_{il}^{\alpha}(T-s) f_{il}^{\alpha}(s, x_{il}^{\alpha}(s))ds \\
&\quad + \widetilde{g}_{il}^{\alpha}\ (\widetilde{g}_{il}^{\alpha})^{-1}\ (x^1 - g(t_1, t_2, \cdots, t_p, x(\cdot)))_{il}^{\alpha} \\
&\quad - S_{il}^{\alpha}(T)(x_0 - g(t_1, t_2, \cdots, t_p, x(\cdot)))_{il}^{\alpha} \\
&\qquad - \int_0^T S_{il}^{\alpha}(T-s) f_{il}^{\alpha}(s, x_{il}^{\alpha}(s))ds\ , \\
&S_{ir}^{\alpha}(T)(x_0 - g(t_1, t_2, \cdots, t_p, x(\cdot)))_{ir}^{\alpha} \\
&\qquad + \int_0^T S_{ir}^{\alpha}(T-s) f_{ir}^{\alpha}(s, x_{ir}^{\alpha}(s))ds \\
&\quad + \widetilde{g}_{ir}^{\alpha}\ (\widetilde{g}_{ir}^{\alpha})^{-1}\ (x^1 - g(t_1, t_2, \cdots, t_p, x(\cdot)))_{ir}^{\alpha} \\
&\quad - S_{ir}^{\alpha}(T)(x_0 - g(t_1, t_2, \cdots, t_p, x(\cdot)))_{ir}^{\alpha} \\
&\qquad - \int_0^T S_{ir}^{\alpha}(T-s) f_{ir}^{\alpha}(s, x_{ir}^{\alpha}(s))ds \\
&= [(x^1 - g(t_1, t_2, \cdots, t_p, x(\cdot)))_{il}^{\alpha}, \\
&\qquad (x^1 - g(t_1, t_2, \cdots, t_p, x(\cdot)))_{ir}^{\alpha}] \\
&= [(x^1 - g(t_1, t_2, \cdots, t_p, x(\cdot)))_i]^{\alpha}
\end{aligned}$$

Therefore

$$\begin{aligned}
[x(T)]^{\alpha} &= \prod_{i=1}^{n}[x_i(T)]^{\alpha} \\
&= \prod_{i=1}^{n}[(x^1 - g(t_1, t_2, \cdots, t_p, x(\cdot)))_i]^{\alpha} \\
&= [x^1 - g(t_1, t_2, \cdots, t_p, x(\cdot))]^{\alpha}.
\end{aligned}$$

We now set

$$\begin{aligned}
&\Phi x(t) \\
&= S(t)(x_0 - g(t_1, t_2, \cdots, t_p, x(\cdot))) \\
&\quad + \int_0^t S(t-s) f(s, x(s))ds \\
&\quad + \int_0^t S(t-s)\widetilde{g}^{-1}\ x^1 - g(t_1, t_2, \cdots, t_p, x(\cdot)) \\
&\quad - S(T)(x_0 - g(t_1, t_2, \cdots, t_p, x(\cdot))) \\
&\quad - \int_0^T S(T-s) f(s, x(s))ds\ ds
\end{aligned}$$

where the fuzzy mappings $\widetilde{g}^{-1}$ satisfied above statements.

Notice that $\Phi x(T) =_{\alpha} x^1 - g(t_1, t_2, \cdots, t_p, x(\cdot))$, which means that the control $u(t)$ steers the (F.C.S.) from the origine to $x^1 - g(t_1, t_2, \cdots, t_p, x(\cdot))$ in time T provided we can obtain a fixed point of the nonlinear operator Φ.

Assume that the following hypotheses:

(H1) (F.C.S. 1) is exact controllable.

(H2) Inhomogeneous term $f : [0, T] \times E_N^n \to E_N^n$ satisfies a global Lipschitz condition, there exists a finite constant $k_i > 0$ such that

$$\begin{aligned}
&d_H\ [f_i(s, x_i(s))]^{\alpha}, [f_i(s, y_i(s))]^{\alpha} \\
&\le k_i\ d_H\ [x_i(s)]^{\alpha}, [y_i(s)]^{\alpha}
\end{aligned}$$

for all $x_i(s), y_i(s) \in E_N$ and $f_i : [0, T] \times E_N \to E_N$ ($i = 1, \cdots, n$) is the i-th projection of f.

(H3) $g : [0,T]^p \times E_N^n \to E_N^n$ satisfies a global Lipschitz condition, there exists a finite constant $L_i > 0$ such that

$$d_H \ [g_i(t_1,t_2,\cdots,t_p,x_i(\cdot))]^\alpha, [g_i(t_1,t_2,\cdots,t_p,y_i(\cdot))]^\alpha \le L_i \, d_H \ [x_i(s)]^\alpha, [y_i(s)]^\alpha$$

for all $x_i(s), y_i(s) \in E_N$ and $g_i : [0,T]^p \times E_N \to E_N$ $(i = 1,\cdots,n)$ is the i-th projection of g.

(H4) $\big((1+2c)L_i + 2ck_iT)\big) < 1$.

Theorem 3.1 Suppose that hypotheses (H1), (H2), (H3) and (H4) are satisfied. Then the state of the (F.I.S.) can be steered from the initial value $x_0 - g(t_1,t_2,\cdots,t_p,x(\cdot))$ to any final state $x^1 - g(t_1,t_2,\cdots,t_p,x(\cdot))$ in time T.

Proof The continuous function from $C([0,T] : E_N^n)$ to itself defined by

$$\begin{aligned}\Phi x(t) =_\alpha & S(t)(x_0 - g(t_1,t_2,\cdots,t_p,x(\cdot))) + \int_0^t S(t-s)f(s,x(s))ds \\ & + \int_0^t S(t-s)\widetilde{g}^{-1} \ x^1 - g(t_1,t_2,\cdots,t_p,x(\cdot)) \\ & - S(T)(x_0 - g(t_1,t_2,\cdots,t_p,x(\cdot))) - \int_0^T S(T-s)f(s,x(s))ds \ ds.\end{aligned}$$

There exist Φ_i $(i = 1,\cdots,n)$ is continuous function from $C([0,T] : E_N)$ to itself. Let $x, y \in C([0,T] : E_N^n)$ there exist x_i, y_i $(i = 1,\cdots,n) \in C([0,T] : E_N)$.

$$\begin{aligned}
& d_H \ [\Phi_i x_i(t)]^\alpha, [\Phi_i y_i(t)]^\alpha \\
= & d_H \quad S_i(t)(x_0 - g(t_1,t_2,\cdots,t_p,x(\cdot)))_i + \int_0^t S_i(t-s)f_i(s,x_i(s))ds \\
& + \int_0^t S_i(t-s)\widetilde{g}_i^{-1} \ (x^1 - g(t_1,t_2,\cdots,t_p,x(\cdot)))_i - S_i(T)(x_0 - g(t_1,t_2,\cdots,t_p,x(\cdot)))_i \\
& - \int_0^T S_i(T-s)f_i(s,x_i(s))ds \ ds \ ^\alpha, \\
& S_i(t)(x_0 - g(t_1,t_2,\cdots,t_p,y(\cdot)))_i + \int_0^t S_i(t-s)f_i(s,y_i(s))ds \\
& + \int_0^t S_i(t-s)\widetilde{g}_i^{-1} \ (x^1 - g(t_1,t_2,\cdots,t_p,y(\cdot)))_i - S_i(T)(x_0 - g(t_1,t_2,\cdots,t_p,y(\cdot)))_i \\
& - \int_0^T S_i(T-s)f_i(s,y_i(s))ds \ ds \ ^\alpha \\
\le & d_H \quad S_i(t)(x_0 - g(t_1,t_2,\cdots,t_p,x(\cdot)))_i \ ^\alpha, \ S_i(t)(x_0 - g(t_1,t_2,\cdots,t_p,y(\cdot)))_i \ ^\alpha \\
& + d_H \quad \int_0^t S_i(t-s)f_i(s,x_i(s))ds \ ^\alpha, \ \int_0^t S_i(t-s)f_i(s,y_i(s))ds \ ^\alpha \\
& + d_H \quad \int_0^T S_i(T-s)\widetilde{g}_i^{-1}(g_i(t_1,t_2,\cdots,t_p,x(\cdot)) + S_i(T)g_i(t_1,t_2,\cdots,t_p,x(\cdot)) + \int_0^T S_i(T-s)f_i(s,x_i(s))ds)ds \ ^\alpha, \\
& \int_0^T S_i(T-s)\widetilde{g}_i^{-1}(g_i(t_1,t_2,\cdots,t_p,y(\cdot)) + S_i(T)g_i(t_1,t_2,\cdots,t_p,y(\cdot)) + \int_0^T S_i(T-s)f_i(s,y_i(s))ds)ds \ ^\alpha \\
\le & d_H \quad S_i(t)(x_0 - g(t_1,t_2,\cdots,t_p,x(\cdot)))_i \ ^\alpha, \ S_i(t)(x_0 - g(t_1,t_2,\cdots,t_p,y(\cdot)))_i \ ^\alpha \\
& + d_H \quad \int_0^t S_i(t-s)f_i(s,x_i(s)) \ ^\alpha ds, \ \int_0^t S_i(t-s)f_i(s,y_i(s)) \ ^\alpha ds \\
& + d_H \quad \widetilde{g}_i(\widetilde{g}_i^{-1}(g_i(t_1,t_2,\cdots,t_p,x(\cdot)) + S_i(T)g_i(t_1,t_2,\cdots,t_p,x(\cdot)) + \int_0^T S_i(T-s)f_i(s,x_i(s))ds)) \ ^\alpha, \\
& \widetilde{g}_i(\widetilde{g}_i^{-1}(g_i(t_1,t_2,\cdots,t_p,y(\cdot)) + S_i(T)g_i(t_1,t_2,\cdots,t_p,y(\cdot)) + \int_0^T S_i(T-s)f_i(s,y_i(s))ds)) \ ^\alpha \\
\le & d_H \quad S_i(t)g_i(t_1,t_2,\cdots,t_p,x(\cdot)) \ ^\alpha, \ S_i(t)g_i(t_1,t_2,\cdots,t_p,y(\cdot)) \ ^\alpha \\
& + \int_0^t d_H \quad S_i(t-s)f_i(s,x_i(s)) \ ^\alpha, \ S_i(t-s)f_i(s,y_i(s)) \ ^\alpha \ ds \\
& + d_H \quad g_i(t_1,t_2,\cdots,t_p,x(\cdot)) \ ^\alpha, \ g(t_1,t_2,\cdots,t_p,y(\cdot)) \ ^\alpha \\
& + d_H \quad S_i(T)g_i(t_1,t_2,\cdots,t_p,x(\cdot)) \ ^\alpha, \ S_i(T)g_i(t_1,t_2,\cdots,t_p,y(\cdot)) \ ^\alpha \\
& + \int_0^T d_H \quad S_i(T-s)f_i(s,x_i(s)) \ ^\alpha, \ S_i(T-s)f_i(s,y_i(s)) \ ^\alpha \ ds \\
\le & cL_i d_H \ [x_i(t)]^\alpha, [y_i(t)]^\alpha + ck_i \int_0^t d_H \ [x_i(s)]^\alpha, [y_i(s)]^\alpha \ ds
\end{aligned}$$

$$+ L_i d_H \ [x_i(t)]^\alpha, [y_i(t)]^\alpha \ + cL_i d_H \ [x_i(t)]^\alpha, [y_i(t)]^\alpha$$
$$+ ck_i \int_0^T d_H \ [x_i(s)]^\alpha, [y_i(s)]^\alpha \ ds$$
$$\leq \ (1+2c)L_i + ck_i(t+T) \ d_H \ [x_i(t)]^\alpha, [y_i(t)]^\alpha$$

Thus

$$\begin{aligned} & d_\infty \ \Phi x, \Phi y \\ = & \sup_{\alpha \in (0,1]} d_H \ [\Phi x]^\alpha, [\Phi y]^\alpha \\ = & \sup_{\alpha \in (0,1]} \left\{ \sum_{i=1}^n d_H([\Phi_i x_i]^\alpha, [\Phi_i y_i]^\alpha) \ ^2 \right\}^{1/2} \\ \leq & \ (1+2c)L_i + ck_i(t+T) \\ & \quad \times \sup_{\alpha \in (0,1]} \left\{ \sum_{i=1}^n d_H([x_i(t)]^\alpha, [y_i(t)]^\alpha) \ ^2 \right\}^{1/2} \\ = & \ (1+2c)L_i + ck_i(t+T) \ d_\infty(x,y) \end{aligned}$$

Hence

$$\begin{aligned} H_1 \ \Phi x, \Phi y \ &= \sup_{t \in [0,T]} d_\infty \ \Phi x, \Phi y \\ \leq & \sup_{t \in [0,T]} \ (1+2c)L_i + ck_i(t+T) \ d_\infty(x,y) \\ \leq & \ (1+2c)L_i + 2ck_i T) \ \sup_{t \in [0,T]} d_\infty(x,y) \\ = & \ (1+2c)L_i + 2ck_i T) \ H_1(x,y). \end{aligned}$$

Since $\big((1+2c)L_i + 2ck_iT)\big) < 1$, Φ is a contraction mapping. By the Banach fixed point theorem, (F.C.S.) has a unique fixed point $x \in C([0,T] : E_N^n)$.

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Bu Young Lee

Professor of Dong-A University

Research Area: Fuzzy mathematics, Fuzzy topology, General topology

E-mail : bylee@dau.ac.kr

Dong Gun Park

Professor of Dong-A University

Research Area: Fuzzy mathematics,Fuzzy differential equation, Partial differential equation

E-mail : dgpark@dau.ac.kr

Gyu Tak Choi

Associate Professor of Kyungnam College of Information & Technonlogy

Research Area: Fuzzy mathematics, Fuzzy Programming, Linear programming

E-mail : gtchoi@kit.ac.kr

Young Chel Kwun

Professor of Dong-A University

Research Area: Fuzzy mathematics, Fuzzy differential equation, System theory and control

E-mail : yckwun@dau.ac.kr