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FATIGUE LIFE PREDICTION OF RUBBER MATERIALS USING TEARING ENERGY

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ABSTRACT–It has been almost impossible to predict the fatigue life in the field of rubber materials by numerical methods. One of the reasons is that there are no obvious fracture criteria and excessively various ways of mixing processes. Tearing energy is considered as a fracture criterion which can be applied to rubber compounds regardless of different types of fillers, relative to other fracture factors. Fatigue life of rubber materials can be approximately predicted based on the assumption that the latent defect caused by contaminants or voids in the matrix, imperfectly dispersed compounding ingredients, mold lubricants and surface flaws always exists. Numerical expression for the prediction of fatigue life was derived from the rate of rough cut growth region and the formulated tearing energy equation. Endurance test data for dumbbell specimens were compared with the predicted fatigue life for verification. Also, fatigue life of industrial rubber components was predicted.

KEY WORDS : Rubber, Tearing energy, Fracture criterion, Fatigue life prediction, Energy release rate, Engine mount

1. INTRODUCTION

Of many possible causes for the failure of vulcanized rubbers, one particular process is fatigue failure. Fatigue failure of rubber is remarkably distinguished from the one of metals in the view of crack generation and growth rate (Steiner *et al.*, 2001). Fatigue theories applied to rubber compounds should be somewhat different from so- called metal fatigue theories. Though fatigue theories about rubber compounds have been studied and developed since 1950, their application to design of rubber components has not been expanded widely since its theoritical difficulty and research were mainly concentrated on the experimental phenomenon. Most of all, non-standardizaion of rubber materials has made a key role to prevent its application.

Tearing energy was proposed as a factor of the fatigue fracture for rubber compounds (Rivlin and Thomas, 1952). Tearing energy T was defined as the amount of the strain energy decrease per unit change in the crack surface area. Tearing energy was experimentally estimated for a two dimensional specimen with given crack configuration and given loading. When considering other complicated specimen geometry such as industrial components, three dimensional modeling of crack propagation process is inevitably needed to estimate tearing energy. In the studies by Andrew, the efforts to find a theoretical way of calculating tearing energy have been limited to the two dimensional geometry (Andrew, 1974). Thomas suggested the first step of the fatigue life prediction for rubber material using tearing energy was proposed in the form of an empirical equation (Thomas, 1958; Gent *et al.*, 1964). The cut growth rate equation which was defined by a dynamic cut growth constant in the rough cut growth region was experimentally verified using tensional test-pieces. However, tearing energy still did not play a key role as the fracture criteria of rubber material due to absence of a theoretical way of measuring the crack length and calculating the energy release rate for industrial rubber components. Fatigue life could have been estimated only by using the rubber component itself. A finite element method was applied to estimate fatigue life of rubber components in some studies (Kim and Kim 1996; Kim *et al.*, 1998, 2004), but the S-N data for fatigue specimen under various conditions was inevitably required. Kim suggested strain and displacement as the factors of fatigue life, and showed a reasonable agreeement between the prediction and test. As mentioned above, if fatigue test data of rubber has been accumulated sufficiently and rubber materials have been standardized, the method in Kim should be very effective and useful. Recently, the logarithmic value of fatigue life for a two dimensional specimen has been expressed as a function of double displacement amplitude, hysteresis and J-value which has

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the same physical quantity as tearing energy, but a theoretical method of tearing energy has not been proposed (Kim and Kim, 2005a).

In this paper, a theoritical and numerical method for the fatigue life prediction derived from the cut growth rate equation under cyclic loading condition is proposed. The fracture source of rubber is reviewed in the foregoing paper, and verified with experimental results. To verify the developed prediction method, fatgue life of a three dimensional dumbell fatigue specimen is compared with the predicted result which has been evaluated by using developed code and static finite element analysis. Various average loading and amplitude conditions are applied. The fatigue life of several identical engine mounts is measured and compared with the predicted fatigue life for the verification of availablity in general rubber components.

2. FATIGUE LIFE PREDICTION OF RUBBER

2.1. Rough Cut Growth Region

Tearing processes with repeated loading and relaxation was studied (Thomas,1958). A typical cut growth curve is shown in Figure 1 for the cut growth rate versus cycles. The rapid cut growth rate of the initial tearing region gradually slows down after a few thousand cycles, and becomes a substantially constant rate. The region of the constant cut growth rate was called the rough cut growth region and the rate equation is known as,

$$\frac{dc}{dN}=\frac{T^2}{G_d} \tag{1}$$

G_d is a dynamic cut growth rate constant, T is tearing energy, c is the cut length and N means the cycles. The cut growth rate right after the rough cut growth region has much faster rate, so most of the fatigue life in rubber material was likely included in rough cut growth region. This fact was verified by the experiments in which the cut growth life was evaluated with various test-pieces of different geometry and recipes (Gent *et al.*, 1964). In these experiments, the evaluated life beyond the rough cut growth region and the calculated life by the equation of the rough cut growth rate showed nearly an identical fatigue life as shown in Figure 2 and Table 1. The solid line is the theoritical result and the symbols indicate the estimated fatigue life for various test-pieces. The theoritical result shows a constant rate of the cut growth whereas a rapid change of the experimental cut growth rate occurs in the vicinities of the complete tearing points. The upward departure of the experimental points at a high T value was expected due to the initial catastrophic tearing. l is the stretch ratio and c_0 means the initial cut length in Table 1. The rough cut growth rate equation can be used in predicting fatigue life for the rubber-like materials

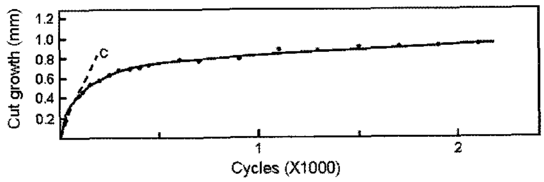

Figure 1. Dynamic cut growth curve of rubber test-pieces.

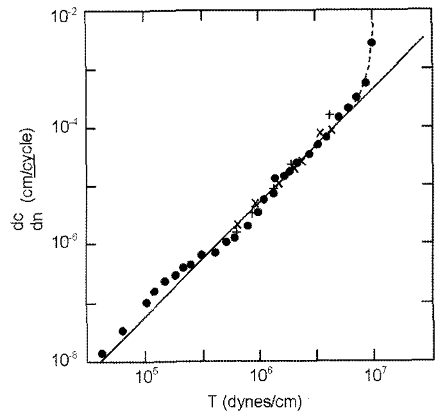

Figure 2. Relationship between the rate of the cut growth *dc/dN* and tearing energy *T* for various forms of test-pieces.

which has the latent defects to be larger than the critical crack length. It is obvious that the fatigue life of the ideal rubber should be longer than the other, general rubber components.

2.2. Basic Assumption

The developed equation for the fatigue life prediction in this paper has several basic assumptions.

2.2.1. Existence of latent defects

Defects above the critical length of 10 μm are assumed to be dispersed uniformly. Rubbers are manufactured using various compounding ingredients, and usually reinforced by mineral particles such as carbon black and sometimes

Table 1. Calculated and observed fatigue life.

l	C_0 ($\times 10^{-3}$)	N, kcycles	
		Observed	Calculated
2.36	36	3.0	4.4
2.08	46	6.5	7.2
1.47	43	78	88
1.205	70	700	760
1.098	78	8600	9500

by long metallic or organic fibers (Bathias and Hegorjujago, 2002). Voids, inclusions or agglomerates result from inhomegeneous mixing during the material processing. These imperfections are the sources of micro-cracks in rubber (Lake and Lindley, 1964; Mars and Fatemi, 2002). Mold lubricants and surface flaws can be other sources. Gent showed that carbon black had a function of crack nucleation during the experimental examination of the tensile strength of the vulcanized rubber cylinders which were bonded to the plane metal end-pieces (Gent and Lindley, 1958). Cavities sized from 5 to 20 μm are observed dispersed at irregular intervals of 20–500 μm by examination of SEM (Scanning Electron Microscopy) pictures of the surface of the engine mount and rubber bush . The other rubber components, door-seal, rubber bush showed an analogus result. Also, the dispersion of carbon black agglomerates of the rubber compounds was examined in a square of 1 μm × 1 μm. The distribution ratio was over 97% and the minimum size of carbon black agglomerates was identified as 9–10 mm (Kim and Kim, 2005b). It has to be noticed that those cavities and agglomerates can hardly be avoided by any type of mixing process (Saintier *et al.*, 2005). Fatigue life which is evaluated by the developed equation in this paper means the life of latent defects. Therefore, fatigue life of the ideal rubber products which have none of the latent defects or cracks should be longer than the expected life.

2.2.2. Non-directional property of the cut growth

Specified geometry of the cracks was not considered in the formulation of the general tearing energy equation. This assumption is very quantitive, and means that the cracks can grow in any direction. An identical phenomenon can be observed in the actual situation, but this assumption can imply an error in the case of a pure compressional loading condition. If the compressional load is applied, the direction of the crack growth should be different from the case of a tensional load. An identical direction and length are assumed in both case of tensional and compressional loading conditions and identical magnitude in this paper. However, the effect of the compressional load may be different from the case of metallic materials which results in the crack closeing effect. When the compressional load is applied, the rubber should be deformed in the perpendicular direction of compression according to the material behavior of large deformation and incompressibility. This phenomenon could be considered as assumed tensional deformation which causes the crack growth in the perpendicular loading direction.

2.2.3. Bonded planes

Automotive rubber components are usually manufatured in the assembly bonded with metal pieces. In the case of inferior goods that has a shape problem or weak bonded plane, the rubber part can be separated from the metal piece occasionally during the term of service. Fatigue life of rubber components should be evaluated by stiffness weakening, not by separation. The life evaluated by the separation case does not considered as fatigue life in general. If one of the fatigue specimens shows a much lower cycle number than the average fatigue life, the majority of the cause is generally due to inferiority of bonding planes and the fatigue test data can not be accounted for.

2.2.4. Assumptions of tearing energy

The basic assumptions of tearing energy independent from geometry and loading are preserved in the cut growth rate equation which includes tearing energy as a factor. Also, the incipient tearing is assumed, so errors can be encountered in the case of catastrophic tearing.

2.3. Equation for Fatigue Life Prediction

The cut growth rate equation (1) could only be evaluated by experimental method because the numerical method to calculate tearing energy had not existed. In this paper, using the simple tearing energy equation (2) suggested in the previous paper (Kim and Kim, 2005b), the numerical method of fatigue life perdiction was developed.

$$T = c\sum_j w_j \tag{2}$$

w means the strain energy density, the subscript j indicates integration point which shows the maximum strain energy density, c is the total crack length. The number of cycles required to cause the crack to grow from an initial length c_0 to a length c_f is obtained by integrating equation (1).

$$\int_0^{N_f} dN = \int_{c_0}^{c_f} \frac{G_d}{T^2} dc \tag{3}$$

Submiting the equation (2),

$$N_f = \int_{c_0}^{c_f} \frac{G_d}{c^2\left(\sum_j w_j\right)} dc \tag{4}$$

The size and magnitude of the maximum strain energy density is assumed to be constant.

$$N_f = \frac{G_d}{\left(\sum_j w_j\right)^2} \int_{c_0}^{c_f} \frac{1}{c^2} dc \tag{5}$$

$$N_f = \frac{G_d}{\left(\sum_j w_j\right)^2} \left[-\frac{1}{c}\right]_{c_0}^{c_f} \tag{6}$$

Therefore, the equation for fatigue life prediction can be obtained as the equation (7).

$$N_f = \frac{G_d}{\left(\sum_j w_j\right)^2}\left(\frac{1}{c_0} - \frac{1}{c_f}\right) \qquad (7)$$

This equation shows that N_f becomes independent of c when the latter is much greater than c_0, and that a finite number of cycles are necessary to cause an indefinite increase in crack length. Thus, according to the above equation, the size of a fatigue specimen will have little effect on the number of cycles to failure if its dimensions are much greater than c_0. Ignoring the effect by the latter crack size, the number of cycles to break or fatigue life will therefore given from the equation (8), on the condition of $c >> c_0$.

$$N_f = \frac{G_d}{c_0\left(\sum_j w_j\right)^2} \qquad (8)$$

2.4. Process of Fatigue Life Prediction

A developed equation for evaluating fatigue life providing the approximate number of cycles to failure uses the result variable from the static finite element analysis, strain energy density, but only in the case of natural rubber whose dynamic cut growth rate constants and the tearing criterion have been known. Otherwise, the dynamic cut growth constants should be acquired for evaluating fatigue life of other rubber material. However, as previously known, the dynamic cut growth constants for all recipes do not need to be considered. The dynamic cut growth constants for a representive recipe can be used in the place of the remains. If the dynamic cut growth constants can be databased, fatigue life of rubber products can be evaluated using only the result of the static finite element analysis and static material test data. Figure 3 shows the process to evaluate fatigue life of rubber components.

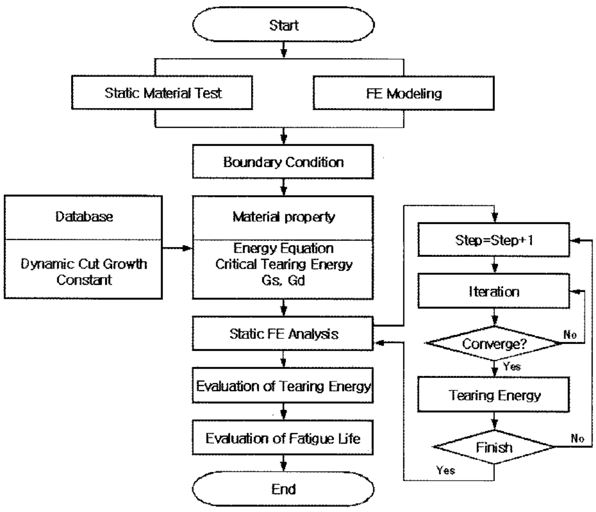

Figure 3. Suggested process for evaluating fatigue life of rubber components.

3. APPLICATION TO FATIGUE SPECIMENS

The predicted fatigue life using the equation (8) was compared with the experimental result of a rubber fatigue specimen tested by Rubber NRL of KIMM for verification.

3.1. Experiments

The hourglass-shaped specimen shown in Figure 4 was designed by Takeychi *et al.* (1993). The 3D dumbbell specimen has an elliptical cross-section, and parting lines are located on the minor axis of the specimen to avoid undesirable failure at the surface discontinuities (Kim *et al.*, 2004). The used material is a carbon filled vulcanized natural rubber with a hardness of IRHD 60. The fatigue test was conducted in an ambient temperature of 20°C. Mean stress effects on the fatigue damage were taken into account by applying the zero and positive mean load of 49 and 98N. He defined fatigue life for the 3D dumbbell specimen as the number of cycles to a complete separa-

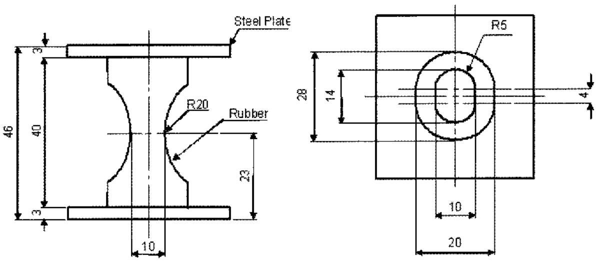

Figure 4. Shape of a rubber fatigue specimen.

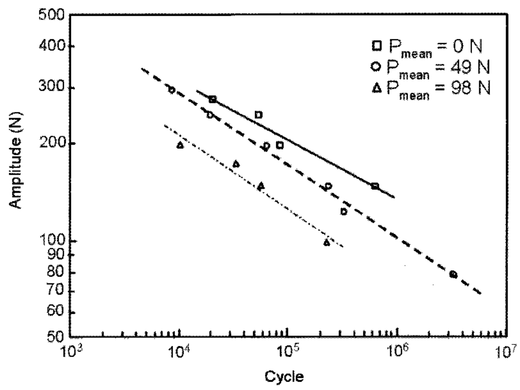

Figure 5. Fatigue life versus amplitude graph with different mean loads tested by KIMM.

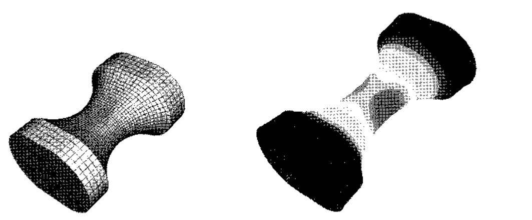

Figure 6. 3D FE model of rubber fatigue specimens.

tion. The relationship between the applied load amplitude and cycles to failure is shown in Figure 5. In his experiment, fatigue life decreased with an increasing amplitude at a fixed mean load, and increased with a decreasing mean load at fixed amplitude.

3.2. Finite Element Analysis

Rubber can be considered as a hyperelasitc material with incompressibility, whose behavior is characterized by the strain energy potential. In order to define the hyperelastic material behavior, material parameters in the Ogeden strain energy potential (Ogden, 1972, 1976) of the order N=3 was determined using the static uniaxial tension data identical with Kim's experiment. The 3D dumbbell specimen was modeled with an 8-noded hexa hyperelastic element, shown in Figure 6. A nonlinear finite element analysis program of ABAQUS was used for the analysis of the rubber specimen.

The static finite element analysis was in the previous step in which the information about the strain energy density was acquired. Figure 6 shows the strain energy density distribution of a dumbbell specimen under a tensile load of 150N. Tearing energy was analyzed from the static analysis result, and fatigue life was calculated with a developed code in which the fatige life equation (8) was built in. Table 2 lists several loading conditions. During the fatigue life analysis, the maximum tearing energy was searched in the whole FE model within the

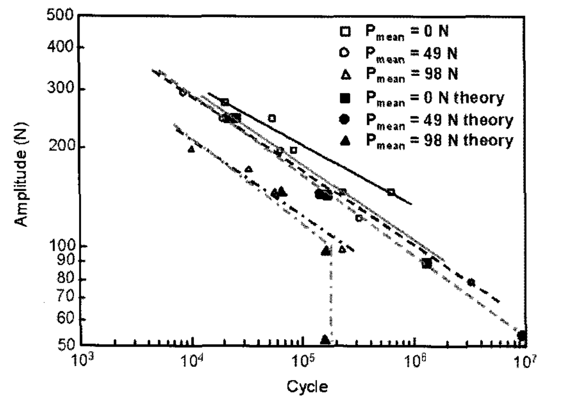

Figure 7. Comparison of the experimental result with the predicted fatigue life.

Table 2. Boundary condition and the result of fatigue life.

Mean load (N)	Amplitude (N)	Test data (cycle×10^4)	Predicted (cycle×10^4)
0	150	80	10.61
0	250	8	4.68
50	150	30	10.25
100	100	30	10.42
100	150	8	8.80

range of 1% average error. The critical crack length of rough cut growth 10 μm and dynamic cut growth constant of natural rubber 200 were substituted in the equation (8).

3.3. Predicted Fatigue Life of Specimens

The predicted result of fatigue life at each load condition is shown in Table 2, and compared with the experimental result in Figure 7. As shown in Table 2, an identical tendency of fatigue life which decreases as the amplitude increases can be observed in both experimental and predicted results even though the number of cycles predicted by the theory is smaller than experimental result. However, predicted fatigue life should be smaller than the experimental life because the experimental result was determined at a complete separation and the predicted result indicates fatigue life before the catastrophic tearing. The case of the mean load 100N and amplitude 150N in which analogous number of cycles with experiments

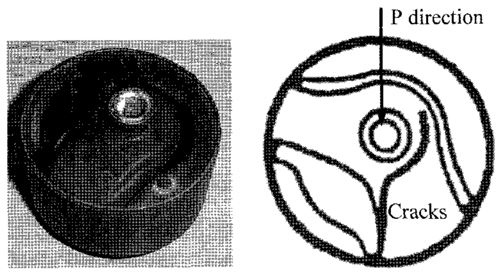

Figure 8. Two-bridge-type engine mount and cracked shape and load direction.

Figure 9. FE model and tearing energy distribution of engine mounts.

Table 3. Endurance load condition of engine mounts.

Load dir.	Load spec.
P dir. (W=520N)	1W 1.5W, 2W, 3W

occur due to large enough tensile load to tear the specimen at once and the compressional load. The difference between the theory and the experiment was generally 1.1–4.0 times, which can occur in an experiment with the identical fatigue specimen.

4. APPLICATION TO ENGINE MOUNT

4.1. Experiments

An identical procedure was applied to predict fatigue life of engine mounts. Existing methods or evaluating fatigue life with empirical equations needs to determine the damage parameter corresponding with the rubber compounds of each rubber component. Suggested methods in this paper can be applied to any NR rubber components without any additional experiments. Engine mount of two- bridge-type is shown in Figure 8. The most severe loading condition 1W±3W is applied among load specifications listed in Table 3. Fatigue tests were conducted in an ambient temperature of 23°C under the load-controlled condition with a sine waveform of 3 Hz, depending on the magnitude of load amplitude. The cracking of the engine mount would cause a decrease in tensile modulus. Failure is defined as the number of cycles at which the tensile modulus dropped by 20% generally. In this paper, test was conducted until the time when significant crack length was observed with four engine mount samples. Experimental results for each sample showed the range of fatigue life 5.0–8.6×10^5 cycles.

4.2. Predicted Fatigue Life of Engine Mounts

In order to define the hyperelastic material behavior, material parameters in the Ogden strain energy potential of the order N=3 was determined using the static uniaxial tension data supplied by the company. Engine mount is modeled with an 8-noded hexa hyperelastic element shown in Figure 9. A nonlinear finite element analysis program ABAQUS was used for the analysis of the engine mount. The maximum load 4W in the +P direction and the minimum load 2W in the –P direction were set as boundary conditions. Two different FE analyses were performed independently, and the results were combined to present the effect of each load condition. During the analyses, the maximum tearing energy was searched in the whole FE model within the range of 1% error. The critical crack length of rough cut growth 10 μm and the dynamic cut growth constant 200 were used. The tearing energy distribution is shown in Figure 9. The bright grey region indicates the predicted teared region corresponding with the experimental results. The maximum strain energy density was displayed at both sides of the bridges, and the magnitude was about 0.169547 kgf/mm^3. The corresponding predicted fatigue life was estimated with 6.957×10^5 cycles located in the mid-range of the experimental fatigue life. The estimated fatigue life and predicted fatigue life are listed in Table 4.

Table 4. Comparison between the fatigue test data and predicted life of engine mounts.

Fatigue test data (cycle)		Predicted life (cycle)
Sample 1	5.0×10^5	6.957×10^5
Sample 2	6.1×10^5	
Sample 3	7.2×10^5	
Sample 4	8.6×10^5	

5. CONCLUSION

An algebric expression of fatigue life prediction for rubber-like materials was developed using a rough cut growth rate equation and a simple tearing energy formulation. The applicablity of the rough cut growth rate equation has also been inquired previously about various cut growth behaviors of each different recipe. Fatigue life of dumbbell specimen was predicted using a developed equation incorporating the static finite element analysis. An identical tendency with experimental results from KIMM in various amplitudes and mean load conditions was shown. The predicted life was practically lower than that of experimental result which was estimated at a complete separation. The difference between the experiments and the prediction was theoretically expected. Fatigue life of automotive rubber component of engine mounts was predicted and in a fairly good agreement as showing the mean value of experimental fatigue life for several identical samples. Assumption about the non-directional property of crack growth imply an error due to neglecting the relationship between the loading and the crack growth direction. However, the predicted fatigue life in the case of apparent compression indicated an inconsiderable difference with the experiments, but was still overestimated. The effects of the compressional load on fatigue life of rubber need to be studied in detail.

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