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CRITICAL SPEED ANALYSIS OF JUDDERING DUE TO CHANGE IN SURFACE TEMPERATURE OF DISK BRAKE

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ABSTRACT−The change in the critical speed due to surface temperature of automotive disk brakes may be analyzed both theoretically as well as experimentally. Juddering of disk brakes is closely related to its critical speed. In analyzing the critical speed, if σ is positive, Disk develops TEI (Thermo-Elastic Instability) resulting in juddering in disk brakes. And σ is affected not only by the critical speed but also by the initial temperature of disk surface. As the initial temperature of the disk surface rises, the critical speed decreases and juddering is developed more easily. Also, when hot spots are developed by TEI, they show large temperature difference in small local range.

KEY WORDS : Thermo-elastic instability, Hot spot, Brake chattering, Juddering, Squeal noise

1. INTRODUCTION

As the use of the asbestos that causes environmental problems was restricted in automotive disk brakes, the juddering has emerged as a serious problem in automotive brake systems. Thermo-elastic Instability (TEI) that is developed by the friction between the brake and the pad contributes to frictional heating, thermo-elastic distortion and brake noise cause by elastic contact during braking. TEI is closely related to rotational speed and initial temperature of the surface of a disk. When the rotational speed increases, the thermal instability becomes high and the heating that occurs between the disk and the pad modifies the distribution of braking pressure randomly. Perturbations of the braking pressure and the surface temperature of the disk put the disk and the pad in local contact. This local contact forms hot spots on the disk surface. When the hot spots are developed on the disk surface, juddering occurs in the system (Dow and Burten, 1972; Lee and Barber, 1993).

In this paper automotive brake system was analyzed theoretically by assuming that the system was an ideal one that moved at a critical speed that develops TEI between two semi-infinite plates of disk pressed with uniform pressure. Critical speed causing the juddering to the automotive brake system was measured experimentally. Also, the changes in juddering and the critical speed that developed due to the initial difference of temperature of disk surface were analyzed.

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2. THEORETICAL BACKGROUND OF JUDDERING IN DISK BRAKES

2.1. Theory of TEI Caused by Sliding Contact

The brake system model assumes that the pad is sliding at a constant speed v against semi-infinite solid (brake). Analyzing the model, contact problem between two surfaces and distributions of temperature and contact pressure were surveyed ignoring the wear of disk and pad due to friction. The temperature of the disk and the contact pressure were analyzed in time. The corner surfaces of the plate and the semi-infinite solid show the thermal distortion because the contact pressure between them, and causes thermal expansion in strips and the solid. The fact that the corner part is very big in size compared to the solid gives us an advantage to calculate

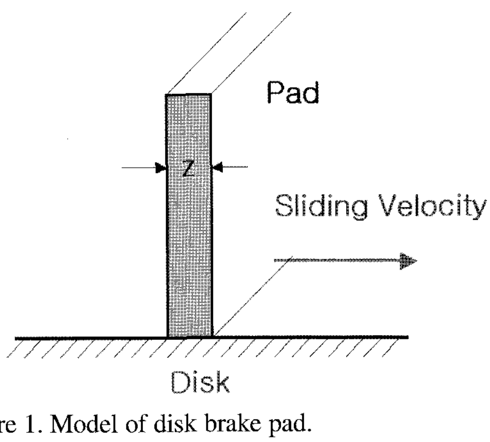

Figure 1. Model of disk brake pad.

the quantity of distortion on the interface easily when the solid is assumed to be a rigid body. The change in distribution of temperature that occurs in the contact of interface, T (x, y, z), can be shown as the following using Fourier series (Dow and Burten, 1972).

$$T = \sum_{n=1}^{\infty} T_0 F_n(y) e^{\sigma t} \sin n\pi \tag{1}$$

The temperature of the parts of the pad far from the contact region should be the same as the ambient temperature. The total heat Q_F due to friction experiences heat transfer because of the separation of the disk and the pad on the contact surface.

$$Q_F = \frac{\mu VPA}{J} \tag{2}$$

Where μ, P, A are the friction coefficients, contact pressure and contact area respectively and J=4.19 kJ/kg is Joule's coefficient that converts mechanical energy to thermal energy.

When Laplace transform is taken in terms of time variant, equation (1) reduces to

$$\int_0^{\infty} e^{-st} \frac{\partial T}{\partial t} dt = s \int_0^{\infty} e^{-st} T dt \equiv sL(T) \tag{3}$$

Thus, the equation of heat transfer is

$$\frac{\partial^2 L(T)}{\partial \bar{y}^2} - \frac{s}{\bar{k}} L(T) \tag{4}$$

And solution for this equation is

$$L(T) = \frac{T_0}{s} \exp\left\{ -\left(\frac{s}{\bar{k}}\right)^{\frac{1}{2}} \bar{y} \right\} \tag{5}$$

Therefore, the transferred heat quantity per strip, Q_s can be expressed as.

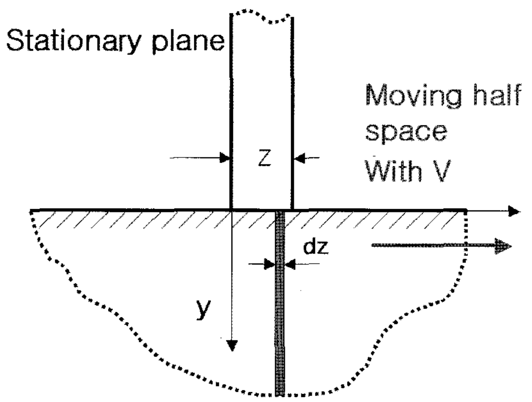

Figure 2. The calculated model that transfers heat quantity to the disk from the pad.

$$Q_s = -\bar{k} A_s \frac{\partial T}{\partial \bar{y}} = \bar{k} A_s T_0 \frac{e^{\bar{y}^2/4\bar{k}t}}{(\pi \bar{k} t)^{1/2}} \tag{6}$$

In equation (1), the index σ indicates the behavior of the changing temperature with time. When σ is positive, the temperature continues to rise in time and thus the system becomes unstable. In the case σ is zero, there is no change in temperature in time. Thus, the stability of the system can be determined at σ=0. When σ has negative values, the system becomes stable. In the present analysis, the temperature change was analyzed only for the following case.

$$T = T_0 F(y) e^{\sigma t} \sin \omega x \tag{7}$$

Where T_0 and ω are the initial temperature of the disk surface and the frequency of a sinusoidal perturbation.

2.2. Theoretical Analysis of TEI Model

The brake pad, which is given the thickness of z, shows drag braking at a constant speed v on the disk imposed as semi-infinite solid. When the thickness of the pad is assumed to be very small compared to the size of the disk, the heat conduction of the pad can be assumed to be two-dimensional and the governing heat-transfer is.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{k} \cdot \frac{\partial T}{\partial t} \tag{8}$$

Where k is thermal diffusivity (k=ρ/c). Since the pad is infinite in y direction, the area of the strip per unit length in x direction is dz and the heat quantity calculated at $\bar{y}$=0 becomes (Hill and Barber, 1985).

$$Qc = \frac{\bar{k} T_0}{(\pi \bar{k} t)} dz \tag{9}$$

And if the heat quantity transferred to the whole strips between 0 and z are summed up, the total heat quantity per unit length transferred from the interface to the semi-infinite solid becomes

$$Qc = \int_0^z \bar{k} T_0 \left(\frac{\upsilon}{\pi \bar{k}}\right)^{1/2} \frac{dz}{\sqrt{z}} = 2\bar{k} T_0 \left(\frac{\upsilon z}{\pi \bar{k}}\right)^{1/2} \tag{10}$$

Thus, if the boundary condition of heat transfer is applied to the interface (y=0)

$$Q_F - Q_C = \frac{\mu \upsilon P z}{J} - 2\bar{k} T_0 \left(\frac{\upsilon z}{\pi \bar{k}}\right)^{1/2} \tag{11}$$

In terms of a condition of developing TEI, the pattern of temperature change in disk can be shown as equation (1). The distribution of temperature that appears on the pad can be written as the following.

$$T = T_0 e^{-by} e^{\sigma t} \sin \omega x \tag{12}$$

Where b is defined as

$$b = \left[\frac{\sigma}{k} + \omega^2\right]^{1/2} > 0 \qquad (13)$$

The distribution of contact pressure P on the interface is

$$-P = E\alpha T_0 e^{\sigma t}\frac{k}{\sigma}\left[\omega^2 - \omega\left[\frac{\sigma}{k} + \omega\right]^{1/2}\right] \qquad (14)$$

Substituting the equation for the distribution of contact pressure on the interface and the equation (12) into (13), we obtain b as

$$b = \frac{\mu\upsilon\alpha E}{kJ}\cdot\frac{k\omega}{\sigma}\left[\left(\frac{\sigma}{k} + \omega^2\right)^{1/2} - \omega\right] - \frac{2\bar{k}}{k}\left(\frac{\upsilon}{\pi k\bar{z}}\right)^{1/2} \qquad (15)$$

Using this equation, σ can be estimated from the moving speed of the solid v. When $\sigma=0$, it means that the temperature distribution of interface belongs to the region of critical speed where temperature distribution changes in time. That is, at a speed of the disk corresponding to $\sigma=0$, the changing temperature obtains the neutral stability where it keeps its initial values. In this case,

$$\omega = \frac{\mu\upsilon\alpha E}{2kJ} \qquad (16)$$

It can be seen that the value of the critical speed increases as the turbulence frequency rises. Two constants C_h and C_c are defined as

$$C_h = \frac{\mu\upsilon\alpha E}{kJ} \text{ and } C_c = 2\frac{\bar{k}}{k}\left(\frac{\upsilon}{\pi\bar{k}z}\right)^{1/2} \qquad (17)$$

Rearranging this equation for b,

$$b = -\frac{\omega + C_c}{2} + \frac{1}{2}[(\omega + C_c)^2 + 4\omega(C_h - C_c)]^{1/2} \qquad (18)$$

The condition that b should always have positive values is $C_h > C_c$ and

$$\upsilon_L = \frac{4\bar{k}^2J^2}{(\mu\alpha E)^2}\frac{1}{\pi k\bar{z}} \qquad (19)$$

Estimating σ with respect to moving speed v_L, σ is given as (Cho and Ahn, 2002)

$$\sigma = \left\{\left[-\frac{\omega + C_c}{2} + \frac{1}{2}[(\omega + C_c)^2 + 4\omega(C_h - C_c)^{1/2}]^{1/2}\right]\right\} \times k$$

$$\sigma = \left\{\frac{(\omega + C_c)^2}{4}\cdot\frac{(\omega + C_c)}{2}[(\omega + C_c)^2 + 4\omega(C_h - C_c)]^{1/2}\right.$$

$$\left. + \frac{1}{4}(\omega + C_c)^2 + 4\omega(C_h - C_c)^2 - \omega^2\right\} \qquad (20)$$

Using the equation (20), the relationship between V_L and σ of the disk made of cast iron can be expressed as a graph (Figure 3). When the disk is made of cast iron, its critical speed is approximately 155 km/h. Thus, σ is negative if it is under the critical speed and this is the condition in which TEI can hardly occur. When σ is positive, the distributed temperature of the disk increases exponentially with respect to time and TEI can occur easily. Thus, the disk brake gets the distribution of temperature that can develop juddering.

The critical speed changes according to the initial temperature of the disk surface. It is high when the initial temperature of the surface is low, while it is low when the temperature of the surface is high. Also, it can be seen that as the temperature of the surface gets higher the juddering gets easier and more serious to be developed (Barber and Zhang, 1990).

Table 1. Brake design values.

	Lining	Pad	Disk	Units
Density (ρ)	2034	2595	7228	kg/m^3
Specific heat	1256	1465	419	Nm/kgK
Conductivity ($\bar{k}$)	4174	4362	174465	Nm/h km
Diffusivity (k)	0.00163	0.0011	0.0576	m^2/h

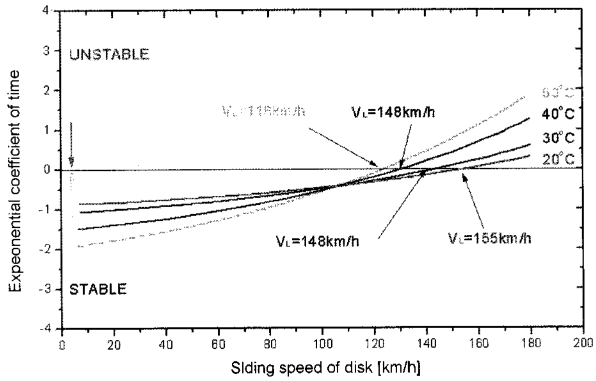

Figure 3. Moving velocity (v_L) of the disk conform with change the time constant (σ).

3. THE EXPERIMENTAL METHOD

The disk of an automotive vehicle was made to rotate at a constant rpm using AC motor. The rotational speed of the disk can be calculated as the rpm of the rotational axis, measured by a tacho meter. Supplying the caliper with the oil pressure that is formed using the vacuum boaster of the automotive vehicle operates brake pad. The oil-pressure applied to brake pad is measured by attaching the pressure gauge to the master cylinder. The oil-pressure is kept constant during the experiment. When the friction is developed in contacting with the disk pad,

Table 2. Specifications of the experimental apparatus.

Motor part	AC motor (3000 rpm, 30 kw) Motor controller (inverter)
Test part	Disk brake Brake pad
Braking part	Vacuum booster Master cylinder Caliper
Sensor part	Infrared thermometer (Ray tek) Tacho meter Pressure gauge
Data acquisition	PC (Data logger)

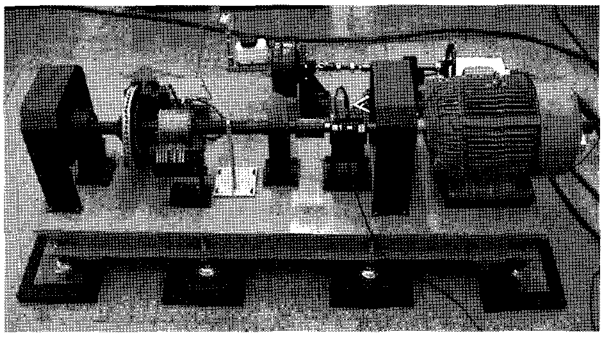

Figure 4. Photograph of experimental apparatus for automotive disk brake.

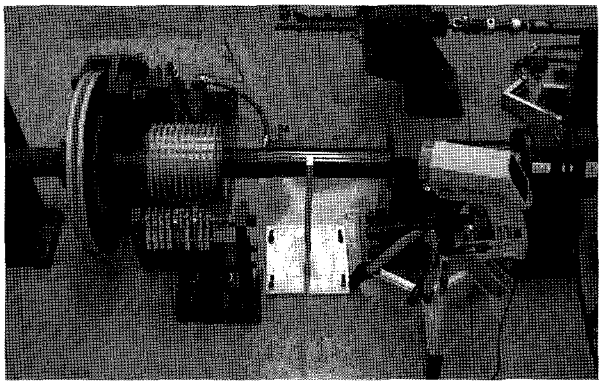

Figure 5. Photograph of temperature measurement system with infrared thermometer.

it can be seen that rpm decreases. To keep the rotational velocity constant in this case, the feedback process using an encoder controls AC motor. Thus, the change in rotational speed hardly occurs during the experiment. The temperature of the disk surface was measured by the non-contact method using the infrared thermometer. The measurement was accomplished at a location 180° away from the fixed part of the caliper.

It is important that the disk should be rotated at a constant speed to be able to measure the critical speed in juddering that is caused by TEI in the disk brake. And the friction is made to appear between the disk and the pad when a constant pressure is applied to the brake pad. The pressure set up in the experiment is 10 bar. The heating is caused by friction and the surface of the disk shows TEI when σ becomes positive due to the rotational speed. And the TEI modifies the distribution of temperature on the surface of the disk, which then causes juddering. Thus, the critical speed at which juddering is developed can be observed as the change in the distribution of temperature on the disk surface, as measured by the infrared thermometer.

4. EXPERIMENTAL RESULTS AND DISCUSSION

The following results were obtained for a brake system whose disk is 15 inches diameter while the pressure on the disk pad was 10 bar. When the initial temperature of the disk surface was 20°C, the critical speed of the disk was 150km/h and the juddering took about 550 seconds to be developed. Figure 6 shows that the temperature difference between the theory and the experiment is about 15 at the critical speed. When the initial temperature of the disk surface was 30°C, the critical speed of the disk reduced to 142 km/h and the juddering took 420 seconds to be developed. Figure 7 shows that the temperature difference between the theory and the experiment is about 12 at the critical speed. When the initial temperature of the

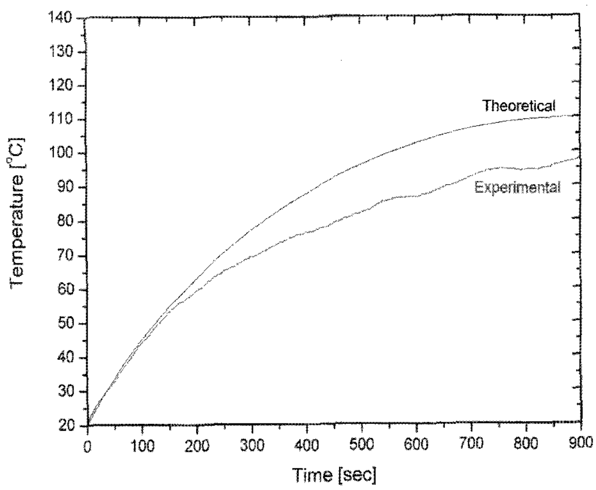

Figure 6. Graph showing the temperature of the disk surface at the critical speed V_L=150 km/h (the initial temperature of the surface: 20°C).

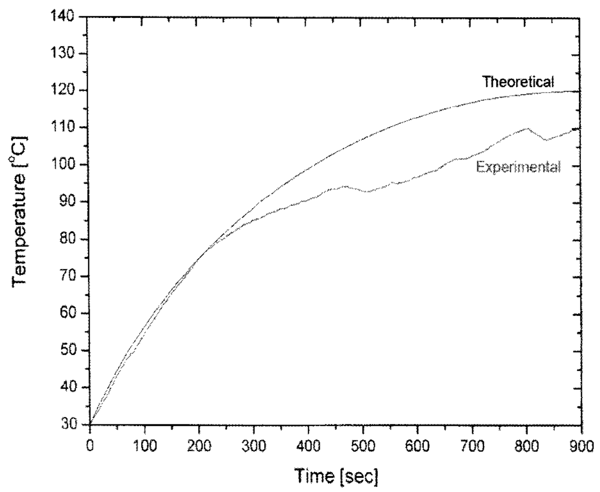

Figure 7. Graph showing the temperature of the disk surface at the critical speed V_L=142 km/h (the initial temperature of the surface: 30°C).

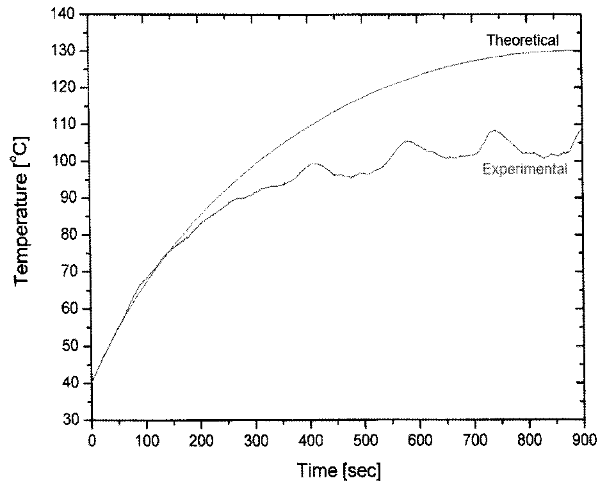

Figure 8. Graph showing the temperature of the disk surface at the critical speed V_L=120 km/h (the initial temperature of the surface: 40°C).

disk surface was 40°C, the critical speed of the disk reduced to 120 km/h and the juddering took 350 seconds to be developed. Figure 8 shows that the temperature difference between the theory and the experiment is about 15 at the critical speed. It can be seen that there was a great disparity of value between the theory and the experiment when juddering occurred. It can be seen that the critical speed of the disk reduced rapidly to 105 km/h when the initial temperature of the surface was 50°C and the juddering took 210 seconds to be developed. Figure 9 shows that the temperature difference between the theory and the experiment is about 25 at the critical speed. It can be seen that the disparity of value between the theory and

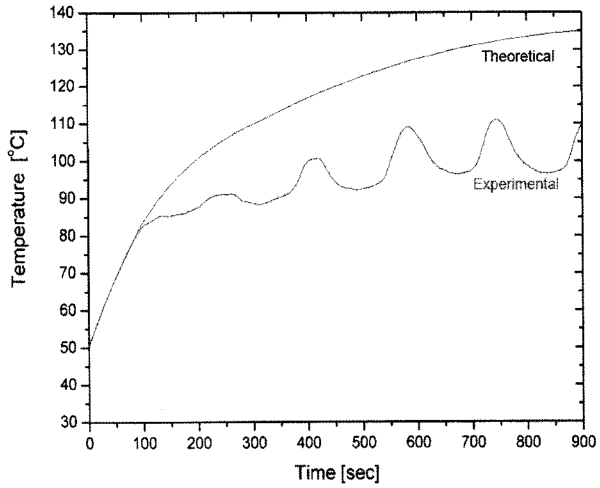

Figure 9. Graph showing the temperature of the disk surface at the critical speed V_L=150 km/h (the initial temperature of the surface: 50°C).

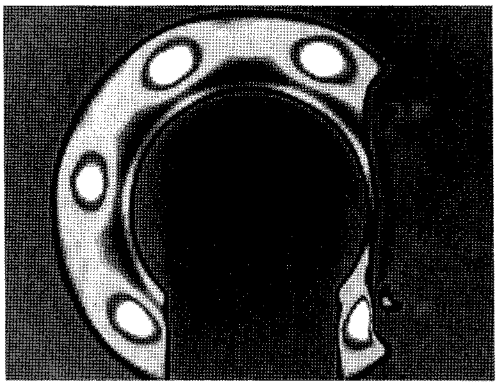

Figure 10. Hot spot on the disk surface during the juddering (Heat Infrared Camera: JADE MWIR).

the experiment grew very large in time after juddering occurred. It can be seen that when the disk reached its critical speed, hot spot was caused on the disk surface by TEI. In general, when hot spot occurs on the disk surface, juddering develops. Figure 10 discover the temperature distribution on the disk surface investigated using a high speed heat infrared camera when juddering occurred.

5. CONCLUSION

When the disk of an automotive vehicle is made of cast iron, σ can be seen to show small change of value in speed regions. And in the case when σ is positive, TEI is developed easily, so the juddering in the disk brake is developed. It has been shown that the value of σ can be seen both theoretically and experimentally to change according not only to the critical speed of the disk but also the initial temperature of the disk surface. If the initial temperature of the disk surface is high, the critical speed decreases and the juddering are developed at a low speed. When the temperature of the surface is high, the juddering takes short time to be developed and is caused easily by TEI. Also, it can be seen that when juddering is developed, the temperature distribution of the disk surface shows large difference in temperature and hot spot is developed on the disk surface. The discrepancy the temperature distribution in time between the theory and the experiment is due to cooling of the disk by convection as the disk rotates.

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