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AUTOMORPHISMS OF A WEYL-TYPE ALGEBRA I

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Abstract. Every non-associative algebra L corresponds to its symmetric semi-Lie algebra $L_{[,]}$ with respect to its commutator. It is an interesting problem whether the equality $Aut_{non}(L) = Aut_{semi-Lie}(L)$ holds or not [2], [13]. We find the non-associative algebra automorphism groups Aut_{non} $(\overline{WN_{0,0,1}}_{[0,1,r_1,\dots,r_p]})$ and $Aut_{semi-Lie}$ $(\overline{WN_{0,0,1}}_{[0,1,r_1,\dots,r_p]})$, where every automorphism of the automorphism groups is the composition of elementary maps [3], [4], [7], [8], [9], [10], [11]. The results of the paper show that the **F**-algebra automorphism groups of a polynomial ring and its Laurent extension make easy to find the automorphism groups of the algebras in the paper.

1. Preliminaries

Let $\mathbf{N}$ be the set of all non-negative integers and $\mathbf{Z}$ be the set of all integers. Let $\mathbf{F}$ be a field of characteristic zero. Let $\mathbf{F}^\bullet$ be the multiplicative group of non-zero elements of $\mathbf{F}$. Let $\mathbf{F}[x_1, \dots, x_{m+s}]$ be the polynomial ring with the variables $x_1, \dots, x_{m+s}$. Let $g_1, \dots, g_n$ be given polynomials in $\mathbf{F}[x_1, \dots, x_{m+s}]$. For $n, m, s \in \mathbf{N}$, let us define the commutative, associative $\mathbf{F}$-algebra $F_{g_n,m,s} = \mathbf{F}[e^{\pm g_1}, \dots, e^{\pm g_n}, x_1^{\pm 1}, \dots, x_m^{\pm 1}, x_{m+1}, \dots, x_{m+s}]$ which is called a stable algebra in the paper [5] with the standard basis

$$\mathbf{B} = \{e^{a_1 g_1} \cdots e^{a_n g_n} x_1^{i_1} \cdots x_{m+s}^{i_{m+s}} | a_1, \dots, a_n, i_1, \dots, i_m \in \mathbf{Z}, \\ i_{m+1}, \dots, i_{m+s} \in \mathbf{N}\}$$

and with the obvious addition and the multiplication [5], [6], [9], where we take appropriate $g_1, \dots, g_n$ so that $\mathbf{B}$ can be the standard basis of $F_{g_n,m,s}$. ∂_w, $1 \le w \le m+s$, denotes the usual partial derivative with respect to x_w on $F_{g_n,m,s}$. For partial derivatives $\partial_u, \dots, \partial_v$ of $F_{g_n,m,s}$,

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the composition $\partial_u^{j_u} \circ \cdots \circ \partial_v^{j_v}$ of them is denoted by $\partial_u^{j_u} \cdots \partial_v^{j_v}$, where $j_u, \ldots, j_v \in \mathbf{N}$. Note that $\partial_v^0(f) = f$ for any $f \in F_{g_n,m,s}$. Let D be the set

$$\{\partial_u^{j_u} \cdots \partial_v^{j_v} | 1 \leq u, \ldots, v \leq m+s, j_u, \ldots, j_v \in \mathbf{N}\}.$$

Let us define the vector space $WN(g_n, m, s)$ over $\mathbf{F}$ which is spanned by the standard basis

$$\{e^{a_1 g_1} \cdots e^{a_n g_n} x_1^{i_1} \cdots x_{m+s}^{i_{m+s}} \partial_u^{j_u} \cdots \partial_v^{j_v} | a_1, \ldots, a_n, i_1, \ldots, i_m \in \mathbf{Z},$$
$$i_{m+1}, \ldots, i_{m+s} \in \mathbf{N}, j_u, \ldots, j_v \in \mathbf{N}, 1 \leq u, \ldots, v \leq m+s\}. \tag{1}$$

Thus we may define the multiplication $*$ on $WN(g_n, m, s)$ as follows:

$$e^{a_{11} g_1} \cdots e^{a_{1n} g_n} x_1^{i_{11}} \cdots x_{m+s}^{i_{1,m+s}} \partial_u^{j_u} \cdots \partial_v^{j_v} * e^{a_{21} g_1} \cdots e^{a_{2n} g_n}$$
$$x_1^{i_{21}} \cdots x_{m+s}^{i_{2,m+s}} \partial_h^{j_h} \cdots \partial_w^{j_w} = e^{a_{11} g_1} \cdots e^{a_{1n} g_n} x_1^{i_{11}} \cdots x_{m+s}^{i_{1,m+s}}$$
$$\partial_u^{j_u} \cdots \partial_v^{j_v} (e^{a_{21} g_1} \cdots e^{a_{2n} g_n} x_1^{i_{21}} \cdots x_{m+s}^{i_{2,m+s}}) \partial_h^{j_h} \cdots \partial_w^{j_w} \tag{2}$$

for any basis elements $e^{a_{11} g_1} \cdots e^{a_{1n} g_n} x_1^{i_{11}} \cdots x_{m+s}^{i_{1,m+s}} \partial_u^{j_u} \cdots \partial_v^{j_v}$ and $e^{a_{21} g_1} \cdots e^{a_{2n} g_n}\ x_1^{i_{21}} \cdots x_{m+s}^{i_{2,m+s}} \partial_h^{j_h} \cdots \partial_w^{j_w} \in WN(g_n, m, s)$ [8]. Thus we can define the Weyl-type non-associative algebra $\overline{WN_{g_n,m,s}}$ with the multiplication $*$ in (2.3) and with the set $WN(g_n, m, s)$ [1], [3], [4], [8], [13], [14]. For $r \in \mathbf{N}$, let us define the non-associative subalgebra $\overline{WN_{g_n,m,s_r}}$ of the non-associative algebra $\overline{WN_{g_n,m,s}}$ spanned by

$$\{e^{a_1 g_1} \cdots e^{a_n g_n} x_1^{i_1} \cdots x_s^{i_s} \partial_u^{j_u} \cdots \partial_v^{j_v} | a_1, \ldots, a_n, i_1, \ldots, i_m \in \mathbf{Z},$$
$$i_{m+1}, \ldots, i_s \in \mathbf{N}, j_u, \ldots, j_v \in \mathbf{N},$$
$$j_u + \cdots + j_v \leq r, 1 \leq u, \ldots, v \leq m+s\}. \tag{3}$$

The the non-associative subalgebra $\overline{WN_{g_n,m,s_1}}$ of the non-associative algebra $\overline{WN_{g_n,m,s}}$ is the non-associative algebra $\overline{N_{g_n,m,s}}$ in the paper [1]. Generally, there is no left or right identity of $\overline{WN_{g_n,m,s}}$. The the non-associative algebra $\overline{WN_{g_n,m,s}}$ is $\mathbf{Z}^n$-graded as follows:

$$\overline{WN_{g_n,m,s}} = \bigoplus_{(a_1,\ldots,a_n)} WN_{(a_1,\ldots,a_n)}, \tag{4}$$

where $WN_{(a_1,\ldots,a_n)}$ is the vector subspace of $\overline{WN_{g_n,m,s}}$ with the standard basis

$$\{e^{a_1 g_1} \cdots e^{a_n g_n} x_1^{i_1} \cdots x_{m+s}^{i_{m+s}} \partial_u^{j_u} \cdots \partial_v^{j_v} | i_1, \ldots, i_m \in \mathbf{Z}, i_{m+1}, \ldots, i_{m+s},$$
$$j_u, \ldots, j_v \in \mathbf{N}, 1 \leq u, \ldots, v \leq m+s\}.$$

An element in $WN_{(a_1,\ldots,a_n)}$ is called an $(a_1, \ldots, a_n)$-homogenous element and $WN_{(a_1,\ldots,a_n)}$ is called the $(a_1, \ldots, a_n)$-homogeneous component. For any basis element $e^{a_1 g_1}\cdots e^{a_n g_n} x_1^{i_1}\cdots x_{m+s}^{i_{m+s}}\,\partial_t$ of $\overline{WN_{g_n,m,s}}$, let us define the homogeneous degree $deg_N(e^{a_1 g_1}\cdots e^{a_n g_n} x_1^{i_1}\cdots\ x_{m+s}^{i_{m+s}}\partial_u^{j_u}\cdots\partial_v^{j_v})$ of it as follows:

$$deg_N(e^{a_1 g_1}\cdots e^{a_n g_n} x_1^{i_1}\cdots x_{m+s}^{i_{m+s}}\partial_u^{j_u}\cdots\partial_v^{j_v}) = \sum_{u=1}^{m+s}|i_u|,$$

where $|i_u|$ is the absolute value of i_u, $1 \le u \le m+s$. Throughout this paper, for any basis element $e^{a_\mu g_\mu}\cdots e^{a_\nu g_\nu} x_\lambda^{i_\lambda}\cdots x_\sigma^{i_\sigma}\partial_u^{j_u}\cdots\partial_v^{j_v}$, we write it such that $1 \le \mu \le \cdots \le \nu \le n$, $1 \le \lambda \le \cdots \le \sigma \le m$, and $1 \le u \le \cdots \le v \le m+s$. For any element $l \in \overline{WN_{g_n,m,s}}$, we may define $deg_N(l)$ as the highest homogeneous degree of the basis terms of l. Thus for any basis elements l_1 and l_2 of $\overline{WN_{0,0,s}}$, we may write $l_1 + l_1$ or $l_2 + l_1$ well orderly with unambiguity. For any element $l \in \overline{WN_{0,0,s}}$, we may define $deg_N(l)$ as the highest homogeneous degree of each monomial of l. For any $l \in \overline{WN_{g_n,m,s}}$, let us define $\#(l)$ as the number of different homogeneous components of l. $\overline{WN_{n,m,s}}$ (resp. $\overline{WN_{g_n,m,s_r}}$) has the subalgebra WT (resp. WT_r) spanned by $\{\partial_u^{j_u}\cdots\partial_v^{j_v}|(\text{resp. } j_u+\cdots+j_v \le r,)\ \ j_u,\ldots,j_v \in \mathbf{N}, 1 \le u,v \le s\}$ which is the right annihilator of $\overline{WN_{g_n,m,s}}$. For a subset D_1 of the set D, let us define the non-associative subalgebra $\overline{WN_{g_n,m,s}}_{D_1}$ of the non-associative algebra $\overline{WN_{g_n,m,s}}$ with the set

$$\{f\partial | f \in F_{g_n,m,s}, \partial \in D_1\}$$

Since the non-associative algebra $\overline{WN_{g_n,m,s}}$ is $\mathbf{Z}^n$-graded, $\overline{WN_{g_n,m,s}}_{D_1}$ is $\mathbf{Z}^n$-graded. A non-associative algebra A is simple, if it has no proper two sided ideal which is not zero ideal [14]. For any element l in a non-associative algebra A, l is full, if the ideal generated by l is A. Generally, the algebra $\overline{WN_{g_n,m,s}}_{[0,r]}$ or $\overline{WN_{g_n,m,s_r}}$ is not Lie admissible [1], [9], since the Jacobi identity does not hold using the commutator of the non-associative algebra $\overline{WN_{g_n,m,s}}_{[0,r]}$ or the non-associative algebra $\overline{WN_{g_n,m,s_r}}$ for $r > 1$. For any $\mathbf{F}$-algebra A and an element $l \in A$, an element $l_1 \in A$ is a left (resp. right) stabilizing element of l, if $l_1 * l = cl$ (resp. $l * l_1 = cl$), where $c \in \mathbf{F}$. For any element $l_1 \in A$, $l \in A$ is a locally left (resp. right) unity of $l_1 \in A$, if $l * l_1 = l_1$ (resp. $l_1 * l = l_1$) holds and throughout the paper, we read it as that l is a left unity of l_1, etc.. A semi-Lie algebra enjoys similar results of a Lie algebra except a result which requires the Jacobi identity [3], [12], [13], [14]. A semi-Lie algebra is self-centralizing, if for any element of A, the dimension of its centralizer in A is one. If $|D_1| \ne 1$, then $\overline{WN_{0,0,1}}_{D_1}$ has no right identity and if $|D_1| = 1$, then $\overline{WN_{0,0,1}}_{D_1}$ has a right identity. $D_1 = \{0\}$ if and only if $\overline{WN_{0,0,1}}_{D_1}$ has the (two-sided) identity, i.e., the algebra is the polynomial ring.

2. Automorphism groups

Throughout this section, we put $r_1 < \cdots$ non-associative algebra $\overline{WN_{0,1,0}}_{[0,r_1,\ldots,r_p]}$ and its subalgebras. The non-associative algebras

$$\overline{WN_{0,0,1}}_{[0,r_1,\ldots,r_p]}, \overline{WN_{0,0,1}}_{[r_1,\ldots,r_p]}, \overline{WN_{0,1,0}}_{[r_1,\ldots,r_p]}, \overline{WN_{0,1,0}}_{[0,r_1,\ldots,r_p]}$$

and their corresponding semi-Lie algebras are simple.

LEMMA 2.1. *For any non-associative algebra isomorphism θ of $\overline{WN_{0,0,1}}_{[0,r_1,\ldots,r_p]}$ (resp. $\overline{WN_{0,1,0}}_{[0,r_1,\ldots,r_p]}$), $\theta(c) = c$ for any $c \in \mathbf{F}$.*

PROOF. Since 1 is the identity element of $\mathbf{F}[x]$ (resp. $\mathbf{F}[x^{\pm 1}]$,) the proof is straightforward. So let us omit it. □

LEMMA 2.2. *For any non-associative algebra automorphism θ of $\overline{WN_{0,0,1}}_{[0,1,r_1,\ldots,r_p]}$, $\theta(\partial^u) = c_u\partial^u$ holds, where c_u is a non-zero scalar for $u \in \{0, 1, r_1, \ldots, r_p\}$.*

PROOF. Let θ be the non-associative algebra automorphism θ of $\overline{WN_{0,0,1}}_{[0,1,r_1,\ldots,r_p]}$. Note that $\theta(\mathbf{F}[x]) \subset \mathbf{F}[x])$ and $\theta(\overline{WN_{0,0,1}}_{[r_1,\ldots,r_p]}) \subset \overline{WN_{0,0,1}}_{[r_1,\ldots,r_p]}$. Since the dimension of the right annihilator of ∂ in $\overline{WN_{0,0,1}}_{[1,r_1,\ldots,r_p]}$ is $1 + r_p$, $\theta(\partial) = c_1\partial$ is obvious, where $c_1 \in \mathbf{F}^\bullet$. Since $\mathbf{F}[x]$ is an integral domain and $\theta(\partial * x) = 1$, we have that $\theta(x) = \frac{x}{c_1} + d$ with appropriate scalars. This implies that

$$\theta(x^k) = (\frac{x}{c_1} + d)^k \tag{5}$$

for $k \in \mathbf{N}$. Because of the dimension of the right annihilator of ∂^u and by (5), we have that $\theta(\partial^u) = c_u\partial^u$, where $c_u \in \mathbf{F}^\bullet$, $u \in \{1, r_1, \ldots, r_p\}$, and $c_0 = 1$. This completes the proof of the lemma. □

Note 1. For any $c_1 \in \mathbf{F}^\bullet$ and $d \in \mathbf{F}$, let us define an elementary $\mathbf{F}$-map $\theta_{c_1,d}$ of $\overline{WN_{0,0,1}}_{[0,1,r_1,\ldots,r_p]}$ as follows:

$$\theta_{c_1,d}(x^u\partial^v) = c_v(\frac{x}{c_1} + d)^u\partial^v \tag{6}$$

then $\theta_{c_1,d}$ can be linearly extended to a non-associative algebra automorphism of $\overline{WN_{0,0,1}}_{[0,1,r_1,\ldots,r_p]}$, where $c \in \mathbf{F}$ and $c_v = c_1^v$, $v \in \{1, r_1, \ldots, r_p\}$.

LEMMA 2.3. *For any non-associative algebra automorphism θ of $\overline{WN_{0,0,1}}_{[0,1,r_1,\ldots,r_p]}$, $\theta = \theta_{c_1,d}$ in Note 1.*

PROOF. Let θ be the automorphism in the lemma. By Lemma 2.2, $\theta(\partial^u) = c_u\partial^u$ holds, where c_u is a non-zero scalar for $u \in \{0,1,r_1,\ldots,r_p\}$. By (5), $\theta(x^k) = (\frac{x}{c}+d)^k$ for $k \in \mathbf{N}$ and $d \in \mathbf{F}$. By $\theta(x^k * \partial^u) = \theta(x^k\partial^u)$, we have that $\theta(x^k\partial^u) = c_u(\frac{x}{c_1}+d)^k\partial^u$. By $\theta(\partial^u * x^u\partial) = u!\theta(\partial)$, we have that $c_u = c_1^u$ for $u \in \{0,1,r_1,\ldots,r_p\}$ and $c_0 = 1$. This completes the proof of the lemma. □

THEOREM 2.1. *The algebra automorphism group*

$$Aut_{\mathbf{F}}(\overline{WN_{0,0,1}}_{[0,1,r_1,\ldots,r_p]})$$

is generated by $\theta_{c_1,d}$ in Note 1 with appropriate scalars.

PROOF. The proof of the theorem straightforward by Lemma 2.2, Lemma 2.3, Lemma 2.4, and Note 1. Let us omit it. □

LEMMA 2.4. *For any θ in $Aut_{\mathbf{F}}(\overline{WN_{0,1,0}}_{[0,1,r_1,\ldots,r_p]})$, $\theta(x\partial) = x\partial$.*

PROOF. Let θ be an automorphism of $\overline{WN_{0,1,0}}_{[0,1,r_1,\ldots,r_p]}$. By (5), for $k \in \mathbf{N}$, we have that $\theta(x^k) = (\frac{x}{c}+d)^k$ for $c \in \mathbf{F}^\bullet$ and $d \in \mathbf{F}$. Since x is invertible in $\mathbf{F}[x^{\pm 1}, y]$ with respect to the usual multiplication of $\mathbf{F}[x^{\pm 1}, y]$, we have that $d = 0$. Thus the remaining proof of the lemma is similar to the proof of Lemma 2.3. Let us omit it. □

PROPOSITION 2.1. *The algebra automorphism group*

$$Aut_{\mathbf{F}}(\overline{WN_{0,1,0}}_{[0,1,r_1,\ldots,r_p]})$$

is generated by $\theta_{c_1,0}$ in Note 1 with appropriate scalars.

PROOF. The proof of the proposition straightforward by Theorem 2.1 and Lemma 2.4. Let us omit it. □

LEMMA 2.5. *For any non-associative algebra automorphism θ of*

$$\overline{WN_{0,0,1}}_{[r_1,r_2]}, \theta(\partial^u) = c_u\partial^u$$

holds, where c_u is a non-zero scalar for $u \in \{r_1, r_2\}$, where r_1 and r_2 are positive integers.

PROOF. Let θ be the non-associative algebra automorphism θ of $\overline{WN_{0,0,1}}_{[r_1,r_2]}$. Since the dimension of the right annihilator of ∂^{r_1}, $\theta(\partial^{r_1}) = c_1\partial^{r_1}$ is obvious, where $c_1 \in \mathbf{F}^\bullet$. Because of the dimension of the right annihilator of ∂^{r_1}, we have that $\theta(\partial^{r_1}) = c_1\partial^{r_1}$, where $c_1 \in \mathbf{F}^\bullet$. By $c_1\partial^{r_1} * \theta(x^{r_1}\partial^{r_1}) = r_1! c_1 \partial^{r_1}$, we also have that

$$\theta(x^{r_1}\partial^{r_1}) = c_1^{r_1}(\frac{x}{c_1} + d)^{r_1} + \#,$$

where # is the sum of the remaining terms of $\theta(x^{r_1}\partial^{r_1})$ and its degree is less than r_1 with appropriate scalars. This implies $\theta(\partial^{r_2}) = c_2\partial^{r_2}$, where $c_2 \in \mathbf{F}^\bullet$. This completes the proof of the lemma.

LEMMA 2.6. *For any non-associative algebra automorphism θ of*

$$\overline{WN_{0,0,1}}_{[1,2]}, \theta(\partial^u) = c_u\partial^u$$

holds, where c_u is a non-zero scalar for $u \in \{1, 2\}$.

PROOF. Let θ be the non-associative algebra automorphism θ of $\overline{WN_{0,0,1}}_{[1,2]}$. Since $x\partial$ is an idempotent and it is a right identity of ∂, by Lemma 2.5, we have that $\theta(x\partial) = c_1(\frac{1}{c_1}x + d)\partial$ with appropriate scalars. By $c_1\partial * \theta(x^2\partial) = 2c_1(\frac{1}{c_1}x + d)\partial$, we also have that

$$\theta(x^2\partial) = c_1^2(\frac{1}{c_1}x + d)^2\partial + d_1\partial + d_2\partial^2 \tag{7}$$

with appropriate scalars. By (7) and $\theta(\partial^2 * x^2\partial) = 2c_1\partial$, we have that $c_1 = c_2$, i.e., $\theta(\partial^2) = c_1\partial^2$. Since $x\partial$ is a right identity and ∂^2 annihilates $x\partial$, we have that $d_2 = 0$. By $\theta(x\partial^2 * x^2\partial) = 2\theta(x^2\partial)$, we have that $d_1 = 0$. Since $x^2\partial$ and ∂ generates the non-associative subalgebra $\overline{WN_{0,0,1}}_{[1]}$ of $\overline{WN_{0,0,1}}_{[1,2]}$, We can prove that

$$\theta(x^k\partial) = c_1^k(\frac{1}{c_1}x + d)^k\partial \tag{8}$$

for any $k \in \mathbf{N}$. Since $x\partial^2$ annihilates itself, by $c_1\partial * \theta(x\partial^2) = c_1\partial^2$, we have that $\theta(x\partial^2) = c_1(\frac{1}{c_1}x + d_4)\partial^2$, where $d_4 \in \mathbf{F}$. By $\theta(x\partial^2 * x^3\partial) = 6\theta(x^2\partial)$, we also have that $d_4 = d$. By induction on k of $x^k\partial^2$, $k \in \mathbf{N}$, we can also prove that

$$\theta(x^k\partial^2) = c_1^k(\frac{1}{c_1}x + d)^k\partial^2. \tag{9}$$

This implies that $\theta = \theta_{c_1,d}$ which is defined in Note 2. This completes the proof of the lemma. □

Note 2. For any $c_1 \in \mathbf{F}^\bullet$ and $d \in \mathbf{F}$, let us define an elementary $\mathbf{F}$-map $\theta_{c_1,d}$ of $\overline{WN_{0,0,1}}_{[1,2]}$ as follows:

$$\theta_{c_1,d}(x^u\partial^v) = c_v(\frac{x}{c_1} + d)^u\partial^v \tag{10}$$

then $\theta_{c_1,d}$ can be linearly extended to a non-associative algebra automorphism of $\overline{WN_{0,0,1}}_{[1,2]}$, where $c_v \in \mathbf{F}^\bullet$ and $c_v = c_1^v$, $v \in \{1,2\}$.

THEOREM 2.2. *The algebra automorphism group*

$$Aut_{non}(\overline{WN_{0,0,1}}_{[1,2]})$$

is generated by $\theta_{c_1,d}$ *in Note 2 with appropriate scalars.*

PROOF. The proof of the theorem is similar to the proof of Theorem 2.1, so let us omit it. □

PROPOSITION 2.2. *Any non-zero algebra endomorphism* θ *of*

$$\overline{WN_{0,0,1}}_{[1,2]}$$

is surjective.

PROOF. Since $\overline{WN_{0,0,1}}_{[1,2]}$ is simple, the endomorphism in the proposition is injective. The remaining proof of the proposition is straightforward by reviewing the proof of Theorem 2.2. So let us omit it. □

Since the semi-Lie algebra $\overline{WN_{0,0,1}}_{[1,2]_{[,]}}$ is self-centralizing [6], the algebra enjoys similar results of Lemma 2.5, Lemma 2.6 and Note 2, thus we have the following theorem.

THEOREM 2.3. *The semi-Lie algebra automorphism group*

$$Aut_{semi-Lie}(\overline{WN_{0,0,1}}_{[1,2]_{[,]}})$$

is generated by $\theta_{c_1,d}$ *in Note 2 with appropriate scalars.*

PROOF. The proof of the theorem is similar to the proof of Theorem 2.1, so let us omit it. □

Since the semi-Lie algebras $\overline{WN_{0,0,1}}_{[0,1,\dots,r]_{[,]}}$, $\overline{WN_{0,0,1}}_{[0,r_1,\dots,r_p]_{[,]}}$ and $\overline{WN_{0,1,0}}_{[1,2]_{[,]}}$ is self-centralizing [6], we have a similar results of Theorem 2.3 of the above semi-Lie algebras. Because of the dimensions of right annihilators of ∂^k, $k \in \mathbf{N}$, it is an interesting problem to find the similar formula of (9). Thus we have the following open questions:

Question 1. Find the non-associative algebra automorphism group $Aut_{non}(\overline{WN_{0,0,1}}_{[1,r_1,\dots,r_p]})$ of the non-associative algebra $\overline{WN_{0,0,1}}_{[1,r_1,\dots,r_p]}$.

Question 2. Find the non-associative algebra automorphism group $Aut_{non}(\overline{WN_{0,0,1}}_{[r_1,\dots,r_p]})$ of the non-associative algebra $\overline{WN_{0,0,1}}_{[r_1,\dots,r_p]}$ such that $r_1 \geq 1$ and $p > 1$.

References

[1] M. H. Ahmadi, K. -B. Nam, and J. Pakinathan, *Lie admissible non-associative algebras*, Algebra Colloquium, Vol. 12, No. 1, World Scientific, (March) 2005, 113–120.

[2] A. A. Albert, *Power-Associative Rings*, Trans. Amer. Math. Soc. **64**, 552–593 (1948).

[3] S. H. Choi and K. -B. Nam, *The Derivation of a Restricted Weyl Type Non-Associative Algebra*, Hadronic Journal **28** (2005), no. 3, 287–295.

[4] ———, *Derivations of a restricted Weyl Type Algebra I*, Accepted, Rocky Mountain Journal of Mathematics, 2005.

[5] V. G. Kac, *Description of Filtered Lie Algebra with which Graded Lie algebras of Cartan type are Associated*, Izv. Akad. Nauk SSSR, Ser. Mat. Tom. **38** (1974), 832–834.

[6] N. Kawamoto, A. Mitsukawa, K. -B. Nam, and M. -O. Wang, *The automorphisms of generalized Witt type Lie algebras*, Journal of Lie Theory, 13 Vol(2), Heldermann Verlag, 2003, 571–576.

[7] K. -B. Nam, *On Some Non-Associative Algebras Using Additive Groups*, Southeast Asian Bulletin of Mathematics, Vol. 27, Springer Verlag, 2003, 493–500.

[8] K. -B. Nam and S. H. Choi, *On Evaluation Algebras*, SEAMS Bull Mathematics **29** (2005), no. 2, 381–385.

[9] ———, *On the Derivations of Non-Associative Weyl-type Algebras*, Appear, Southeast Asian Bull. Math., 2005.

[10] K. -B. Nam, Y. Kim and M. -O. Wang, *Weyl-type Non-Associative Algebras I*, IMCC Proceedings, 2004, SAS Publishers, 147–155.

[11] K. -B. Nam and M. -O. Wang, *Notes on Some Non-Associative Algebras*, Journal of Applied Algebra and Discrete Structured, Vol 1, No. 3, 159–164.

[12] A. N. Rudakov, *Groups of Automorphisms of Infinite-Dimensional Simple Lie Algebras*, Math. USSR-Izvestija **3** (1969), 707–722.

[13] R. M. Santilli, *An introduction to Lie-admissible algebras*, Nuovo Cimento Suppl. **6** (1968), no. 1, 1225–1249.

[14] R. D. Schafer, *Introduction to nonassociative algebras*, Dover, 1995, 128–138.

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