

## A NOTE ON GENERALIZED NET MODEL OF E-LEARNING EVALUATION ASSOCIATED WITH INTUITIONISTIC FUZZY ESTIMATIONS

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### Abstract

A generalized net is used to construct a model which describes the process of evaluation of the problems solved by students. The model utilizes the theory of intuitionistic fuzzy sets. The model can be used to simulate some processes, related to estimation of students' background.

**Key words :** E-Learning, Generalized Net, Intuitionistic Fuzzy Sets, University

### 1. Introduction

In a series of research, the authors study some of the most important processes of functioning of universities. In the present paper the process of evaluation of the problems solved by students is described by Generalized Nets (GNs, see [1]).

Let us have  $m$  in number students and let each of them have to solve  $n$  in number problems. The evaluations corresponding to the students background about some theme are represented by intuitionistic fuzzy form (for the concept of Intuitionistic Fuzzy Set (IFS) see [2]). They have the form  $\langle \mu^s(i), \nu^s(i) \rangle$ , where  $\mu^s(i)$  and  $\nu^s(i)$  determine the degrees of validity and non-validity of the problems, solved by  $i$ -th student. Of course, the way of evaluation of the different themes is vary, but for some groups of themes (e.g., mathematics, informatics, physics, chemistry) the evaluations of the students solutions of the problems can be obtained, in general, by three ways:

**Version 1.** The solution of a problems is evaluated as valid or non-valid only on the bases of the obtained answer or on the basis of validity of full proof. For example, both problems below will be evaluated as wrong:

*Problem:*  $1 + 2 + 3 + 4 = ?$

*Answer 1:*  $1 + 2 + 3 + 4 = 3 + 3 + 4 = 6 + 4 = 11.$

*Answer 2:*  $1 + 2 + 3 + 4 = 4 + 3 + 4 = 7 + 4 = 11.$

In this case the evaluation of the  $i$ -th student is

$$\langle \mu^s(i), \nu^s(i) \rangle = \left\langle \frac{r}{n}, \frac{s}{n} \right\rangle,$$

where:  $r$  is the number of right solved problems,  $s$  is the number of wrong solved problems. Therefore, the degree of uncertainty here is determined by the number of the problems which the student had not worked over.

**Version 2.**  $j$ -th problem is divided into subproblems and the solution of each subproblem is evaluated independently. Each problem of the  $i$ -th student is evaluated by

$$\langle \mu^s(i), \nu^s(i) \rangle = \left\langle \frac{\sum_{j=1}^n z_j}{n}, \frac{\sum_{j=1}^n y_j}{n} \right\rangle,$$

where

$z_j$  is the number of the first correctly solved subproblems that precede a wrong subproblem

(if such one exists),

$y_j$  is the number of the wrongly solved subproblems that are follow a correctly solved subproblem (if such one exists).

For example, if the student's solution of the above problem is

$$1 + 2 + 3 + 4 = 3 + 3 + 4 = 7 + 4$$

then his evaluation will be

$$\left\langle \frac{1}{3}, \frac{1}{3} \right\rangle.$$

**Version 3.**  $j$ -th problem is divided into subproblems and the solution of each unique subproblem is evaluated independently. Each problem of the  $i$ -th student is evaluated by

$$\langle \mu^s(i), \nu^s(i) \rangle = \left\langle \frac{\sum_{j=1}^n z_j}{n}, \frac{\sum_{j=1}^n y_j}{n} \right\rangle$$

where  $x_j$  is the number of all right solved subproblems (if such one exists),  $y_j$  is the number of all wrong solved subproblems (if such one exists).

For example, if the student's solution of the above problem is

$$1 + 2 + 3 + 4 = 3 + 3 + 4 = 7 + 4 = 11$$

then his evaluation will be

$$\left\langle \frac{2}{3}, \frac{1}{3} \right\rangle.$$

In the paper we shall describe some processes related to examination and evaluation of students. But, the model gives additional possibilities that we will discuss below. One of them is that we can evaluate how precise are the teachers, how suitable are the problems over which the students have to work and other related issues.

## 2. A GN-model

The GN-model (see Figure. 1) contains 6 transitions and 18 places, collected in three groups and related to the three types of the tokens that will enter respective types of places:

$\alpha$  - tokens and  $a$ -places represent the teachers and their activities,

$\beta$  - tokens and  $b$ -places represent the problems,

$\gamma$  - tokens and  $c$ -places represent the students and their solutions of the problems.

For brevity, we shall use the notation  $\alpha$ -,  $\beta$ - and  $\gamma$ -tokens instead of  $\alpha_i$ -,  $\beta_j$ - and  $\gamma_k$ - tokens, where  $i, j, k$  are numerations of the respective tokens.

In the beginning  $\alpha$ -,  $\beta$ - and  $\gamma$ -tokens stay, respectively, in places  $a_2, b_2$  and  $c_2$  with initial characteristics:

$x_0^a$  = "name, speciality and score of a teacher",

$x_0^b$  = "text of a problem, theme, level of difficulty",

$x_0^y$  = "name, speciality and current evaluations of a student".

If we would like the model to be more detailed, the first and the latest characteristics can have, e.g., the following larger forms

$x_0^a$  = "name, speciality and score of a teacher

variant of evaluation that the teacher uses",

$x_0^y$  = "name, speciality and current evaluations of a student,

name of the student's teacher who will give the problems and/or examine the student".

All  $\alpha$ -tokens, all  $\beta$ -tokens, and all  $\gamma$ -tokens have equal priorities, but the priority of  $\alpha$ -tokens is higher than the priority of  $\beta$ -tokens, that is higher than the priority of  $\gamma$ -tokens.

Let  $x_{cu}^a, x_{cu}^b$  and  $x_{cu}^y$  are current  $\alpha$ -,  $\beta$ -, and  $\gamma$ -tokens' characteristics, respectively.

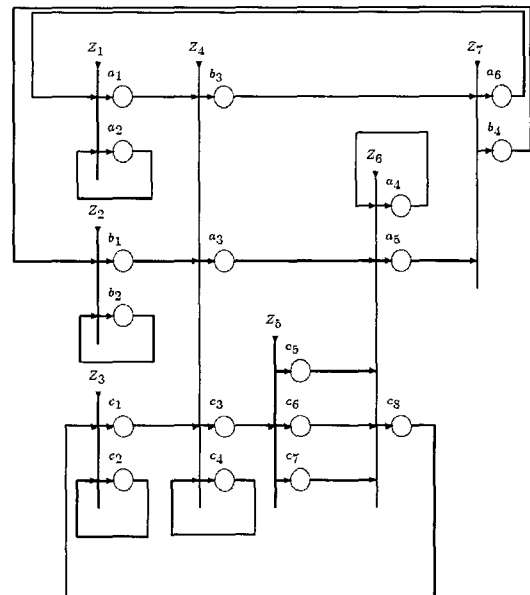


Figure 1

The forms of the transitions are the following.

$$Z_1 = \langle \{a_2, a_6\}, \{a_1, a_2\} \rangle,$$

where:

	a <sub>1</sub>	a <sub>2</sub>
a <sub>2</sub>	W <sub>2,1</sub> <sup>a</sup>	W <sub>2,2</sub> <sup>a</sup>
a <sub>6</sub>	false	true

W<sub>2,1</sub><sup>a</sup> = "The teacher must examine",

W<sub>2,2</sub><sup>a</sup> = ¬W<sub>2,1</sub><sup>a</sup>.

where ¬P is the negation of predicate P.

The α-tokens do not obtain any characteristic in place a<sub>2</sub> and they obtain the characteristic "list of the problems that the student must solve" in place a<sub>1</sub>.

$$Z_2 = \langle \{b_2, b_4\}, \{b_1, b_2\} \rangle,$$

	b <sub>1</sub>	b <sub>2</sub>
b <sub>2</sub>	W <sub>2,1</sub> <sup>b</sup>	W <sub>2,2</sub> <sup>b</sup>
b <sub>4</sub>	false	true

where:

W<sub>2,1</sub><sup>b</sup> = "The problem is included in x<sub>cu</sub><sup>a</sup>",

W<sub>2,2</sub><sup>b</sup> = ¬W<sub>2,1</sub><sup>b</sup>.

The β-tokens do not obtain any characteristic in place b<sub>2</sub> and they obtain the characteristic "texts of the problems that the student must solve" in place b<sub>1</sub>.

$$Z_3 = \langle \{c_2, c_7\}, \{c_1, c_2\} \rangle,$$

	c <sub>1</sub>	c <sub>2</sub>
c <sub>2</sub>	W <sub>2,1</sub> <sup>c</sup>	W <sub>2,2</sub> <sup>c</sup>
c <sub>7</sub>	false	true

where:

W<sub>2,1</sub><sup>c</sup> = "The student must have examination",

W<sub>2,2</sub><sup>c</sup> = ¬W<sub>2,1</sub><sup>c</sup>.

The γ-tokens do not obtain any characteristic in places c<sub>1</sub> and c<sub>2</sub>.

$$Z_4 = \langle \{a_1, b_1, c_1\}, \{a_3, b_3, c_3\} \rangle,$$

	a <sub>3</sub>	b <sub>3</sub>	c <sub>3</sub>
a <sub>1</sub>	true	false	false
b <sub>1</sub>	false	false	false
c <sub>1</sub>	false	false	false

The α- and β-tokens do not obtain any characteristic in places a<sub>3</sub> and b<sub>3</sub>, respectively, while γ-tokens obtain characteristic "student's solutions of the problems" in place c<sub>3</sub>.

$$Z_5 = \langle \{c_3\}, \{c_4, c_5, c_6\} \rangle,$$

	c <sub>4</sub>	c <sub>5</sub>	c <sub>6</sub>
c <sub>3</sub>	W <sub>3,4</sub> <sup>c</sup>	W <sub>3,5</sub> <sup>c</sup>	W <sub>3,6</sub> <sup>c</sup>

where:

W<sub>3,4</sub><sup>c</sup> = "The teacher that will examine the present research prefers First way for evaluation",

W<sub>3,5</sub><sup>c</sup> = "The teacher that will examine the present research prefers Second way for evaluation",

W<sub>3,6</sub><sup>c</sup> = "The teacher that will examine the present research prefers Third way for evaluation".

The γ-tokens enter one of the three output places of transition Z<sub>5</sub> obtaining characteristic "estimation of the current student's problems".

$$Z_6 = \langle \{a_3, a_4, c_4, c_5, c_6\}, \{a_4, a_5, c_7\} \rangle,$$

	a <sub>4</sub>	a <sub>5</sub>	c <sub>7</sub>
a <sub>3</sub>	W <sub>3,4</sub> <sup>a</sup>	W <sub>3,5</sub> <sup>a</sup>	false
a <sub>4</sub>	W <sub>4,4</sub> <sup>a</sup>	W <sub>4,5</sub> <sup>a</sup>	false
c <sub>4</sub>	false	false	true
c <sub>5</sub>	false	false	true
c <sub>6</sub>	false	false	true

where:

W<sub>3,4</sub><sup>a</sup> = W<sub>4,4</sub><sup>a</sup> = "There are students whose research must be evaluated by the current teacher",

$$W_{3,5}^a = W_{4,5}^a = \neg W_{3,4}^a.$$

The α- and γ-tokens do not obtain any characteristic in places a<sub>4</sub>, a<sub>5</sub> and c<sub>7</sub>.

$$Z_7 = \langle \{a_5, b_3\}, \{a_3, b_4\} \rangle,$$

	a <sub>6</sub>	b <sub>4</sub>
a <sub>5</sub>	true	false
b <sub>3</sub>	false	true

When the aim of the model is to describe the process of students' evaluation, tokens α will not obtain any characteristic in place a<sub>6</sub>, while β-tokens obtain the characteristic "estimation of the difficulties of the problems"

in place b<sub>4</sub>. This estimation for the j-th problem can be obtained in the following intuitionistic fuzzy form

$$\langle \mu^p(j), \nu^p(j) \rangle = \left\langle \frac{r}{m}, \frac{s}{m} \right\rangle,$$

where:

r is the number of the students who correctly solved j-th problem,

s is the number of the students who gave a wrong solution to j-th problem.

This estimation can be more precise, if we add information about the current status of the students. For example, if some or all prize students (at least in the area of the problems) have not solved the problem, then it has been really very difficult. In the opposite case, if all or a big part of the students have solved some problem, it has obviously been easy.

We can use the accumulated information for evaluation of teachers, too. Now,  $\alpha$ -tokens can obtain the characteristic "estimation of the teacher's score for the current examination" in place  $a_6$ .

This characteristic can be obtained on the basis of the current and previous students' estimations and by estimating the difficulties of the separate problems.

For example, if some problems have been solved by either all or none of the students, then they have not been determined suitably - their being very difficult or very easy can reflect on the score of the teacher who determined these problems for the examination.

### 3. Conclusions

The so-constructed GN-model gives possibility to simulate some processes, related to estimation of students' background. The present model is an element of a more general model describing different processes, flowing in a university. The authors, together with some colleagues have been preparing an extensive research on this theme.

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