Definition and Correlation for Spray Angle in Non-Reacting Diesel Fuel Sprays

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Key Words: Macroscopic spray characteristics, Definition of spray angle, Correlation for spray angle, Diesel fuel spray

Abstract

Of the macroscopic spray characteristics of non-reacting diesel fuel sprays, the spray angle reflects directly the atomization and air entrainment processes downstream the nozzle. In addition, spray angle is important because it will be closely related to the spray penetration. The existing definitions for the measurement of spray angle as well as the correlations for the prediction of spray angle are, therefore, summarized and reviewed. The existing definition of spray angle can be classified into four groups: distance based on orifice diameter, distance based on spray tip penetration, definition based on surface wave, and definition based on atomization. It is strongly required to specify the definition and measurement method when the data for spray angle is reported. The existing correlations for spray angle can be classified into two groups: theoretical and empirical correlations. The study on the evaluation of the existing correlations for spray angle is required.

1. Introduction

A clean and efficient combustion of fuel in a Diesel engine depends on the quality of fuel-air mixture prepared in the fuel injection process. The fuel-air mixture process is mainly affected by the spray characteristics, which depends on several parameters. Spray characteristics can be classified into three groups: macroscopic, microscopic and atomization characteristics. The macroscopic spray characteristic involves the global spray features such as spray (cone) angle, spray (tip) penetration and air entrainment by the spray. The internal dynamics of the spray such as mean diameter, velocity, density and size distribution of droplet belong to the microscopic spray characteristic. Atomization characteristic includes breakup length and breakup time of liquid jet.

Of the macroscopic spray characteristics, the spray angle reflects directly the atomization and air entrainment processes downstream the nozzle. The spray angle is important because it will be closely related to the spray penetration. Therefore, most of theoretical correlations for the prediction of spray penetration such as fuel spray model by Wakuri et al.(25), modified fuel spray model by Naber and Siebers(13), cone model by Schihl et al.(21), two-phase flow model by Sazhin et al.(22) and momentum flux conservation model by Desantes, et al.(14) include the spray angle term as well as the ambient gas density, the injection pressure or initial velocity of the spray, the time after the start of injection, the orifice diameter etc.

Arregle et al.(11) pointed out in their conclusion for the effect of the injection parameters on macroscopic and microscopic diesel spray characteristics generated by a common-rail injection system that even though a simple dimensional analysis and the existing correlation gave a reasonably fair prediction of the spray penetration, accurate measurement of the spray angle is crucial.

The possible causes for the different measurement result of spray angle in non-reacting diesel fuel

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sprays will be the photographic techniques, image analysis methods and the definition of spray angle. In addition, the origin for the different prediction results of spray angle in non-reacting diesel fuel sprays will be the parameters considered in the correlations and the derivation background (theoretical or empirical) for the prediction of spray angle.

There exist several different definitions for the measurement of spray angle determined from spray produced by plain-orifice atomizers: In addition, the correlations for the prediction of spray angle are not well summarized. Therefore, it is required to evaluate the existing definition of spray angle as well as the correlations for the spray angle. The purpose of this study is to review the existing definitions and correlations for spray angle in non-reacting diesel fuel sprays.

2. Definition of spray angle

Photographic methods are usually employed to determine spray angle. There exist several different photographic methods such as shadowgraphy, schlielen etc. Even though several photographic methods are summarized by ILASS-Japan(10), it is required to compare the uncertainty from the results for different photographic methods.

It is clear from Ballester and Dopazo(12) that the exposure time in the photographic technique for the measurement of spray angle strongly affects the spray angle for the same spray. It is also evident from Naber and Siebers(13) that the selection of intensity threshold level in the film image analysis remarkably affects the measurement of spray angle.

The spray angle of a plain-orifice atomizer is normally defined as the angle formed by two straight lines drawn from the discharge orifice to the outer periphery of the spray at a fixed distance downstream of the nozzle. The spray angle is clearly influenced by a fixed distance downstream of the nozzle which can be classified into two groups: distance based on orifice diameter and distance based on spray tip penetration. On the other hand, there are several studies regarding the definitions of spray angle based on surface wave of spray and based on atomization of spray. Therefore, the existing definition of spray angle can be summarized as four groups as follows.

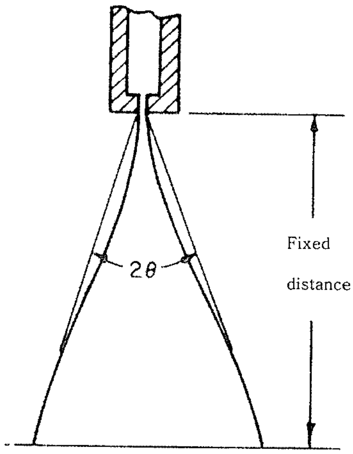

Fig. 1 Definition of spray angle based on orifice diameter

2.1 Distance based on orifice diameter

Hiroyasu and Arai(8) had measured the spray angle at the distance 100 diameters downstream of the nozzle orifice and suggested the empirical correlation shown in Table 2 in this paper. Later Arai et al.(19) defined the spray angle as the angle formed by two straight lines drawn from the discharge orifice to the outer periphery of the spray at a distance 60 diameters downstream of the nozzle.

In the study for the effect of nozzle geometry, flow conditions, and manufacturing imperfections on spray angle, Ohrn et al.(23) have measured the spray angle and the width of the spray at an axial distance of 350 diameters downstream of the nozzle exit. They maintained the orifice diameter constant at 0.254 mm. They reported the uncertainty in the spray angle as ± 0.5.

On the other hand, an experimental study was conducted by Varde(17) to determine the effects of nozzle orifice size and operating parameters on the spray angle of diesel spray injected into gaseous environment. From his experiment, an average spray angle for each operating condition for the fully developed spray, where the axial length of the spray was at least

80 mm, was evaluated. When this distance is converted to distance based on orifice diameter, the maximum is 444 diameters (orifice diameter of 0.18 mm) and the minimum is 151 diameters (orifice diameter of 0.53 mm). High fuel pressures in the range of 55 to 130 MPa and ambient gas density ranging from atmospheric to 40 kg/m^3 were introduced in this study.

From the above discussion, existing distances based on orifice diameter for the measurement of spray angle are 60, 100, 151, 350 and 444 diameters. Even though it is easy to measure spray angle by this definition, it is well known that the results for spray angle are widely different according to the measurement point from the nozzle tip. In addition, the background for the selection of the fixed distance from the nozzle tip is not clearly explained.

2.2 Distance based on spray tip penetration

The spray angle reflects directly the atomization and air entrainment processes downstream the nozzle hole. Hence it will be closely linked to the global spray behaviour and particularly to its penetration. Therefore, there exist several studies to introduce the fixed distance downstream of the nozzle based on spray tip penetration.

According to Naber and Siebers(13), the spray angle θ is defined by the following relationship:

$$\tan\theta = A_{p,S/2}/(S/2)^2 \qquad (1)$$

where $A_{p,S/2}$ is the projected spray area of the upstream half of the spray in an image. The spray angle and penetration require an iterative process to evaluate, since the definition used for each depends on the other. The penetration is defined as the distance along the spray axis to a location where 1/2 of the pixels on an arc of $\theta/2$ centered on the spray axis are dark.

In the definition by Pastor et al.(15), two straight lines are fitted to the first 60% of the spray contour closest to the nozzle, where the spray is assumed to behave like a steady spray. The spray angle is defined as the angle of these two straight lines. The spray tip penetration is defined as the distance from the origin point of the spray to the intersection point between the contour and the bisector of the spray angle. It is well known that there are two different regions in non-reacting diesel fuel spray. One region is a conical-shape region from the nozzle to 60% or 70% of the total penetration and so-called steady region. The other region is an elliptical-shape region, which is called the transient region or unsteady region(14).

Recently, Delacourt et al.(2) reported the different definition of distance based on spray tip penetration as shown in Fig. 2.

Their method consist of describing a circle arc C_r for each value of r ($r_{min} < r < r_{max}$) starting from the point M_r and on both sides of the spray axis. They selected r_{min} with 150% of the needle radius for starting radius and r_{max} with 70% of spray tip penetration for ending radius. The intersection point (Q_r and P_r) of C_r with spray periphery are the detected by using a luminance threshold. By summing up the two angles obtained by θ'_r and θ'_{r+}, they calculate the spray aperture angle θ'_r which means the projected spray angle for the radius r. The angle θ' is the average of all the θ'_r obtained. Then they introduced the spray cone angle of the injector needle α as follows.

$$\theta = 2\arcsin\left(\sin\left(\frac{\theta'}{2}\right)\sin\left(\frac{\alpha}{2}\right)\right)$$

$$\theta' = \frac{1}{(Nr_{max} - Nr_{min})}\sum_{r=r_{min}}^{r_{max}} \theta'_r \qquad (2)$$

where $(Nr_{\max} - Nr_{\min})$ represents the equivalent number of pixels to the distance $(r_{\max} - r_{\min})$.

In this case, the background of selection for starting radius (150% of the needle radius) and ending radius (70% of spray tip penetration) is not clearly

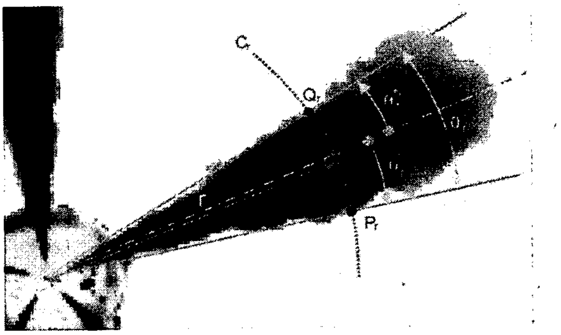

Fig. 2 Definition of spray angle by Delacourt et al.(2)

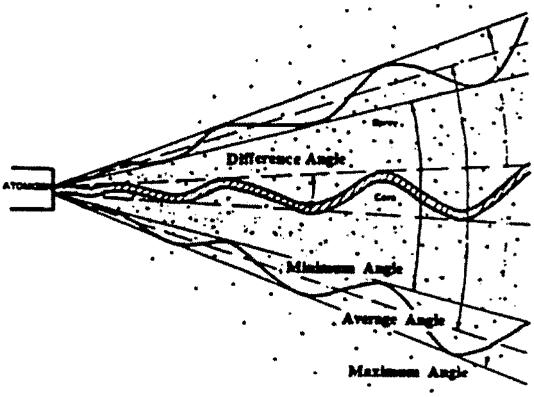

Fig. 3 Definition of spray angle by Ruiz and Chigier(3)

explained. In addition, the designation of spray cone angle and spray angle in this study is not widely accepted one.

2.3 Definition based on surface wave

The average of the angle that follows the peaks of surface waves in the outer periphery of the spray and the angle that follows the valleys was defined as the representative spray angle by Ruiz and Chigier(3) as shown in Fig. 3. This concept is introduced by Ishigawa and Tsujimura(20) for the measurement of two spray angles near the nozzle exit.

2.4 Definition based on atomization

Hiroyasu and Arai(9) defined the spray angle θ as the maximum angle of a cone which is fixed at the nozzle exit as shown in Fig. 4. Angle of a deformation cone θ_d and angle of a spray cone θ_s was also defined from the effective origins of the deformation and spray cones located at the distance Ld and Ls respectively from the nozzle exit for the incomplete and complete sprays.

On the other hand, Dan et al.(24) proposed that diesel spray is divided into two spatial regime, namely, "Spray cone angle" regime and "Spray angle" regime based on the internal structure of spray. The spray cone angle regime, defined with 2 < z < 10 mm, is formed by the fuel jet in which turbulence is initiated in the nozzle. The spray angle regime, defined with z > 20~40 mm, is formed by the gas vortex stream in which motion is induced by the liquid jet. In addition, characteristic length Lm, defined by the length from the nozzle tip to the 90% intensity of the scattering signals on the spray axis, was proposed as the divergence point of the spray cone angle regime and the spray angle regime.

Furthermore, Ishikawa and Tsujimura(20) defined two spray spreading angles depending on the observation distance from the nozzle orifice exit (0 < z < 7

Table 1 Theoretical correlations for spray angle in plain-orifice atomizer

Investigators	Correlations	Remarks
Wakuri et al. (1960)	$\tan\frac{\theta}{2}=\frac{1}{A^2}\left(\frac{c\rho_l}{\rho_a}\right)^{0.5}$ $A=K/(V_{inj}d)^{0.5}$ $K=S/t^{0.5}$	ρ_a : ambient density ρ_l : liquid jet density c : coefficient of contraction t : time V_{inj} : jet velocity at the nozzle outlet d : nozzle diameter S : spray penetration L/d : length/diameter ratio of the nozzle A : constant for a given nozzle geometry c_d : discharge coefficient(=0.65) S_1 : spray tip penetration at 1 ms K_w=2.65, t_1=1 ms, α_3=0.2 Δp : fuel pressure drop across the hole
Abramovich (1963)	$\tan\theta=0.13\left(1+\frac{\rho_a}{\rho_l}\right)$	
Bracco et al (1985)	$\tan\frac{\theta}{2}=\frac{4\pi}{A}\left(\frac{\rho_a}{\rho_l}\right)^{0.5}\frac{\sqrt{3}}{6}$ $A=3.0+0.28(L/d)$	
Renner & Maly (1994)	$\tan\frac{\theta}{2}=K_w\left[\frac{V_{inj}d\cdot t_1}{(\rho_a/c_d\rho_l)^{\alpha_3}S_1^2}\right]$ $V_{inj}=(2\Delta p/\rho_l)^{0.5}$	

Table 2 Empirical correlations for spray angle in plain-orifice atomizer

Investigators	Correlations	Remarks
Sitkei (1964)	$\theta = 0.03\left(\frac{L}{d}\right)^{-0.3}\left(\frac{\rho_a}{\rho_l}\right)^{0.1} Re_l^{0.7}$ $Re_l = V_{inj} \cdot d/\nu_l$	ρ_a : ambient density ρ_l : liquid jet density Re_l : liquid jet Reynolds member ν_l : Kinematic viscosity of liquid jet V_{inj} : jet velocity at the nozzle outlet μ_a : ambient viscosity We_l : liquid jet Weber number d_o : sac chamber diameter of nozzle L/d : length/diameter ratio of the nozzle. d : nozzle diameter
Yokota and Matsuoka (1977)	$\theta = 0.0676 Re_l^{0.64}\left(\frac{L}{d}\right)^{-n} / \left[1 - exp\left(-0.023\frac{\rho_l}{\rho_a}\right)\right]$ $n = 0.0284\left(\frac{\rho_l}{\rho_a}\right)^{0.39}$	
Hiroyasu and Arai (1980)	$\theta = 0.0413\left(\frac{\rho_a}{\rho_l}\right)^{0.25}\left(\frac{V_{inj} d \cdot \rho_l}{\mu_a}\right)^{0.5}$	
Varde (1985)	$\tan\theta = A_1\left(\frac{\rho_a}{\rho_l}\right)^{\frac{1}{3}} (Re_l)^{\frac{1}{3}} (We_l)^{A_2}$ $A_1 = 0.0001\left(\frac{L}{d}\right)^5 \quad for \quad \frac{L}{d} < 6, \quad A_2 = \left(3 - \frac{L}{d}\right) / \left(3\frac{L}{d}\right)$ $A_1 = 0.7 \quad for \quad \frac{L}{d} > 6$	
Hiroyasu and Arai (1990)	$\theta = 83.5\left(\frac{L}{d}\right)^{-0.22}\left(\frac{d}{d_0}\right)^{0.15}\left(\frac{\rho_a}{\rho_l}\right)^{0.26}$	
Ishikawa and Tsujimura (1999)	$\tan\frac{\theta}{2} = (1.4 \times 10^{-7} V_{inj} + 3.6 \times 10^{-3})\rho_a$ $+ 2.9 \times 10^{-5} V_{inj} + 1.9 \times 10^{-3}$	

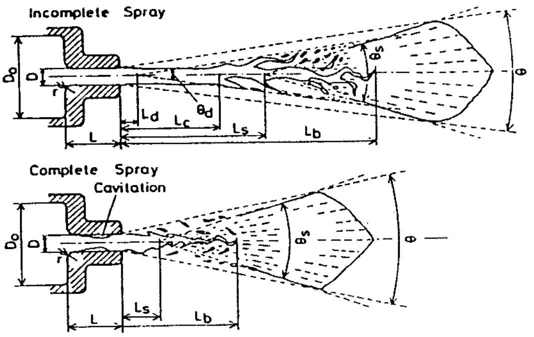

Fig. 4 Different spray angles of incomplete and complete sprays

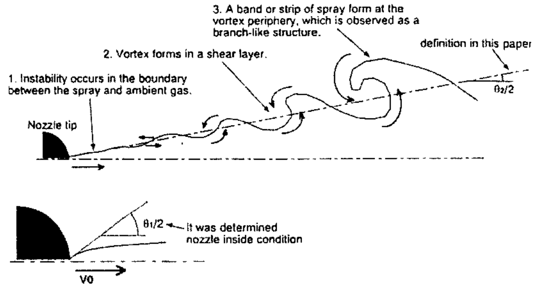

Fig. 5 Definition of two spray spreading angles by Ishigawa and Tsujimura(20)

mm)as shown in Fig. 5.

One angle (θ_1) is near the nozzle orifice outlet (0 < z < 3d) which is defined by spray cone angle. The other one (θ_2) is downstream from the nozzle orifice outlet (20d < z < 7 mm). They found that θ_1 depends mainly on the nozzle geometry and θ_2 was affected by the injection velocity and ambient pressure.

3. Correlations for spray angle

Several correlations have been derived to express spray angle produced by plain-orifice atomizer in terms of nozzle dimensions and the relevant air and liquid properties.

The existing correlations for spray angle produced by a plain-orifice atomizer can be classified into two groups: theoretical and empirical correlations as summarized in Tables 1 and 2 respectively.

3.1 Theoretical correlations

As a theoretical correlation, one of expression which is not well known is given by the momentum theory by Wakuri et al.(25) as

$$\tan\frac{\theta}{2}=\frac{1}{A^2}\left(\frac{c\rho_l}{\rho_a}\right)^{0.5} \qquad (3)$$

where $A=K/(V_{inj}d)^{0.5}$, c the coefficient of contraction, ρ_l the fuel density and the ambient density, $K=S/t^{0.5}$, S the spray penetration, t the time, V_{inj} is the jet velocity at the fuel nozzle outlet, d the diameter of the fuel nozzle.

In addition, Renner and Maly(6) had proposed the modified correlation of Wakuri et al.(25)'s one as follows.

$$\tan\frac{\theta}{2}=K_w[V_{inj}dt_1/((\rho_a/\mu\rho_l)^{\alpha_3}S_1^2)] \qquad (4)$$

where and $V_{inj}=(2\Delta p/\rho_l)^{0.5}$ and $K_w=2.65$, $\mu=0.65$ (discharge coefficient), $t=1$ ms and S_1=tip penetration at 1ms and Δp fuel pressure drop across the hole. The original density exponent of 1/2 was too large for their experimental results. A good fit was obtained only if the same exponent was used for all density ratios in the model, i.e. $\alpha_3= 0.2$

The theoretical correlation by Bracco et al.(4) in Table 1 was tested for the range of validity by Wu et al.(16). They concluded that the spray angle is found to be a strong function of the nozzle geometry and the gas-liquid density ratio and a weak function of the injection velocity. The other theoretical correlations for spray angle based on review by Lefebvre(1) and Dan et al.(24) are summarized in Table 1.

3.2 Empirical Correlations

As an empirical correlation, Varde(17) had derived the correlation to predict spray cone angle in terms of Reynolds and Weber numbers as

$$\tan\theta=A_1(\rho_a/\rho_l)^{1/3}(Re_l)^{1/3}(We_l)^{A_2} \qquad (5)$$

where $A_1= 0.001(L/d)^5$ for $L/d< 6$ and $A_1= 0.7$ for $L/d > 6$, $A_2= (3-L/d)/(3L/d)$.

In the study of measurement of diesel spray angle near the nozzle exit, Ishikawa and Tsujimura(20) suggested the empirical correlation based on the θ_2 explained in the above as follows.

$$\tan\frac{\theta}{2}=(1.4\times10^{-7}V_{inj}+3.6\times10^{-3})\rho_a$$
$$+2.9\times10^{-5}V_{inj}+1.9\times10^{-3} \qquad (6)$$

The spray angles obtained by their empirical correlation were compared with the results from the existing correlations such as one by Wakuri et al.(25) in Table 1, one by Yokota and Matsuoka(18) and one by Hiroyasu and Arai(8) in Table 2. They concluded that the tendency of their results was similar with one from the correlation by Wakuri et al.(25).

The other empirical expressions for spray angle based on review by Lefebvre(1) and Dan et al.(24) are summarized in Table 2. It is found that correlations for the prediction of spray angle involve the same parameters such as ambient density/liquid jet density ratio, nozzle orifice diameter or length/diameter ratio of nozzle, Reynolds number of liquid jet with more or less the similar weights. The study on the evaluation of existing correlations for spray angle including the one by Wakuri et al.(25) and Varde(17) is required.

4. Conclusions

The existing definition of spray angle as well as the correlations for the spray angle are summarized and reviewed. The existing definition of spray angle can be classified into four groups: distance based on orifice diameter, distance based on spray tip penetration, definition based on surface wave, and definition based on atomization. It is strongly required to specify the definition and measurement method when the data for spray angle is reported. The existing correlations for spray angle can be classified into two groups: theoretical and empirical correlations. The study on the evaluation of the existing correlations for spray angle is required, especially for the very high injection pressure and other fuels than diesel etc.

Acknowledgement

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