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An Integer Programming Approach to Packing Lightpaths on WDM Networks

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파장분할다중화망의 광경로 패킹에 대한 정수계획 해법

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We consider a routing and wavelength assignment (RWA) for the efficient operation of WDM networks. For a given physical network, a set of selected pairs of nodes, the number of required connections for each selected pair of nodes, and a set of available wavelengths, the RWA is to realize as many connections as possible without wavelength collision. We give an integer programming formulation and an algorithm based on column generation. Though the proposed algorithm does not guarantee optimal solutions, test results show that the algorithm gives probably good solutions.

Keywords: WDM, RWA, Integer Programming, Column Generation

1. Introduction

Wavelength division multiplexing (WDM) technology is used to accommodate several wavelength channels on a fiber. An all-optical network based on WDM is considered as a very promising approach for the realization of future large bandwidth networks (Lee *et al*., 2000). In a WDM network without wavelength conversion, an optical path (lightpath) with a dedicated wavelength is established for each required connection and no two paths using the same wavelength pass through the same link to avoid collision. The RWA problem is how to realize the required connection among nodes without wavelength collision.

There are two versions of RWA. The first problem RWA is to realize as many connections as possible under the constraint that the paths which share a common link of the given network should be assigned to different wavelengths. The second problem RWA2 is to realize all the given required connections on the given network under the same constraint as that in the first version. The objective is to minimize the number of required wavelengths. Many researches on RWA and RWA2 have been performed. Most of them have concentrated on the development of heuristic algorithms (Chlamtac *et al*., 1992; Dzongang *et al*., 2005; Kurma and Kurma, 2002). Lee and Li (1996), Wuttisittikuljij and O'Mahony (1995). Their solution approaches include choosing a path by some predetermined rule and assigning a wavelength to it in a greedy manner, decoupling the problem into the routing problem and the wavelength assignment problem and the use of meta-heuristics. Several researchers have presented mathematical formulations of the RWA and RWA2. Ramaswami and Sivarajan (1995)

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presented two integer programming formulations of RWA. One is a path-variable based formulation which is the same as our formulation, DP, presented in section 2.2. The other is a node-packing (stable set) based formulation on a path graph in which each node corresponds to a path in the given network and two nodes are connected if and only if the corresponding two paths share a common link. This formulation has exponentially many columns (node- packings in the path graph) in addition to the exponentially many number of rows (nodes in the path graph). They showed that the linear programming relaxation of the formulation gives a strong upper bound on the optimal objective value of RWA. Theoretically, the linear programming relaxation of this formulation gives the same bound as that of our formulation, MP, presented in section 2.3. However, they did not provide any systematic method to solve the linear programming relaxation of their formulation. Moreover, they did not use the formulation to devise an algorithm for RWA. They just presented a shortest path based greedy heuristic procedure for RWA. Lee *et al*. (2000) gives an integer programming formulation for RWA2 and an optimal algorithm based on branch- and-price on ring networks. Lee *et al*. (2002) devised an algorithm based on column generation on general topology for RWA2. Ozdaglar and Bertsekas (2003) gives an integer programming formulation and an algorithm based on LP which gives near optimal solution for RWA2.

The main reason for the use of the heuristic solution approaches is that RWA might be computationally intractable. Chlamtac *et al*. (1992) showed that RWA with the number of available wavelengths greater than or equal to 3 is NP-hard. It can be shown that RWA is NP-hard even if there is only one available wavelength.

In this paper, we consider the first version of RWA without wavelength conversion. We devise efficient algorithm for RWA using a column generation and branch-and-bound scheme. To do this, we develop a formulation of RWA which is decomposed into a master problem and a column generation problem.

In section 2, we analyze the computational complexity of RWA and we give integer programming formulations for RWA. In section 3, our algorithms for RWA is presented which are based on the column generation and branch-and-bound scheme, and the performance of the algorithms are given in section 4. Conclusions are given in section 5.

2. Complexity and Formulations

In this section, we define RWA more specifically and then give mathematical formulations. First, we introduce notations and definitions. Let us assume that the physical network $G=(V,E)$, a set of available wavelengths, a set of selected pairs of nodes, and the number of required connections for each selected pairs of nodes are given. Without loss of generality, we assume that *G* is connected.

V : set of nodes,
E : set of undirected links,
W : set of available wavelengths,
K : set of selected pairs of nodes,
o_k, t_k : two nodes of k for all $k \in K$
r_k : the number of connections required by *k* for all $k \in K$

Then RWA can be stated as follows :
Input : $G=(V,E)$, *W*, *K* and $r_k \in Z_+$ for each $k \in K$.
Output : Set of paths on *G* for realized connections with a wavelength assignment to the paths, such that no two paths of the same wavelength pass through the same link.
Objective : Maximize the number of realized connections.

2.1 Computational complexity

We analyze the computational complexity of the problems defined above. Consider the following UNDIRECTED TWO COMMODITY INTEGRAL FLOW problem.

Instance : Graph $G=(V,E)$ specified nodes s_1, s_2, d_1, and d_2, a positive capacity $c(e) \in Z_+$ for each $e \in E$, positive requirements $R_1, R_2 \in Z_+$.
Question : Are there two flow functions f_1, f_2 : $\{(u,v),(v,u) : \{u,v\} \in E\} \rightarrow Z_+$ such that
(1) for all $\{u,v\} \in E$ and $i \in \{1,2\}$, either $f_i((u,v)) = 0$ or $f_i((v,u)) = 0$,
(2) for each $\{u,v\} \in E$, $\max\{f_1((u,v)), f_1((v,u))\} + \max\{f_2((u,v)), f_2((v,u))\} \le c(\{u,v\})$,
(3) for each $v \in V - \{s_i, d_i\}$ and $i \in \{1,2\}$ flow f_i is conserved at v, and
(4) for $i \in \{1,2\}$, the net flow into d_i under flow f_i is at least R_i ?

The above problem is NP-complete even if the capacity of each link of the given graph is equal to 1 (Garey and Johnson). Now, consider the following decision problem (D) associated with (RWA) :

Instance : $G=(V,E)$, W, K, $W, r_k \in Z_+$ for all $k \in K$, and a positive integer *L*.
Question : Is it possible to realize at least *L* connections ?

It can be easily shown that every instance of UNDIRECTED TWO COMMODITY INTEGRAL FLOW problem with $c(e)=1$ for each $e \in E$ can be polynomially transformed into an instance of (D) with $|W|=1, |K|=2, L=R_1+R_2$, and for each $k \in \{1,2\}, o_k=s_k, t_k=d_k$, and $r_k=R_k$. Therefore, (D) is NP-complete and consequently (RWA) is NP-hard.

2.2 Direct formulation

We present an integer programming formulation of RWA which can be directly obtained from the definition of the problem. Let us introduce additional notations and definitions to be used in the formulation.

$P(k,w)$: set of paths between o_k and t_k which use wavelength w, for all $w \in W$ and $k \in K$

$P(k,w;e)$: set of paths between o_k and t_k which use wavelength w and pass through the link e, for all $e \in E$, $w \in W$ and $k \in K$

Now, we present a formulation of RWA :

$$(\mathrm{DP}) \max \sum_{k \in K} \sum_{w \in W} \sum_{h \in P(k,w)} x_h$$

$$s.t. \sum_{w \in W} \sum_{h \in P(k,w)} x_h \le r_k, \text{for all } k \in K \quad (1)$$

$$\sum_{k \in K} \sum_{h \in P(k,w;e)} x_h \le 1, \text{for all } w \in W \text{ and } e \in E \quad (2)$$

$$x_h \in \{0,1\}, \text{for all } h \in \bigcup_{k \in K} \{ \bigcup_{w \in W} P(k,w) \}$$

The binary decision variable, $x_h=1$ if the corresponding path h is selected, 0, otherwise, for all $h \in \bigcup_{k \in K} \{ \bigcup_{w \in W} P(k,w)\}$. Constraints (1) mean that the number of realized connections is less than or equal to the number of required connections. Constraints (2) ensure that at most one path can use one wavelength on one link. Note that, in the above formulation, the number of available wavelength is given.

DP has many constraints and exponentially many variables. In spite of the huge number of variables, the LP relaxation of DP can be solved efficiently using column generation technique (Barnhart *et al.* (1994). Though the LP relaxation of DP can be solved efficiently, there remains other difficulties. DP has symmetric structure that cause branch-and-bound to perform poorly because the problem barely change after branching. This symmetric structure essentially result from the indexing of wavelengths in the formulation, so that the set of paths $P(k,w)$ is actually the same as $P(k,w')$, for all $w,w' \in W$ with $w \neq w'$ and $k \in K$. If the number of available wavelengths becomes larger, the side effect of the symmetric structure would become more apparent.

In addition to the symmetric structure of the formulation, the bounds on the optimal objective value of RWA provided by the LP relaxation of DP can be very weak.

It might be possible to overcome the above difficulties by devising clever solution methods using DP. In this study, however, we present one possible way to overcome the above difficulties by using a different formulation.

2.3 Alternative formulation

In this section, we present an alternative formulation of RWA, which may be able to remedy the demerits of DP. First, we introduce the concept of an *routing configuration* which was used in Lee, K. *et al.* (2002). A *routing configuration* c is represented by a nonnegative vector $a_c \in Z_+^{|K|}$ such that $a_{ck} \le r_k$, for all $k \in K$. A routing configuration c is *independent* if we can realize c on the given network using only one wavelength. Let C be the set of independent routing configurations. Then, we can reformulate RWA as follows :

$$(\mathrm{MP}) \max \sum_{c \in C} p_c z_c$$

$$s.t. \sum_{c \in C} z_c \le w, \quad (3)$$

$$\sum_{c \in C} a_{ck} z_c \le r_k, \text{ for all } k \in K \quad (4)$$

$$z_c \text{ nonnegative integer, for all } c \in C,$$

where $w=|W|$ and $p_c=\sum_{k \in K} a_{ck}$, for all $c \in C$.

Each decision variable $z_c=l$, $c \in C$ if an independent routing configuration c is realized on the network l times using l wavelengths, where l is a nonnegative integer. p_c is the number of paths in independent routing configuration c. Thus, the objective function means the total number of realized connections. Constraints (3) ensure that the independent routing configurations should be realized at most w times in total, where w is the number of available wavelengths. Constraints (4) means the number of realized connections for a pair of nodes $k \in K$ is less than or equal to the given number of required connections.

In the above alternative formulation, we do not explicitly consider the assignment of wavelengths. After a solution is obtained, we have only to assign wavelengths to realized routing configurations. That can be done by assigning l wavelengths to each independent routing configuration if it is realized l times, where l is a positive integer, such that each wavelength is used at most one time.

MP have no symmetric structures, but it also has exponentially many variables. However, the LP relaxation can be solved efficiently by using the column generation techniques (Barnhart *et al*., 1994). Let MPL be the LP relaxation of MP. As it is customary to column generation techniques, we assume that a subset $C' \subset C$ of independent routing configurations is given. Replacing C by C' in MPL yields the restricted linear program MPL', whose solutions are suboptimal to MPL. Let μ be the scalar dual variable associated with (3), and let α_k be the dual variables associated with constraints (4). The constraints in the dual of MPL' are,

$$\mu + \sum_{k \in K} a_{ck}\alpha_k \geq p_c, \ c \in C', \mu \geq 0, \alpha_k \geq 0, k \in K.$$

Let $(\overline{\mu}, \overline{\alpha})$ be an optimal solution to the dual of MPL'. Then, it is also optimal to the dual of MPL if

$$\overline{\mu} + \sum_{k \in K} a_{ck}\overline{\alpha_k} \geq p_c, \ c \in C \setminus C'$$

Using $p_c = \sum_{k \in K} a_{ck}$, we may write the optimality condition for MPL :

$$\max\{ \sum_{k \in K} (1 - \overline{\alpha_k}) a_{ck} | c \in C\} \leq \overline{\mu} \quad (5)$$

Using (5), we can derive the formulation of the column generation problem for MP. Recall that each column of MP is an independent routing configuration on the network. To formulate column generation problems, let $P(k)$ be the set of paths between o_k and d_k, for each $k \in K$, and let $P(k;e)$ be the set of paths which pass through the link e, for each $k \in K$ and $e \in E$. Then the column generation problem for MP can be formulated as follows :

$$(\text{SP}) \max \sum_{k \in K} \sum_{h \in P(k)} (1 - \overline{\alpha_k}) x_h$$

$$\text{s.t.} \sum_{h \in P(k)} x_h \leq r_k, \text{ for all } k \in K \quad (6)$$

$$\sum_{k \in K} \sum_{h \in P(k;e)} x_h \leq 1, \text{ for all } e \in E \quad (7)$$

$$x_h \in \{0,1\}, \text{ for all } h \in \bigcup_{k \in K} P(k)$$

Note that $a_{ck} = \sum_{k \in K} x_h$, for all $k \in K$ and $c \in C$. Constraints (6) and (7) ensure that a solution to SP should be an independent routing configuration. If the optimal objective value of SP is greater than $\overline{\mu}$, then the independent routing configuration corresponding to the optimal solution can be added to MPL', otherwise, no routing configuration is generated. We mention that SP is a hard integer programming problem. It is clear that RWA with $|W| = 1$ is a special case of SP such that $\overline{\alpha_k} = 0$ for all k. We showed that RWA with $|W| = 1$ is NP-hard in section 2. Therefore, SP is NP-hard.

We discuss the theoretical strength of the bounds provided by LP relaxations of MP. Let Q be the set of binary vectors which satisfy constraints (2) of DP. That is,

$$Q = \{x \in B^{|P|} | \sum_{k \in K} \sum_{h \in P(k,w;e)} x_h \leq 1, \text{for all } w \in W \text{and } e \in E\}$$

where $P = \bigcup_{k \in K} \{ \bigcup_{w \in W} P(k,w)\}$. Let $conv(Q)$ be the convex hull of Q. Consider the following linear program DP' :

$$(\text{DP}')\max \sum_{k \in K} \sum_{w \in W} \sum_{h \in P(k,w)} x_h$$

$$\text{s.t. (1) and } x \in conv(Q)$$

Then, every feasible solution to (DP') is also a feasible solution to the LP relaxation of DP. If we let $Z_{DP'}$ and Z_{DLP} be the optimal objective value of (DP') and that of the LP relaxation of DP, then $Z_{DP'} \leq Z_{DPL}$. There also exists an instance of RWA in which $Z_{DP'} < Z_{DPL}$.

Now, we are to discuss the strength of the bound on the optimal objective value of RWA obtained by MPL. It can be easily shown that for every feasible solution to MPL, we can construct a feasible solution to (DP') with the same objective value. Therefore, we have the following result.

Proposition 1. Let Z_{MPL} be the optimal objective value of MPL. Then $Z_{MPL} \leq Z_{DP'}$.

The above proposition implies that the bound on the optimal objective value of RWA obtained by the LP relaxation of MP is at least as strong as that obtained by the LP relaxation of DP.

3. Algorithm for RWA

To present our algorithms for RWA, we assume that a network $G = (V, E)$, set of available wavelengths W, and the number of required connections r_k, for all $k \in K$ are given. Then, our column generation algorithm for RWA is as follows.

Algorithm COLGEN :

Step 1 : Initialization. Use a greedy heuristic algorithm to obtain a feasible solution to RWA with a set of independent routing configurations C_{HEU}. Set $C' \leftarrow C_{HEU}$ and initialize the restricted linear program MPL' with constraints (3), (4) and independent routing configurations $c \in C'$;

Step 2 : MPL' Solution. Solve MPL'. Let z^* be the obtained optimal solution, and let (μ^*, α^*) be the obtained optimal solution to the dual of MPL' ;

Step 3 : Column Generation. Solve SP. Let c^* be the independent routing configuration corresponding to the obtained optimal solution to SP. By the condition (5), if the optimal objective value of SP is greater than μ^*, then add c^* to the current MPL' and goto Step 2. Otherwise, we found an optimal solution to MPL, which is the LP relaxation of MP, goto Step 4 ;

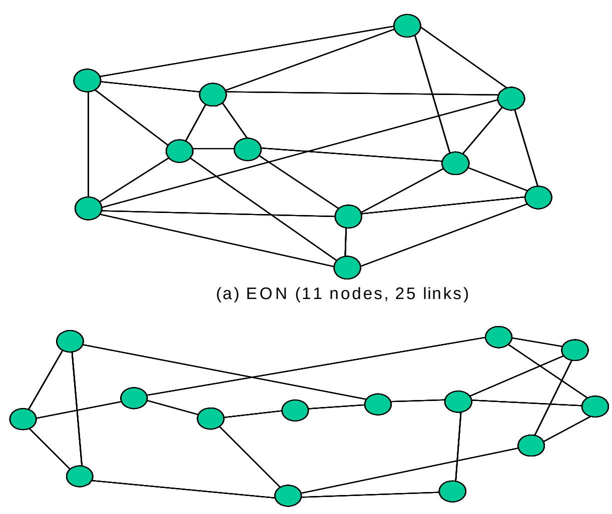
Step 4 : Termination. If z^* is integral, we have an optimal solution to RWA, stop. Otherwise, perform the branch-and-bound without further column generation ;

In Step 1, we develop a greedy heuristic for RWA and use the solution as initial columns. We omit the detail procedure of the heuristic. In Step 3, we must solve SP optimally. As shown in the previous section, SP is an NP-hard problem. But, SP is the same problem as the subproblem in Lee K. *et al*. (2002) and we can use the branch-and-price algorithm in Lee K. *et al*. (2002) to solve SP. Refer to Lee K. *et al*. (2002) for the more detail branch-and-price procedure. In Step 4, we perform a standard branch-and-bound procedure to MP with its existing columns, which will not guarantee an optimal solution to RWA. However, in our computational experiments, the quality of solutions obtained by the algorithm COLGEN was very good. There are some difficulties in devising a branch-and-bound procedure with further column generation to obtain an optimal solution to RWA using the formulation MP. The main reason for that is resulted from the fact that it is difficult to devise branching rules which do not destroy the structure of the column generation problems after branching. First, suppose that we have an optimal solution z^* to the LP relaxation of MP which has fractional coordinates and we are to use the branching rule of branching on a variable of a fractional value. We would choose a variable with a fractional value z_c^* and make two new nodes in the branch-and-bound tree, one with $z_c \le \lfloor z_c^* \rfloor$ and the other with $z_c \ge \lceil z_c^* \rceil$. If we are to optimize the LP relaxation of MP for the first node, it is possible that the optimal solution to the column generation problem SP will be the independent routing configuration represented by z_c. Then, in order to find a column which violates (5) and satisfies the branching decision, or prove that no such column exists, we must identify the solution to SP with the next highest objective value. At depth *n* in the branch-and-bound tree, we may need to find the n^{th} optimal solution to SP to generate columns. If the column generation problem is easy enough to find the n^{th} optimal solution efficiently, we may use this branching rule. However, the column generation problem SP itself is NP-hard, so the above branching rule may not be a viable option.

Vanderbeck and Wolsey (1996) presented a branching rule that can also be used for MP. However, their branching rule must be enforced by explicitly adding a new constraint to the LP relaxation of MP at each node in the branch-and-bound tree. Thus, each branching decision creates a new dual variable which must be incorporated into the column generation problem in order to generate columns correctly. At depth *n* in the branch-and-bound tree there will be *n* additional dual variables. Therefore, the structure of the column generation problem is destroyed due to the branching.

4. Computational experiments

In this section, we present the performance of our algorithms for RWA. We first give the characteristics of the test problems. Then, we give the test results of our algorithm for RWA on Pentium PC (700MHz). We applied our algorithm for RWA to a set of problems defined on two networks, European Optical Network (EON) and the NSFNET. <Figure 1> shows the topologies of the two networks.



(a) EON (11 nodes, 25 links)

(b) NSFNET (14 nodes, 21 links)

Figure 1. Topologies of two networks.

We randomly generated twenty test problems on each network. In each test problem, the number of required connections of each pair of nodes equal to 1 with the probability 0.5 and equal to 2 with the same probability.

The computational results for RWA on the two networks are summarized in <Table 1> ~ <Table 2>. In those tables, $|W|$, # of Cols, and # of B&B refer to the number of available wavelengths specified, the number of columns generated by the algorithm COLGEN including those used to initialize the initial formulation of the LP relaxation of MP, and the number of nodes explored in the branch-and-bound procedure, respectively. $\lfloor Z_{LP} \rfloor$ and Z_{COL} refers to the optimal objective value of the LP relaxation of MP and the number of realized connections obtained by COLGEN, respectively. Then, Z_{LP} and Z_{COL} give an upper bound and a lower bound on optimal objective value of RWA. *Gap* is defined $\lfloor Z_{LP} \rfloor - Z_{COL}$ and it means the upper bound on the difference between optimal objective value of RWA and the solution of obtained by our algorithm, COLGEN. Finally, the time (seconds) to solve RWA by COLGEN is reported under the heading of Time.

From the results, we can see that the LP relaxation of MP gives very sharp upper bounds on the optimal objective value of test problems for RWA. Also, our algorithm performs very well. The average difference between the upper bounds provided by the LP relaxation of MP and the solution obtained by COLGEN is 0.55 for the test problems on EON. For those on NSFNET, the average difference is 1.45.

Table 1. Computational results on EON

No.	|W|	# of cols	# of B&B	$\lfloor Z_{LP} \rfloor$	Z_{COL}	*Gap*	Time(Sec.)
1	5	38	0	76	76	0	57.5
2	6	31	0	85	85	0	40.6
3	6	49	2	85	84	1	60.9
4	6	57	2	84	83	1	142
5	5	29	0	73	73	0	44
6	5	75	1	77	77	0	133.2
7	5	38	0	76	76	0	47
8	5	48	0	72	72	0	68.8
9	5	45	0	73	73	0	60.8
10	5	60	2	77	77	0	90.4
11	5	45	2	76	75	1	64.8
12	5	53	0	77	77	0	73.8
13	5	64	12	73	71	2	119
14	5	51	0	79	79	0	66.1
15	6	48	10	87	85	2	79.2
16	6	61	2	84	83	1	123.6
17	5	53	2	80	79	1	59.9
18	5	49	1	72	72	0	78.1
19	5	48	8	75	74	1	65
20	5	35	10	73	72	1	56.6

Table 2. Computational results on NSFNET

No.	|W|	# of cols	# of B&B	$\lfloor Z_{LP} \rfloor$	Z_{COL}	*Gap*	Time(Sec.)
1	19	66	9	143	141	2	98
2	19	67	4	140	138	2	100.8
3	20	60	4	140	139	1	81.4
4	19	66	2	135	134	1	86.9
5	18	67	24	132	131	1	100.7
6	19	60	33	138	137	1	96.5
7	18	72	48	130	129	1	138.4
8	18	61	8	136	135	1	98.3
9	18	59	0	132	132	0	87.5
10	16	69	3	120	118	2	11.3
11	18	68	71	130	128	2	135.4
12	19	66	6	136	134	2	100.4
13	19	62	3	136	135	1	89.4
14	17	56	0	127	127	0	80.6
15	17	64	4	128	127	1	95.1
16	18	67	8	133	131	2	99.4
17	18	76	12	140	138	2	132.5
18	18	70	11	136	135	1	122.2
19	18	81	99	138	135	3	162
20	19	73	144	139	136	3	6211

5. Conclusions

In this paper, we proposed an integer programming formulation, MP, of RWA whose LP relaxation gives tight upper bounds. And we presented an algorithm based on the formulation, which gives good solutions to RWA.

In the proposed algorithm, we solve the LP relaxation of MP by the column generation technique and apply branch-and-bound procedure with existing columns because the difficulty mentioned in section 3. So, the algorithm cannot guarantee optimal solutions. A study on the method to overcome the difficulty might be a worthwhile effort. If the structure of the column generation problem should be inevitably destroyed after branching, we should devise algorithms which can solve the modified column generation problems efficiently after branching.

It also might be possible to devise clever solution methods for RWA using the formulation DP. To do so, it is necessary to devise methods which can destroy the symmetric structure of the formulation.

References

Barnhart, C., Johnson, E. L., Nemhauser, G. L., Savelsbergh,

M. W. P., and Vance, P. H. (1994), Branch-and-Price ; Column Generation for Solving Huge Integet Programs, Mathematical Programming : State of the Art, J. R. Birge and K. G. Murty eds., University of Michigan.

Chlamtac, I., Ganz A., and karmi, G. (1992), Lightpath Communications : An approach to high bandwidth optical WAN's, *IEEE Transactions on Comminication*, **40**, 1171-1182.

Dzongang, C., Galinier, P., and Pierre, S. (2005), A tabu search heuristic for the routing and wavelength assignment problem in optical networks, *IEEE communications letters*, **9**(5), 426-428.

Garey, M. R. and Johnson, D. S. (1979), *Computers and intracterbility - A guide to the theory of NP-Completeness*. San Francisco : W.H.Freeman and Company.

Kurma, M. S. and Kurma, P. S. (2002), Static lightpath establishment in WDM-networks - New ILP formulations and heuristic algorithms, *Computer Communications*, **25**, 109-114.

Lee, K. C. and Li, V. O. K. (1996), A wavelength routing algorithm in wide-area all-optical networks, *Journal of Lightwave Technology*, 1218-1229.

Lee, K., Kang K. C., Lee T., and Park S. (2002), An Optimization Approach to Routing and Wavelength Assignment in WDM All-Optical Mesh Networks without Wavelength Conversion, *ETRI Journal*, **24**(2), 131-141.

Lee, T., Lee, K., and Park, S. (2000), Optimal routing and wavelength assignment in WDM ring networks, *IEEE Journal on Selected Areas in Communications*, **18**, 2146-2154.

Ozdaglar, A. E. and Bertsekas, D. P. (2003), Routing and wavelength assignment in optical networks, *IEEE/ACM transactions on networking*, **11**(2), 259-272.

Ramaswami, R. and Sivarajan, K. N. (1995), Routing and Wavelength Assignment in All-Optical Networks, *IEEE/ACM transactions on networking*, **3**, 489-500.

Vanderbeck F. and Wolsey L. A. (1996), An exact algorithm for IP column generation, *Operations Research Letters*, **19**, 151-159.

Wuttisittikulkij L. and O'Mahony M. J. (1995), Algorithms for Wavelength Assignment in All-Optical Networks, Performance Engineering of Computer and Telecommunication Systems, M. Merabti, M. Carew and F. Ball eds., Springer.