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Precise Geoid Model for Korea from Gravity and GPS Data

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ABSTRACT The data, methodology, and the resulting accurate gravimetric geoid model for the Korean Peninsula (latitude from 32° N to 40° N and longitude from 124° E to 131° E) are presented in this study. The types of used data were a high degree geopotential model (the EGM96 spherical harmonic coefficient set), a set of 12,615 land gravity observations, 1,056,075 shipborne gravity observations, and KMS2002 gravity anomalies from satellite altimetry. The remove-restore technique was successfully applied to combining the above mentioned data sets using up to degree and order 112 of the EGM96 coefficient. The residual geoid was calculated with residual Free-Air anomaly values using the spherical Stokes' formula with a 37-km integration cap radius. The geoid model was referred to WGS84 geodetic system and was tested using a set of GPS/levelling geoid undulations. The absolute accuracy is 0.132 m and some improvement compared to the PNU95 geoid model was found.

Key words gravimetric geoid, gravity, EGM96, TCFA, GPS/levelling

1. Introduction

The height of GPS is based on WGS84 ellipsoid and the height for daily life is referred to sea level, geoid, one of the equipotential surfaces. To use GPS effectively, we need to know accurate geoidal height, the difference between local geoid and earth ellipsoid.

Even EGM96, which is the most accurate model in accounting for geoidal height at one point using global gravity model, has some ±1 m error (Sideris, 1997b). This error is considered a big burden to generalize GPS technique. Therefore, more precise geoid model is required in and around the Korean Peninsula. Precise geoid is computed, taking in account global gravity model and gravity data of studied regions, this geoid model being verified by GPS/levelling geoid of GPS observed benchmarks (Featherstone, 2001; Kuroishi, 2001).

In this study, in order to improve the accuracy to make precise geoid model in and around the Korean Peninsula, gravity and GPS data were measured and analyzed. We collected several existing geophysical data such as EGM96 at the same time, verifying South Korea's precise geoid. Precise geoid from this study will be actively used in civil and geodetic field, as well as offer basic geoid-related data to understand geophysical phenomena better.

2. THEORETICAL CONCEPTS

With the established gravity data around the world, land and marine gravity data measured in and around the Korean Peninsula and satellite altimetry gravity data, gravity anomalies to calculate geoid were evaluated. Regional geoid was calculated using EGM96, and residual geoid was obtained from remove-restore technique (Sideris, 1997a). The combination of regional and residual geoid contributed to draw up the final precise geoid.

2.1 Calculation of the regional geoid

Global gravity data can be expressed in a lot of forms, and among them, global gravity model is composed of spherical coefficient of global gravity potential anomalies, which is relatively easy to handle and accurate. By improving global gravity model consistently, long-wavelength regional geoid can be easily determined by global gravity model's coefficient and geocentric coordinates used (Rapp, 1971, 1997a, 1997b).

To M degree and order, gravity potential anomaly T can be calculated as:

$$T(\phi,\lambda,r) = \frac{GM}{r}\left[\sum_{n=2}^{M}\left(\frac{a}{r}\right)^{n}\sum_{m=0}^{n}(\overline{C}_{nm}\cos m\lambda + \overline{S}_{nm}\sin m\lambda)\overline{P}_{nm}(\sin\phi)\right]$$

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ϕ, λ, r = Geocentric coordinates for points located

GM = Geocentric gravitational constant

a = Scaling factor (using equator radius of earth ellipsoid)

$\overline{C}_{nm}, \overline{S}_{nm}$ = Potential coefficients of degree n and order m

$\overline{P}_{nm}$ = Fully normalized associated Legendre functions

With coefficients $\overline{C}_{nm}$ and $\overline{S}_{nm}$ given by gravity model, gravity potential anomaly is calculated at one point.

Gravity potential anomaly T and geoidal height N were obtained by Bruns' formula (Heiskanen and Moritz, 1967).

$$N = \frac{T}{\gamma}$$

γ = Normal gravity

Through global gravity model, it is possible to calculate geoidal undulation N at one point (ϕ, λ, r).

$$N = \frac{GM}{\gamma\, r}\left[\sum_{n=2}^{M}\left(\frac{a}{r}\right)^{n}\sum_{m=0}^{n}(\overline{C}_{nm}\cos m\lambda + \overline{S}_{nm}\sin m\lambda)\overline{P}_{nm}(\sin\phi)\right]$$

The resolution of the earth gravity model depends on coefficient's expansion. Current global gravity models have maximum degree and order 360 and about 50 km of resolution. Hence, due to resolution limit of earth gravity model, this geoid became the regional geoid that implies long-wavelength geoid change.

2.2 Calculation of the residual geoid

When studied regions's gravity data applied, short-wavelength geoid, in other words, residual geoid can be obtained by several methods such as Stokes' integral method, least squares collocation, and Fast Fourier Transform (FFT) techniques (Tscherning, 1997; Sideris, 1997a). Stokes' integral method was used to get residual geoid in this study.

Considering gravity anomaly(Δg) and Stokes' formula, gravity potential anomaly T can be described as follows:

$$T = \frac{R}{4\pi}\iint_{\sigma} \Delta g\, S(\psi)\, d\sigma$$

Stokes formula is as below:

$$S(\psi) = 1 + \operatorname{cosec}\frac{\psi}{2} - 6\sin\frac{\psi}{2} - 5\cos\psi - 3\cos\psi\ \ln\left(\sin\frac{\psi}{2} + \sin^{2}\frac{\psi}{2}\right)$$

ψ is spherical distance between points to calculate geoidal height and gravity anomaly data at the points, and expressed in angular distance.

With Bruns' formula, geoid undulation(N) was calculated with normal gravity(γ), as:

$$N = \frac{R}{4\pi\gamma}\iint_{\sigma} \Delta g\, S(\psi)\, d\sigma$$

This formula is called Stokes' formula or Stokes' integral after being released by George Gabriel Stokes in 1849 (Heiskanen and Moritz, 1967; Moritz, 1980; Torge, 2001).

Residual geoid can be calculated from residual gravity anomaly by Stokes' integral method. In this case, the reason of using residual gravity anomaly was that the regional effects of gravity data of studied areas were included in earth gravity model and these had to be removed. Accordingly, to obtain residual geoid, regional gravity anomaly using earth gravity model should be removed from gravity anomalies around studied areas. This is called remove-restore technique (Sideris, 1997a) and with this, calculating residual gravity anomalies were done. Through the calculated residual gravity anomalies by Stokes' integral technique, residual geoid was obtained.

3. DATA DESCRIPTION

3.1 EGM96

EGM96 was developed by NIMA, NASA, OSU and other institutions of the U.S for 3 years to improve accuracy of earth gravity model. This is global potential coefficient which consists of a total number of 65,338 of cosine and sine coefficients, ranging from degree and order 2, 0 to 360, 360. Only by these coefficients, various physical data related with earth gravity, such as geoidal height, gravity anomaly, and vertical deflection, can be calculated.

In terms of data analysis and the number of the coefficients, this model is basically similar to the existing global gravity models, OSU91A and JGM-3, but it includes more various and latest gravity data. Thanks to the more accurate earth-related physical data from satellites, EGM96 is referred to EGM96 ellipsoid that is more precise than GRS80 and WGS84 ellipsoid. Furthermore, EGM96 model took into account tidal effects caused by the presence of the sun and the moon. (Lemoine *et al.*, 1998).

For regional geoid in this study, the latest global gravity model, EGM96 was used. Unlike OSU91A, EGM96 reflects gravity data around the world including southeast Asia. Therefore, the accuracy of EGM96 geoid model is within some ±1 m anywhere in the world (Sideris, 1997b).

3.2 Land gravity data

In a topographically undulated area like South Korea, the lack of gravity data on mountainous areas caused a lot of problems with terrain sensitive gravity anomalies. This also brought about inaccuracy in gravity data analysis in geodetic research. That's why gravity has been constantly measured, considering the need of densely observed gravity data in mountainous areas and flat lands in South Korea (Choi, 1986; Choi and Kim, 1996; Choi and Park, 1999; Choi and Lee, 1997; Choi *et al.*, 1996; Choi *et al.*, 2001; Choi *et al.*, 2003).

South Korea's gravity data used in this study are those constantly measured until now. Especially, via PNU95 geoid, gravity measurement using GPS has been conducted since 1997 (Choi *et al.*, 1998), so the number of measurements increased dramatically. In South Korea, there are 12,615 gravity observations now (Fig. 1).

To obtain gravity data of North Korea, we applied regional Bouguer anomaly map from gravity observations at 2,308 stations in North Korea and residual Bouguer anomaly map. In these Bouguer anomalies, the terrain correction was conducted using 1:50,000 and 1:100,000 topography maps. The boundary depth is 10 km between regional and residual Bouguer anomalies (Paek *et al.*, 1996). Both regional and residual Bouguer anomaly map were transformed into numerical values by digitization and were combined into Bouguer anomaly in North Korea for this study.

3.3 TCFA (Terrain-Corrected Free-Air Anomaly)

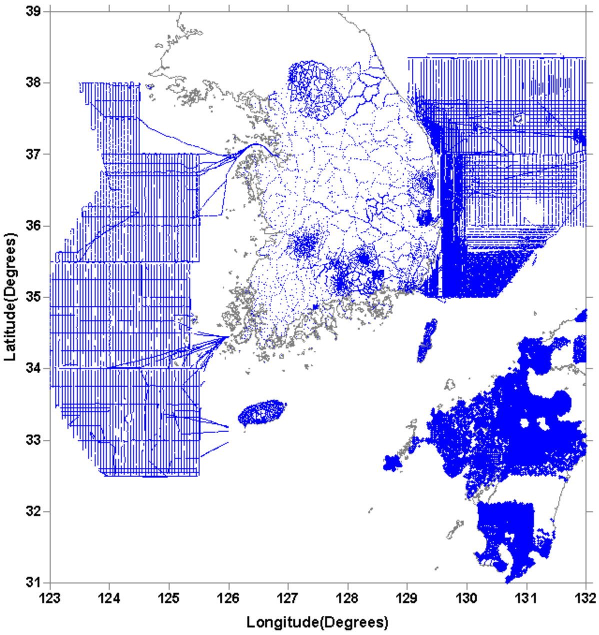

Fig. 1 Distribution of gravity observations in and around the Korean Peninsula.

To solve the problems with unequal distribution of observed land gravity data, Bouguer anomaly and terrain data are applied in this study. Bouguer anomaly usually adopted in the interpretation of underground structure, is gravity anomaly excluding terrain gravity effects, so undulation is relatively smooth against free-air anomaly. If simple Bouguer anomaly and terrain data are used, relatively precise Free-air anomaly considering terrain can be calculated even in the areas where gravity was not observed.

In this case, this free-air anomaly calculated with simple Bouguer anomaly and terrain data is named TCFA, and this is widely used in geoid calculation (Featherstone and Kirby, 2000; Featherstone *et al.*, 2001).

First, simple Bouguer anomaly was determined using all kinds of South and North Koreas' gravity data, then terrain correction effect was calculated up to the radius 50 km of each Bouguer anomaly point using 3-second interval terrain data in and around the Korean Peninsula. TCFA of this study was calculated by combining simple Bouguer anomaly with above gravity effect of terrain.

3.4 Gravity data in Japan

Gravity data of southwest Japanese areas in this study were actually measured and released by Geological

Survey of Japan. These absolute gravity values are standardized by IGSN71 gravity network, which is identical to that of Korea (GSJ, 2000), and coordinates are standardized by Bessel1841, Japanese geodetic system.

We deducted WGS84 normal gravity value using this absolute gravity value of Japan, and got free-air anomaly in Japan, conducting air-mass and free-air corrections. Also, Bessel1841 coordinates were transformed into WGS84 coordinate system. Among free-air anomalies obtained, the data of 23,317 points in southwestern Japan were applied in this study (Fig. 1).

3.5 Marine gravity data

Marine gravity was obtained shipborne gravity by research vessel ‘Haeyang 2000’, the 2,500-ton probe ship of National Oceanic Research Institute. This ship has measured shipborne gravity data at 1,056,075 points from 1996 to 2000 (Fig. 1).

In the case of shipborne gravity, the data are precise but limited only to surveyed areas. In order to get marine gravity of areas where there are no available shipborne gravity data, gravity data calculated from satellite altimetry data were used. KMS2002 altimetry gravity data has been applied, after analyzing several altimetry gravity data from some institutions with shipborne gravity data by 'Haeyang 2000'.

KMS2002 gravity anomaly was released in March, 2003 and the altimetry data used were about 50,000,000 points from GEOSAT, ERS-1 Geodetic mission, ERS-2 Geodetic mission and Exact repeat mission. KMS2002 was 2-minute interval gravity anomaly, ranging from the south latitude 82° to the north latitude 82°(Andersen and Knudsen, 1998; Andersen *et al.*, 2003).

Land gravity data, TCFA were combined with sea gravity data. The Draping method was applied to offset the errors in altimetry gravity data near coastline (Kirby and Forsberg, 1998).

3.6 Combination of gravity data

Final free-air anomaly is the result from the combination of actually observed gravity data, with TCFA, shipborne gravity data, and KMS2002 gravity data. Referring to gravity data of southwestern Japanese areas which are located southeast from studied regions, final free-air anomaly was calculated.

Final free-air anomalies of the east longitude of 123° to 132° and the north latitude of 31° to 41° ranged from -50 mGal to 238 mGal around Korea (Fig. 2) and was up to minimum -142 mGal at the southeast of Japan sea. The Mean of final free-air anomaly was 17.4 mGal (Table 1).

3.7 GPS/levelling data

GPS has an advantage of observing geoid-related physical data directly, which are difficult to be collected, so it is used to calculate and verify geoid model using gravity anomaly data (Featherstone, 2001).

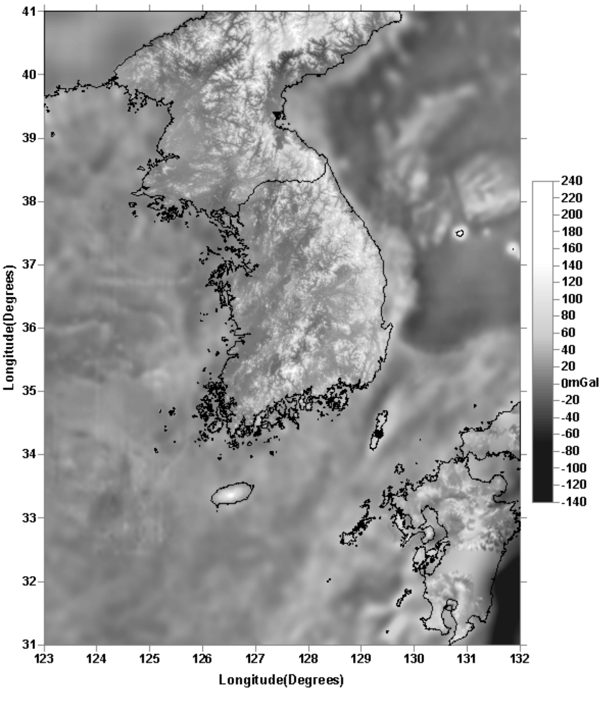

Fig. 2 The final Free-air gravity anomaly map combining all kinds of gravity data in and around the Korean Peninsula.

Table 1 Statistics of the PNU03 geoidal heights and final Free-air anomaly values in and around the Korean Peninsula.

	Geoid (meter)	Free-Air (mGal)
Mean	24.965	17.441
Minimum	14.915	-148.270
Maximum	33.421	238.040

Height obtained by GPS is based on ellipsoid surface, while an altitude is on geoid surface. Therefore, at benchmarks with an exact altitude, the geoidal height, the difference between ellipsoidal height and altitude can be calculated. This calculated geoid is named GPS/levelling geoid. Until now, GPS observation has been conducted at 182 stations of Korea provided by National Geographic Information Institute (Fig. 5) and was adopted in this study.

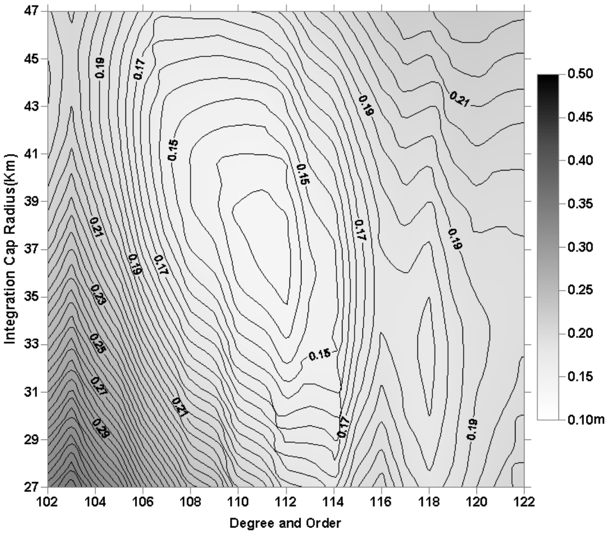

Fig. 3 RMS differences between GPS/levelling geoidal heights and the geoidal heights computed from EGM96 with different degree and various integration radius of local residual gravity anomalies.

4. COMPUTATION OF THE PRECISE GRAVIMETRIC GEOID

Computing regional geoid using EGM96 global gravity model and final free-air anomaly in and around the studied regions, short-wavelength residual geoid was calculated and then, precise geoid was obtained from these.

To this end, both optimum degree and order for regional geoid and integration radius for residual geoid were required. We changed those values until the RMS difference between precise geoid of each degree and order, integration radius and GPS/levelling geoid became minimum. Through this process, we could get the optimum degree and order and integration radius, and the improved precise

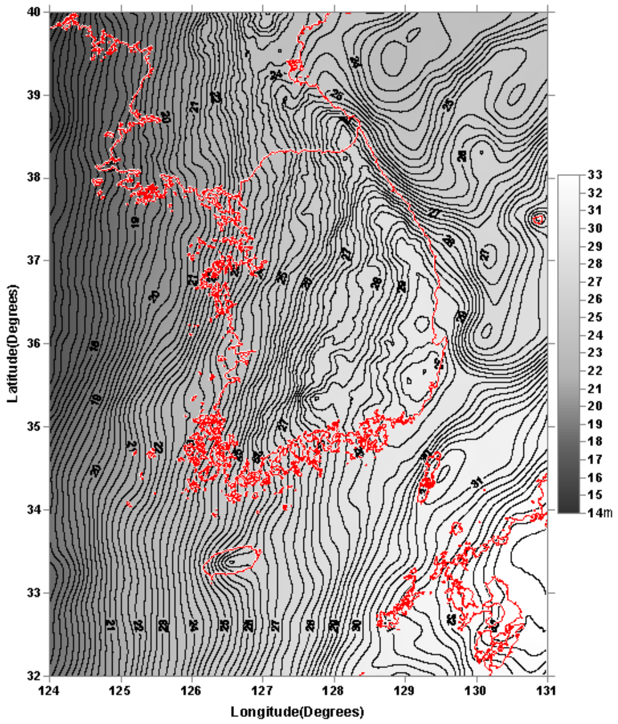

Fig. 4 The PNU03 geoid undulation reconstructed by combining regional with residual geoid in and around the Korean Peninsula.

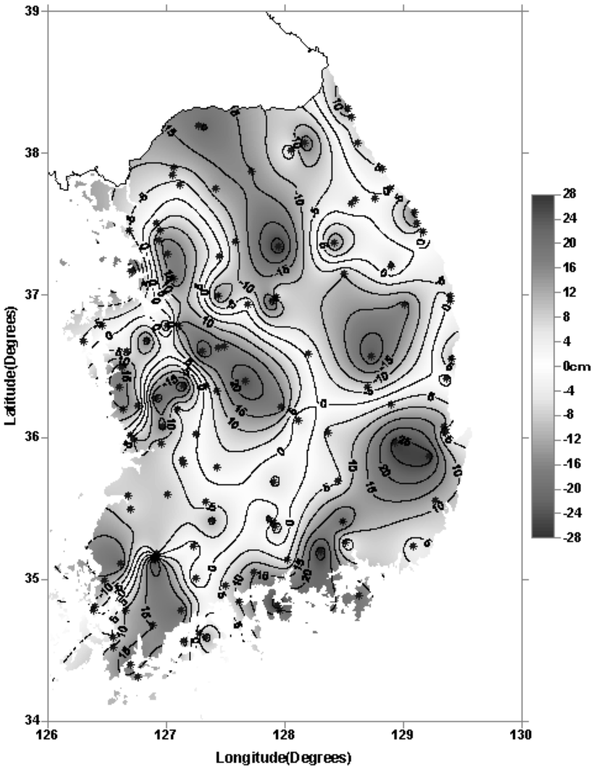

Fig. 5 Differences between PNU03 geoidal heights and GPS/levelling geoidal heights: Solid star represented benchmark location.

geoid of the southern Korean Peninsula was obtained.

When optimal degree and order was 112 and optimum integration radius was 37 km(Fig. 3), RMS difference was minimum 0.1324 m. At this moment, average difference between GPS/levelling geoid and precise geoid from gravity data was 0.5296 m. This was corrected, and it was possible to get the most suitable precise geoid for benchmark network on the Korean surface.

Precise geoid within studied regions ranged from southwest 14.92 m to southeast 33.42 m(Fig. 4), and from minimum 21.43 m around Kangwha-Do to maximum 30.29 m around Mt. Shinbul, Ulsan in the inland South Korea (Table 1).

5. EVALUATION

To verify final precise geoid (PNU03), among 182 GPS data at benchmarks, GPS/levelling geoid of only 121 land points in South Korea was adopted to calculate absolute error, and final precise geoid was compared with previous PNU95.

PNU95 geoid is precise geoid model around South Korea, using OSU91A global gravity model, surface gravity data about 4,500 stations in South Korea, gravity data from Tokyo university, satellite altimetry gravity data (Choi *et al.*, 1997).

As a result, RMS difference turned out to be 0.132 m, the difference value between PNU03 geoid and GPS/levelling geoid ranged from -0.296 m to 0.274 m (Fig. 5). But in case of PNU95, RMS was 0.162 m and the difference values between this and GPS/levelling geoid were -0.559 m to 0.348 m. Through absolute error, this precise geoid proved to be improved more than PNU95 geoid. This fact was a result of precise terrain corrections using accurate terrain data, with more gravity data. This PNU03 geoid is more appropriate, reflecting short-wavelength effects (Table 2).

6. CONCLUSIONS

From the previous gravity data, we adopted land gravity data in 12,615 stations, shipborne gravity in 1,056,075 stations, 2-minute interval KMS2002 altimety gravity, and gravity data in North Korea and southwestern Japan. Three-second interval terrain data in and around the Korean Peninsula were made, and through the terrain correction, TCFA was calculated, taking into account terrain effect of the land, especially mountainous area in free-air anomaly.

The remove-restore technique was successfully applied to combining the above mentioned data sets using up to degree and order 112 of the EGM96 coefficient. The residual geoid was calculated with residual Free-Air anomaly values using the spherical Stokes' formula with a 37-km integration cap radius. For verification, compared with GPS/levelling geoid at 121 points, the difference of RMS is 0.132 m.

By combining regional geoid with residual geoid, precise geoid being calculated, and residual geoid undulation in and around the Korean Peninsula ranges from -1.51 m to maximum 1.84 m. Based on WGS84, precise geoidal height varies from 14.92 m of Kangwha-Do off the west coast to 30.29 m of Mt. Shinbul around Ulsan.

Previous precise geoid of PNU95 was applied to verifying PNU03 and accuracy analysis. Against GPS/levelling geoid, RMS difference of PNU95 is 0.162 m and RMS of PNU03 is 0.132 m, so it was improved.

In the future, through consistent measurement of gravity and the addition of GPS/levelling data in and around

Table 2 Comparison of used data sets to compute the PNU03 and the PNU95 geoid.

	PNU03	PNU95
Global geopotential model	EGM96	OSU91A
Land gravity in South Korea	12,615 points	4,500 points
Shipborne gravity	1,056,075 points	-
Gravity from altimetry	KMS2002	Sandwell(1993)
Topography data	3" by 3"	15" by 15"
Benchmark data	182 points	71 points

the Korean Peninsula, more precise geoid is required, so precise geoid from this study is expected to be applied to comparing and verifying the obtained data.

The previous measuring technique using GPS in Korea mostly gave information at horizontal points only, and conventional surveying techniques are still used for accurate altitude measurement. This is because there are no available precise geoid and some misunderstanding of adopting exiting geoid. Therefore, precise geoid obtained from this study will offer not only sufficient accuracy but also more precise and convenient altitude when GPS used.

References

Andersen, O. B. and Knudsen P., 1998, Global marine gravity field from the ERS-1 and Geosat geodetic mission altimetry, J. Geophys. Res. V.103, p.8129-8137.

Andersen, O. B., Knudsen, P., Kenyon, S., and Trimmer, R., 2003, KMS2002 Global Marine Gravity Field, Bathymetry, and Mean Sea Surface, Poster, IUGG 2003, Sapporo, Japan, June 30-July 11, 2003.

Choi, K. S., 1986, A study on the gravity in the southern part of the Korean Peninsula , Ph. D. thesis, Seoul National University, Seoul, 110p. (in Korean)

Choi, K. S. and Kim, K. O., 1996, Underground Structure by Analysing Gravity Data in and Around the Chindong Area, Kyoungnam, Journal of the Korean Earth Science Society, V.17, p241-253. (in Korean with English abstract)

Choi, K. S. and Lee, J. M., 1997, A Study on the Orthometric Height Correction in Mt. Hangye Area, Journal of the Korean Earth Science Society, V.18, p.522-528. (in Korean with English abstract)

Choi, K. S. and Park, M. H., 1999, A study on the Underground Structure by Analysing Gravity Data in and around Mt. Chum, Sunchun, Chellanam-Do, Journal, College of Education, V.37, p.167-186. (in Korean)

Choi, K. S., Kim, J. H., and Yang, C. S., 1996, Gravity Survey over Mt. Jiri Area, Journal of the Korean Earth Science Society, V.17, p.422-428. (in Korean with English abstract)

Choi, K. S., Kim, J. H., and Yang, C. S., 1997, PNU95 Geoid, Korean Journal of Geophysical Research, V.25, p.1-13.

Choi, K. S., Park, P. H., and Shin, Y. H., 1998, Gravity Surveying with GPS, Journal of the Korean Earth Science Society, V.19, p.120-126. (in Korean with English abstract)

Choi, K. S., Ok, S. S., and Won, J. H., 2001, Establishment of Gravimeter Bases in Korea, Journal of the Korean Geophysical Society, V.4, p.121-131. (in Korean with English abstract)

Choi, K. S., Shin, Y. H., Yang, C. S., and Ok, S. S., 2003, On the Improvement of Precision in Gravity Surveying and Correction, and a Dense Bouguer Anomaly in and Around the Korean Peninsula, Journal of the Korean Earth Science Society, V.24, p.205-215.

Featherstone, W. E., 2001, Absolute and relative testing of gravimetric geoid models using Global Positioning System and orthometric height data, computaters & Geosciences, V.27, p.807-814.

Featherstone, W. E. and Kirby, J. F., 2000, The reduction of aliasing in gravity anomalies and geoid heights using digital terrain data, Geophys. J. Int., V.141, p.204-212.

Featherstone, W. E., Kirby, J. E., Kearsley, A. H. W., Cilliland, J. R., and Johnston, G. M., 2001, The AUSGeoid98 geoid model of Australia: data treatment, computations, and comparisons with GPS-levelling data, J. of Geodesy, V.75, p.313-330.

GSJ, 2000, Gravity CD-ROM of Japan, Digital Geoscience Map P-2, Geological Survey of Japan.

Heiskanen, W. A. and Moritz, H., 1967, Physical Geodesy, W. H. Freeman and Company, San Francisco and London, 364p.

Kirby, J. F. and Forsberg, R., 1998, A comparison of techniques for the integration of satellite altimeter and surface gravity data for geoid determination, in Geodesy on the move, Forsberg, Feissel, and Dietrich (eds.), IAG symposia, Brazil, V.119, Springer-Verlerg, p.207-212.

Kuroishi, Y., 2001, An improved gravimetric geoid for Japan, JGEOID98, and relationships to marine gravity data, J. of Geodesy, V.74, p.745-755.

Lemoine, F. G., Kenyon, S. C., Factor, J. K., Trimmer, R. G., Pavlis, N. K., Chinn, D. S., Cox, C. M., Klosko, S. M., Luthcke, S. B., Torrence, M. H., Wang, Y. M., Williamson, R. G., Pavlis, E. C., Rapp, R. H., and Olson, T. R., 1998, The Development of the Joint NASA GSFC and the Mational Imagery and mapping Agency(NIMA) Geopotential Model EGM96, NASA/TP-1998-206861, 575p.

Moritz, H., 1980, Advanced physical geodesy, Herbert Wichmann Berlag, Karlsruhe, 500p.

Paek, R. J., Kang, H. G., and Jon, G. P., 1996, Geology of Korea, Institute of Geology, State Academy of Sciences, DPR of Korea, Foreign Languages Books Publishing House, Second edition, 631p.

Rapp, R. H., 1971, Methods for the computation of geoid undulations from potential coefficients, Bulletin Geodesique, V.101, p.283-297.

Rapp, R. H., 1997a, Use of potential coefficient models for geoid undulation determination using a spherical harmonic representation of the height anomaly/geoid undulations difference, J. of Geodesy, V.71, p.282-299.

Rapp, R. H., 1997b, Global models for the 1cm geoid-present status and near term prospects, in Geodetic Boundary Value

Problems in View of the One Centimeter Geoid, Sansò and Rummel (eds.), Lecture Notes in Earth Science, Springer-Verlag, Berlin, p.272-311.

Sideris, M. G., 1997b, International tests of the new GSFC/DMA geopotential models, in Gravity, Geoid, and Marine Geodesy, Segawa, Fujimoto, and Okubo (eds.), IAG Symposia, Japan, V.117, Springer-Verlag, p.478-485.

Torge, W., 2001, Geodesy, Third edition, Walter De Gryuter & Co., Berlin, 416p.

Tscherning, 1997, Geoid determination by least squares collocation using GRAVSOFT, in Lecture Notes of the International School for the Determination and Use of the Geoid, Rio de Janeiro (ed.), International Geoid Service, p.183-212.