종속형제어기의 영점의 영향을 고려한 3-파라미터 제어기의 설계: 특성비지정 접근법

論 文
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A Design Method Reducing the Effect of Zeros of a Cascaded Three-Parameters Controller: The Characteristic Ratio Assignment Approach

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Abstract - This paper presents a new approach to the problem of designing a cascaded three-parameters controller for a given linear time invariant (LTI) plant in unity feedback system. We consider a proportional-integral-derivative (PID) and a first-order controller with the specified overshoot and settling time. This problem is difficult to solve because there may be no analytical solution due to the use of low-order controller. Furthermore, the zeros of controller just appear in the zeros of feedback system. The key idea of our method is to impose a constraint on the controller parameters so that the zeros of resulting controller are distant from the dominant pole of closed-loop system to the left as far as the given interval. Two methods realizing the idea are suggested. We have employed the characteristic ratio assignment (CRA) in order to deal with the time response specifications. It is noted that the proposed methods are accomplished only in parameter space. Several illustrative examples are given.

Key Words : Three-Parameters Controller, Time Response, Characteristic Ratio, Generalized Time Constant, Pseudo Break Frequency

1. Introduction

Under the structure of controller cascaded with a LTI plant in unit feedback system, we consider a problem of designing a three-parameters controller that meets the given time response specifications such as overshoot and settling time, if any. This is simple but not easy to tackle. The reason is that the existence of such controller can not be analytically solved for the case where the order of controller is lower than $n-2$, where n is the order of plant. Furthermore, the other difficulty comes up from the fact that the zeros of the closed-loop system must include the zeros of controller. These zeros generally affect the overall system in its damping. However, the zeros of controller in the two parameter configuration do not appear in the numerator of the overall system. The transient response control for the case has been investigated in [1].

In this paper, we present a new design method that will be able to reduce the effect of zeros of controller in cascade structure. We begin with finding all stabilizing set of PID/first-order controllers by means of Datta[2] and Tantaris[3]. Let the stabilizing set be S. Then we will investigate a way that extracts a subset of controllers from S, which satisfies the time response specifications. The key idea of this approach is to impose a constraint on the controller parameters so that zeros of the controller are distant from the dominant pole of closed-loop system to the left in the s-plane as far as the given interval. Both the dominant pole and the constraint can be approximately represented in terms of plant parameters and some design parameters, characteristic ratios α_i and a generalized time constant τ. We will give several examples.

2. Definitions and Preliminaries

Consider a polynomial

$$\delta(s) = a_n s^n + \cdots + a_2 s^2 + a_1 s + a_0. \tag{1}$$

The *characteristic ratios* α_i and the *generalized time constant* τ are defined as [1,4]

$$\alpha_1 := \frac{a_1^2}{a_2 a_0}, \alpha_2 := \frac{a_2^2}{a_3 a_1}, \cdots, \alpha_{n-1} := \frac{a_{n-1}^2}{a_n a_{n-2}}. \tag{2}$$

$$\tau := \frac{a_1}{a_0}. \tag{3}$$

It was shown in [1,4] that α_is of the denominator polynomial of rational model closely relate to the damping

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and the settling time of the system can be controlled by τ. These two parameters will be used when we compose a proper target polynomial for time response requirements.

The *characteristic pulsatances* β_i and the *pseudo break frequencies* ω_i^p are defined by [4,5]

$$\beta_0 := \frac{a_0}{a_1}, \beta_1 := \frac{a_1}{a_2}, \cdots, \beta_{n-1} := \frac{a_{n-1}}{a_n}. \quad (4)$$

$$\omega_i^p := \sqrt{\frac{\eta_i}{\eta_{i+1}}}\frac{\Delta_1^i}{\tau}, i=0,1,\cdots,n-1. \quad (5)$$

where $\eta_k := 1 - \frac{2}{\alpha_k} + \frac{2}{\alpha_k \Delta_{k-1}^{k+1}} - \cdots + (-1)^k \frac{2}{\alpha_k \Pi_{j=1}^{k-1} \Delta_{k-j}^{k+j}}$. and

$\Delta_i^j := \begin{cases} \Pi_{k=i, i<j}^{j} \alpha_k, & \text{if } 0<i<j \\ \alpha_i, & \text{if } 0<i=j \end{cases}$. $\eta_0 = 1,\ \Delta_1^0 = 1.$

Both definitions are used as approximate break points in Bode plot. It has been observed in [5] that the pseudo break point is better approximation comparing with pulsatance. The lowest break frequencies ω_0^p and β_0 correspond to the equivalent real poles which are placed nearest from the origin in complex plane. Here, the negative real pole nearest from the origin is defined as the dominant pole. Therefore, we take the ω_0^p of a characteristic polynomial as its dominant pole.

The design objectives considered on the time response are: (i) slow response acceptable but strictly small overshoot, and (ii) small overshoot admissible but strict settling time. In next section we will address how we design a PID/first-order controller satisfying the above objectives.

3. Controller design with fixed zeros

Fig. 1 shows a cascaded feedback configuration. It is possible to consider a general transfer function model. However, for the sake of explaining the effect of zeros of controller more explicitly, we consider an all-pole plant:

$$G(s) = \frac{N(s)}{D(s)} = \frac{n_0}{d_n s^n + \cdots + d_1 s + d_0}. \quad (6)$$

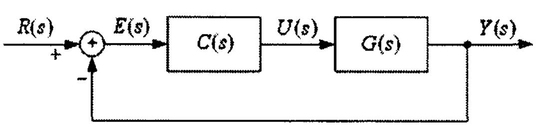

Fig.1. A unit feedback system with cascaded controller.

We first obtain a set of all stabilizing PID or first-order controllers using the algorithms by Datta[2] and Tantaris[3]. The detailed algorithms are omitted here. Let the set be

$$S := \{x \mid \delta(s,x) \text{ is Hurwitz}\}. \quad (7)$$

where x is the vector of controller parameters and $\delta(s,x)$ is the characteristic polynomial. Now, we present the design approaches for PID and first-order controllers respectively.

To reduce the effect of the zeros of controller, the following inequality are strongly required.

$$\gamma - \omega_o^p > 0 \quad (8)$$

where γ denotes a fixed zero of controller to be selected.

From (5) and (8), the bound of α_1 can be calculated.

$$\alpha_1 > \frac{2\gamma^2\tau^2}{\gamma^2\tau^2 - 1}. \quad (9)$$

3.1 PID controller design

The transfer function of PID controller is

$$C(s) = \frac{B(s)}{A(s)} = \frac{k_d s^2 + k_p s + k_i}{s}. \quad (10)$$

Let the vector of control parameters be

$$x_1 = \{k_p, k_i, k_d\}. \quad (11)$$

Then the closed-loop transfer function is described by

$$T(s) = \frac{B(s)N(s)}{A(s)D(s) + B(s)N(s)} = \frac{n_0(k_d s^2 + k_p s + k_i)}{\delta(s, x_1)}. \quad (12)$$

where

$$\delta(s, x_1) = d_n s^{n+1} + \cdots + d_2 s^3 + (d_1 + n_0 k_d)s^2 + (d_0 + n_0 k_p)s + n_0 k_i. \quad (13)$$

It is clear that the design parameter is only α_1, τ since the controller parameters are only included in $\delta_0, \delta_1, \delta_2$.

From (13), we have

$$\alpha_1 = \frac{(d_0 + n_0 k_p)^2}{n_0 k_i (d_1 + n_0 k_d)}, \quad \tau = \frac{d_0 + n_0 k_p}{n_0 k_i}. \quad (14)$$

The zeros of closed-loop system are identical to the roots of numerator of (10). Here we impose the following constraint on the controller parameters so that the nearest zero from origin is placed at γ, where γ is properly chosen on the basis of the poles of the plant. In addition, the other constraint (16) is required to guarantee (15).

$$\gamma = \frac{k_p - \sqrt{k_p^2 - 4k_d k_i}}{2k_d}. \quad (15)$$

$$2\gamma k_d - k_p < 0 \quad (16)$$

From (14) and (15), we can derive that

$$k_p = f_1(\alpha_1, \tau, \gamma) = \frac{\gamma^2 \tau \alpha_1 d_1 - \gamma^2 \tau^2 d_0 - \alpha_1 d_0}{(\gamma^2 \tau^2 + \alpha_1 - \gamma \tau \alpha_1) n_0}. \quad (17a)$$

$$k_i = f_2(\alpha_1, \tau, \gamma) = \frac{\gamma^2 \alpha_1 d_1 - \gamma \alpha_1 d_0}{(\gamma^2 \tau^2 + \alpha_1 - \gamma \tau \alpha_1) n_0}. \quad (17b)$$

$$k_d = f_3(\alpha_1, \tau, \gamma) = \frac{\gamma \tau \alpha_1 d_1 - \gamma \tau^2 d_0 - \alpha_1 d_1}{(\gamma^2 \tau^2 + \alpha_1 - \gamma \tau \alpha_1) n_0}. \quad (17c)$$

According to (17), we see that the set (k_p, k_i, k_d) is obtained by the design parameters (α_1, τ, γ). Note that this set can not guarantee the stability. Thus, we have to check whether the set is in the stabilizing set S.

Using the necessary condition of *Hurwitz* stability $\delta_i(s) > 0$, for $i = 0, 1, \cdots, n+1$, and substituting (17) into (14), the following inequality should be held.

$$\alpha_1 < \frac{\gamma^2\tau^2}{\gamma\tau - 1}. \tag{18}$$

Combining (9) and (18), the admissible range of α_1 becomes

$$\frac{2\gamma^2\tau^2}{\gamma^2\tau^2 - 1} < \alpha_1 < \frac{\gamma^2\tau^2}{\gamma\tau - 1}. \tag{19}$$

3.2 First-order controller design

A first-order controller is

$$C(s) = \frac{B(s)}{A(s)} = \frac{k_1 s + k_0}{s + l_0}. \tag{20}$$

The vector of control parameters is

$$x_2 = \{l_0, k_1, k_0\}. \tag{21}$$

The closed-loop transfer function can be described by

$$T(s) = \frac{B(s)N(s)}{A(s)D(s) + B(s)N(s)} = \frac{n_0(k_1 s + k_0)}{\delta(s, x_2)}. \tag{22}$$

where

$$\delta(s, x_2) = d_n s^{n+1} + \cdots \tag{23}$$
$$+ (d_1 + d_2 l_0)s^2 + (d_0 + d_1 l_0 + n_0 k_1)s + (d_0 l_0 + n_0 k_0).$$

Then α_1 and τ of (23) become

$$\alpha_1 = \frac{(d_0 + d_1 l_0 + n_0 k_1)^2}{(d_1 + d_2 l_0)(d_0 l_0 + n_0 k_0)}, \quad \tau = \frac{d_0 + d_1 l_0 + n_0 k_1}{d_0 l_0 + n_0 k_0}. \tag{24}$$

We impose the following constraint so that the zero of controller is identical to the given zero γ, i.e.,

$$\gamma = \frac{k_0}{k_1}. \tag{25}$$

From (24) and (25), l_0, k_1, k_0 can be derived as

$$l_0 = f_1(\alpha_1, \tau, \gamma) = \frac{\gamma\tau\alpha_1 d_1 - \gamma^2 d_0 - \alpha_1 d_1}{\gamma^2 d_1 - \tau^2 d_0 + \alpha_1 d_2 - \gamma\tau\alpha_1 d_2}. \tag{26a}$$

$$k_1 = f_2(\alpha_1, \tau, \gamma) = \frac{(\tau^2 d_0^2 - \tau\alpha_1 d_1 d_0 + \alpha_1 d_1^2 - \alpha_1 d_2 d_0)}{(\gamma^2 d_1 - \tau^2 d_0 + \alpha_1 d_2 - \gamma\tau\alpha_1 d_2)n_0}. \tag{26b}$$

$$k_0 = f_3(\alpha_1, \tau, \gamma) = \frac{\gamma(\tau^2 d_0^2 - \tau\alpha_1 d_1 d_0 + \alpha_1 d_1^2 - \alpha_1 d_2 d_0)}{(\gamma^2 d_1 - \tau^2 d_0 + \alpha_1 d_2 - \gamma\tau\alpha_1 d_2)n_0}. \tag{26c}$$

Equation (26) shows that the set (l_0, k_1, k_0) is obtained by (α_1, τ, γ). This set can not guarantee the stability. Thus, we have to check whether the set is in S. To satisfy the necessary condition of *Hurwitz* stability, substituting (26) into (24), we obtain

$$\alpha_1 < \frac{\tau^2(\gamma d_1 - d_0)}{(\gamma\tau - 1)d_2}. \tag{27}$$

Combing (9) and (27), we have

$$\frac{2\gamma^2\tau^2}{\gamma^2\tau^2 - 1} < \alpha_1 < \frac{\tau^2(\gamma d_1 - d_0)}{(\gamma\tau - 1)d_2}. \tag{28}$$

4. Illustrative examples and simulation results

In this section, several illustrative examples are given.

Example 1 (PID controller design):

A plant is assumed to be

$$G(s) = \frac{1}{s^2 + 7s + 12}. \tag{29}$$

According to (17), the admissible region of (k_p, k_i, k_d) to $\tau \in [0.5, 1.5]$ when $\alpha_1 = 2.8, \gamma = 4$ and to $\gamma \in [4, 20]$ when $\alpha_1 = 2.8, \tau = 1$ are depicted in Fig.2.

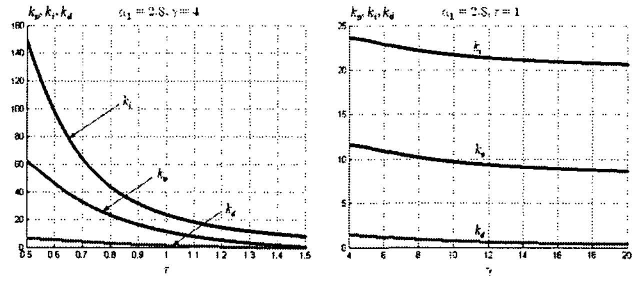

Fig. 2. The admissible regions of (k_p, k_i, k_d) vs τ and γ.

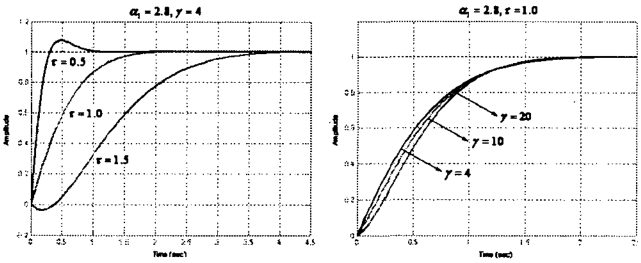

Fig. 3. The step responses of overall system vs τ and γ.

Fig. 3. shows the step response for the fixed (α_1, γ) and (α_1, τ), respectively. The numerical comparisons about overshoot and settling time are represented in Table 1 and Table 2.

Table 1. Overshoot and settling time to different τ.

$\alpha_1 = 2.8, \gamma = 4$	$\tau = 0.5$	$\tau = 1.0$	$\tau = 1.5$
overshoot	7.6430%	0.1363%	0.0966%
settling time	0.8282s	1.6486s	3.4272s

Table 2. Overshoot and settling time to different γ.

$\alpha_1 = 2.8, \tau = 1$	$\gamma = 4$	$\gamma = 10$	$\gamma = 20$
overshoot	0.1363%	0.1131%	0.0975%
settling time	1.6486s	1.6047s	1.5661s

Now, assume that we want to design a PID whose step response satisfies the 1% overshoot and the 2% settling time of 2 sec.

The poles of plant (29) are $-3, -4$. Put $\gamma = 4$, $\tau = 1$. From (19), we have $2.133 < \alpha_1 < 5.333$. From this analysis, we select $\alpha_1 = 2.8$, then the PID controller results in

$$C(s) = \frac{1.421s^2 + 11.58s + 23.58}{s}.$$

Then its closed-loop poles are $-4, -2.2105 \pm j1.004$ and zeros are $-4.1481, -4$. As shown in Fig. 3, the overshoot and settling time are given by 0.1363% and $1.6486s$.

Therefore, the design is achieved successfully.

Example 2 (First-order controller design):

Suppose that a plant is

$$G(s) = \frac{30}{0.01s^3 + 0.25s^2 + s}. \qquad (30)$$

Fig.4. illustrates the admissible region of (l_0, k_1, k_0) to $\tau \in [0.6, 1.0]$ when $\alpha_1 = 2.5, \gamma = 20$ and to $\gamma \in [20, 100]$ when $\alpha_1 = 2.5, \tau = 0.8$ in accordance with (26). The time response behaviors for the fixed (α_1, γ) and (α_1, τ) are shown in Fig. 5, Table 3 and Table 4.

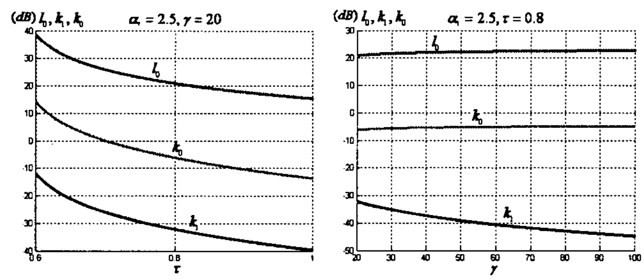

Fig. 4. The admissible regions of (l_0, k_1, k_0) vs τ and γ.

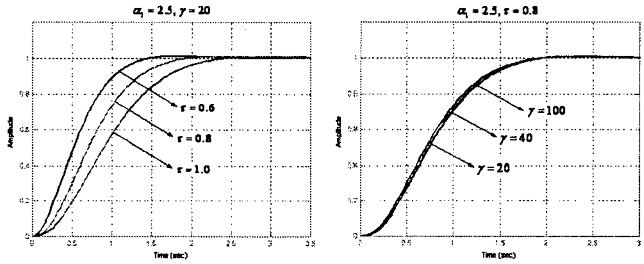

Fig. 5. The step responses of overall system vs τ and γ.

Table 3. Overshoot and settling time to different τ.

$\gamma=20, \alpha_1=2.5$	$\tau=0.6$	$\tau=0.8$	$\tau=1.0$
overshoot	1.2868%	0.9734%	0.9053%
settling time	1.2882s	1.7236s	2.1408s

Table 4. Overshoot and settling time to different γ.

$\tau=0.8, \alpha_1=2.5$	$\gamma=20$	$\gamma=40$	$\gamma=100$
overshoot	0.9734%	1.0220%	1.0522%
settling time	1.7236s	1.7363s	1.7590s

Now, let us consider a problem of finding a first-order controller that meets the following specifications:

(i) overshoot is smaller than 1%,

(ii) 2% settling time is shorter than 2 sec.

From (30), the poles of plant are $0, -5, -20$. Choose $\gamma=20$ and $\tau=0.8$. According to (28), we obtain $2.008 < \alpha_1 < 3.41$.

When we select $\alpha_1 = 2.5$, the resulting first-order controller is given by

$$C(s) = \frac{0.0243s + 0.4866}{s + 10.95}.$$

The poles of closed-loop system are $-20, -11.8485, -2.0502 \pm j1.3990$ and zero at -20. The overshoot and the settling time are 0.9734% and $1.7236s$ respectively. Therefore, we conclude that the design is successfully achieved.

5. Concluding remarks

Subject to a unit feedback structure, a new approach for reducing the effect of controller's zeros has been proposed. We have considered a PID and first-order controller with the specified overshoot and settling time. The main idea of our method was to impose a constraint on the controller parameters so that the zeros of resulting controller are distant from the dominant pole of closed-loop system to the left as far as the given interval. We have employed the CRA in order to deal with the time response specifications. As illustrated in examples, we conclude that the proposed method works well.

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