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PREVIEW CONTROL OF ACTIVE SUSPENSION WITH INTEGRAL ACTION

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ABSTRACT−This paper is concerned with an optimal control suspension system using the preview information of road input based on a quarter car model. The main purpose of the control is to combine good vibration isolation characteristics with improved attitude control. The optimal control law is derived with the use of calculus of variation, consisting of three parts. The first part is a full state feedback term that includes integral control acting on the suspension deflection to ensure zero steady-state deflection in response to static body forces and ramp road inputs. The second part is a feed-forward term which compensates for the body forces when they can be detected, and the third part depends on previewed road input. The performance of the suspension is evaluated in terms of frequency domain characteristics and time responses to ramp road input and cornering forces. The effects of each part of the suspension controller on the system behavior are examined.

KEY WORDS : Preview information, Integral action, Attitude control, Body force

1. INTRODUCTION

Recently active suspensions with preview of road disturbances originally proposed by Bender (1968) have been subject to renewed interest because of advances in microprocessor and sensor technologies that make this type of strategy realizable in practice. Preview information about the road disturbances may be obtained either from preview sensors located in front of the vehicle that measure the road unevenness (relative to the vehicle body), or by measuring motions of the front suspension and utilizing this information to control the rear suspension under the assumption that the rear wheels traverse the same path as the front wheels (Hac and Youn, 1993; Louam *et al.*, 1992). The control laws for preview suspensions are typically synthesized by minimizing a performance index that trades-off measures of ride comfort, road holding, suspension working space, and control effort as expressed by the mean-square values of body acceleration, tire deflection, suspension deflection, and control force, respectively, for a random road input (Hac, 1992; Louam *et al.*, 1992). It has been demonstrated that the presence of preview information can simultaneously improve all of the above aspects of performance (Hac, 1992). A drawback of this approach to suspension design is that it is mainly concerned with improving control of the vehicle vibration, in particular, improvements in isolation of the vehicle body (ride comfort) and wheel tracking performance (road holding). Good ride and handling, however, depend not only on vibration characteristics, but also on the ability of a suspension system to counteract body forces resulting from changes in payload, aerodynamic forces, braking, acceleration, or cornering maneuvers. It is known that optimally controlled suspensions (without preview) do not have zero steady-state deflection in response to static body forces or ramp road inputs (Davis and Thompson, 1988; ElMadany, 1990). Since active suspensions usually have lower body natural frequencies and therefore smaller stiffness than passive systems, in order to ensure better ride comfort, large suspension deflections may be expected in response to quasi-static loads. As a result, a useful working space of the suspension may be considerably reduced. Davis and Thomson (1988) and Thompson and Davis (1988) have shown in their studies of suspensions without preview that by including an additional term involving an integral of the suspension deflection in the performance index, it is possible to obtain a system with infinite stiffness against static loading. Hence the natural frequency of the body can be made lower without reducing the static stiffness required to resist body and maneuvering forces. It has been shown by ElMadany (1990) that when the integral action is not very strong, the performance of the suspension in terms of vibration isolation, suspension travel and tire deflection is only marginally reduced as compared to an optimal system without integral control, while vehicle attitude

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control is vastly improved. The rms values depend on the vehicle velocity and road roughness and the computations need the solution of Riccati and Lyapunov matrix equations. For a preview active suspension, Thompson and Pearce (2001a, 2001b, 2003) have derived a convenient matrix expression for the performance integral and the identical and found techniques for the direct computation of rms values for control force, suspension stroke and tire deflection on a random road. Youn *et al.* (2006) applied the preview control to a fast moving tracked vehicle to improve the ride comfort charactristics.

The purpose of this paper is to examine the improvements in attitude control of an active suspension with preview that can be achieved by including an integral action in the preview controller and by adding a feed-forward term to reduce the effect of body forces when they can be measured or estimated. Integral action is achieved by inclusion of an additional term involving an integral of the suspension deflection in the performance index, the state vector, and the corresponding state equation. It is assumed that in addition to finite preview information about the road input, information about the current body forces may be available. This assumption is justified because the body force can usually be determined from existing measurements. For example, the forces resulting from braking and acceleration could be obtained as functions of measured forward acceleration or brake fluid pressure.

The optimal suspension force is synthesized using a variational approach. The optimal control law consists of three parts: a full state feedback term (including the feedback of the integrated suspension deflection), a feedforward term that depends on the present body force, and a preview term that depends on previewed read input. The results of this analysis are applied to quarter-car models with various types of suspensions. The responses of the model to a ramp road input and to a body force corresponding to an outside quarter of a car during cornering were simulated. Frequency domain characteristics in regard to vibration isolation of the body, road-holding ability and suspension travel are obtained.

2. SYSTEM MODEL

In this section an optimal preview and integral control problem for a quarter car model with an active suspension is formulated. The vehicle system model considered here is shown in Figure 1. In the Figure, m_1, m_2, k_1, b, and k_2 denote a quarter of the body mass, the mass of the wheel with a semi-axle, the suspension stiffness, the coefficient of viscous damping, and the tire stiffness, respectively. The variables $z_1(t)$ and $z_2(t)$ denote absolute vertical displacements of the body and the wheels measured with respect ot the position of equilibrium,

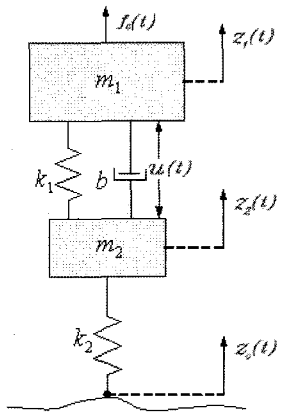

Figure 1. A quarter car model with active suspension.

while $u(t)$ describes the suspension control force, which may be manipulated by the designer. The system is under the influence of two types of disturbances: an external force $f_0(t)$ acting directly on the vehicle body, and a road disturbance described by the road elevation, $z_0(t)$. It is assumed that the road elevation, z_0, and its rate of change, $\dot{z}_0$, can be measured at a given distance L ahead of the vehicle, i.e. $z_0(\tau)$ and $\dot{z}_0(\tau)$ for $\tau \in [t, t+t_p]$ are known, where $t_p=L/v_0$ is the preview time and v_0 is forward velocity of the vehicle. It is considered that the body force, $f_0(t)$, is also known but no preview information about it is available. This assumption is reasonable since the body force can usually be detected through measurements that are used in feedback controlled suspensions. For example, the variation of the payload can be obtained by calculating the difference between the static suspension deflections with and without the payload and multiplying it by a given spring constant. The body force in braking and acceleration could be obtained from the measured vehicle forward acceleration, or brake circuit pressure, or the angular acceleration of body pitch if it is measured for other purposes. Leaning force in cornering can be calculated using the angle of the steering wheel and the forward speed of vehicle. The aerodynamic drag force at high speeds can be computed from the vehicle speed.

The dynamics of the system in Figure 1 is described by

$$m_1\ddot{z}_1+k_1(z_1-z_2)+b(\dot{z}_1-\dot{z}_2)=u+f_0 \tag{1a}$$

$$m_2\ddot{z}_2+k_1(z_2-z_1)+b(\dot{z}_2-\dot{z}_1)+k_2(z_2-z_0)=-u \tag{1b}$$

It is noteworthy that adding equations (1a) and (1b) on both sides yields

$$f_0(t)=m_1\ddot{z}_1(t)+m_2\ddot{z}_2(t)+k_2[z_2(t)-z_0(t)] \tag{2}$$

Hence, the body force $f_0(t)$ can be obtained from the above equation, at least in principle, when all the quantities on the right handside are directly measured or estimated from measurements. These measured variables, however, include the tire deflection $z_2(t)-z_0(t)$ which is not easy to obtain in practice. Therefore, one of the methods of calculating $f_0(t)$ described earlier appears to be more practical.

To synthesize the control force $u(t)$, we use optimal control theory. In this approach an optimal suspension force is synthesized that minimizes a performance index that trades-off measures of ride comfort, suspension rattle space, road holding and control effort. Therefore the variances of the body acceleration $\ddot{z}_1$, the body to axle displacement $z_1(t)-z_2(t)$, dynamic tire force, which is proportional to the tire deflection $z_2(t)-z_0(t)$, and the magnitude of the control force $u(t)$, are included in the performance index. In addition, a term depending on an integral of the suspension deflection is included in the index to achieve a zero steady-state deflection against static body forces or ramp road inputs. The performance index to be minimized can be written as

$$J=\lim_{T\to\infty}\frac{1}{2T}\int_0^T\{\ddot{z}_1^2+\rho_1(z_1-z_2)^2+\rho_2(z_2-z_0)^2$$
$$+\rho_3\left[\int_0^T(z_1-z_2)d\tau\right]^2+\rho_4u^2\}dt \tag{3}$$

where ρ_i, i=1, 2, 3, and 4, are the weighting constants reflecting the designer's preferences regarding the relative importances of various aspects of performances.

We introduce the following state vector

$$\text{x}=\left[(z_1-z_2)\ \dot{z}_1(z_2-z_0)\ \dot{z}_2\ \int_0^t(z_1-z_2)d\tau\right]^T \tag{4}$$

where the last state variable $x_5=\int^t(z_1-z_2)d\tau$ satisfies the following differential equation[0]

$$\dot{x}_5=x_1 \tag{5}$$

The equations of motion (1) along with equation (5) can be expressed as a system of five first order differential equations which can be written as the following matrix state equation

$$\dot{\text{x}}=\text{Ax}+\text{B}u+\text{Dw} \tag{6}$$

where $\text{w}\in\Re^5$ is the state vector, $u\in\Re^1$ the control input and $\text{w}\in\Re^2$ the disturbance. Matrices **A**, **B**, and **D** and vector **w** are given by

$$\text{A}=\begin{bmatrix}0 & 1 & 0 & -1 & 0\\ -k_1/m_1 & -b/m_1 & 0 & b/m_1 & 0\\ 0 & 0 & 0 & 1 & 0\\ k_1/m_2 & b/m_2 & -k_2/m_2 & -b/m_2 & 0\\ 1 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\text{B}=\begin{bmatrix}0 & 1/m_1 & 0 & -1/m_2 & 0\end{bmatrix}^T,$$

$$\text{D}=\begin{bmatrix}0 & 1/m_1 & 0 & 0 & 0\\ 0 & 0 & -1 & 0 & 0\end{bmatrix}^T,\quad \text{w}=\begin{bmatrix}f_0(t)\\ w_0(t)\end{bmatrix} \tag{7}$$

where $w_0(t)=\dot{z}_0(t)$, i.e. $w_0(t)$ is the ground velocity input.

Using equation (1a) to substitute for $\ddot{z}_1$, along with equations (4) and (5), the performance index (3) can be expressed as a quadratic form of the state vector, **x**, the control input u, and the disturbance **w**. That is

$$J=\lim_{T\to\infty}\frac{1}{2T}\int_0^T\{\text{x}^\text{T}\text{Qx}+2\text{x}^\text{T}\text{N}_1u+2\text{x}^\text{T}\text{N}_2\text{w}$$
$$+2\text{x}^\text{T}\text{M}_1u+\text{w}^\text{T}\text{M}_2\text{w}+Ru^2\}dt \tag{8}$$

where

$$\text{Q}=\frac{1}{m_1^2}\begin{bmatrix}k_1^2+\rho_1m_1^2 & k_1b & 0 & -k_1b & 0\\ k_1b & b^2 & 0 & -b^2 & 0\\ 0 & 0 & \rho_2m_1^2 & 0 & 0\\ -k_1b & -b^2 & 0 & b^2 & 0\\ 0 & 0 & 0 & 0 & \rho_3m_1^2\end{bmatrix}$$

$$\text{N}_1=\frac{1}{m_1^2}\begin{bmatrix}-k_1 & -b & 0 & b & 0\end{bmatrix}^T,$$

$$\text{N}_2=\frac{1}{m_1^2}\begin{bmatrix}-k_1 & -b & 0 & b & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}^T,$$

$$\text{M}_1=\begin{bmatrix}1/m_1^2 & 0 & 0 & 0 & 0\end{bmatrix}^T,$$

$$\text{M}_2=\begin{bmatrix}1/m_1^2 & 0\\ 0 & 0\end{bmatrix},\quad R=\frac{1}{m_1^2}+\rho_4 \tag{9}$$

Our objective now is to find an optimal input $u(t)$, that minimizes the given performance index J subject to dynamic constraint (6).

3. ANALYTICAL SOLUTION

We begin this section with a precise definition of the problem.

Definition 1. Consider the linear system described by equation (6) with the given initial condition $\text{x}_0(t)=\text{x}_0$. Assume that $w_0(t)$ is an *a priori* unknown disturbance input which is modeled as a white noise process with zero mean, and $w_0(\tau)$, $\tau\in[t,t+t_p]$ where t_p is a preview time, is given deterministically. The body force $f_0(t)$ is known at the present time instant t but its future is unknown and $f_0(t)$ is assumed to have white noise characteristics. Consider also the performance index (8) in which **Q** is a symmetric and nonnegative definite weighting matrix, and R is a positive scalar such that

$$\text{Q}_\text{n}=\text{Q}-\text{N}_1R^{-1}\text{N}_1^\text{T} \tag{10}$$

is nonnegative definite. Then the problem of determining an input

$$u(t)=f[x(t), f_0(t), w_0(t+\sigma), 0 \le \sigma \le t_p], \ t_0 \le t \le T \quad (11)$$

that minimizes the performance index (8) can be called an *optimal preview control problem with integral action.*

As indicated in the above definition. The controller has preview information with respect to $w_0(t)$ from the present time t up to f_p time units in the future beyond t. Since the time duration of the problem $T \rightarrow \infty$, the preview time $t_p < T$. Since $w_0(t)$ denotes the ground velocity input, treating it as a white noise process is reasonable, especially when accurate statistical description of road unevenness is not abailable. Note that the control law (11) may depend on the disturbing body forces, $f_0(t)$ at the present time. This force can be estimated from measurements as described in the previous section. The assumption that $f_0(t)$ has a white noise characteristic implies that the values of the process at two time instants t and $t+\Delta t$ are uncorrelated no matter how small Δt is. Hence our knowledge of $f_0(t)$ at time t does not provide any information about $f_0(t)$ for $\tau > t$. This assumption is again quite reasonable since the body forces resulting from braking or cross-winds may vary rapidly as compared with time constants related to body dynamics. Since matrix Q_n is symmetric and positive definite it can be factored as $Q_n=Q_n^{1/2}Q_n^{1/2}$. In order to shorten the subsequent equations, we introduce the following notation:

$$A_n=A-BR^{-1}N_1^T, \ N_n=N_2-N_1R^{-1}M_1^T, \\ D_n=D-BR^{-1}M_1^T \quad (12)$$

We can now state the main result of this section.

Theorem 1. Consider the system described by equation (6) under the performance index (8). Suppose that the pair (A_n, B) is stabilizable and $(A_n, Q_n^{1/2})$ is detectable. Assume that the components of the disturbance vector $\mathbf{w}(t)$ satisfy the following: $w_0(t)$ is a stochastic white noise process with zero mean value and $w_0(\tau)$, $\tau \in [t, t+t_p]$ is known deterministically, while $f_0(t)$ is also white noise process with a mean value of zero and is known at the present time t. Then the control law that minimizes the performance index J (equation 8) for the system (6) is given by

$$u=-R^{-1}[(N_1^T+B^TP)x+M_1^Tw+B^Tr] \quad (13)$$

where $\mathbf{P}$ is a positive definite solution of the algebraic Riccati equation

$$Q_n+A_n^TP+PA_n-PBR^{-1}B^TP=0 \quad (14)$$

and the vector $\mathbf{r}(t)$ is

$$r(t)=\int_0^{t_p} e^{A_c^T\sigma}(PD_n+N_n)\begin{bmatrix}0\\ w_0(t+\sigma)\end{bmatrix}d\sigma \quad (15)$$

The closed loop system is described by

$$\dot{x}=A_cx+D_nw+BR^{-1}B^Tr \quad (16)$$

where

$$A_c=A-BR^{-1}(N_1^T+B^TP) \quad (17)$$

is the closed loop system matrix which is asymptotically stable (i.e. all of its eigenvalues have negative real parts).

Proof uses calculus of variation and is a relatively straightforward extension of that given by Hac (1992) for a system without an external body force and without integral action. It is omitted.

The control law consists of three parts: 1) the feedback part $-R^{-1}(N_1^T+B^TP)x(t)$ which uses full state feedback. This part is exactly the same as in corresponding LQ problem without preview. It reduces system sensitivity to parameter variations. Unmodeled dynamics and unmodeled external disturbances: 2) the feedforward part $R^{-1}M_1^Tw$ which reduces the effect of the body force disturbance, $f_0(t)$ on the body motion (see the following explanation) and 3) the preview part $-R^{-1}B^Tr(t)$ which utilizes all available information about future road disturbances on the system dynamics.

From the form of M_1 given by equation (9) and the definition of $\mathbf{w}(t)$ (equation 7) it follows that the feedforward term, $R^{-1}M_1^Tw$, is equal to

$$R^{-1}M_1^Tw=\frac{1}{1+\rho_4 m_1^2}f_0(t) \quad (18)$$

That is, it depends only on $f_0(t)$. Furthermore, if the weighting constant in the performance index corresponding to the control input, $\rho_4=0$, then $R^{-1}M_1^Tw=f_0(t)$ and the suspension fully compensates for the body force. Otherwise only partial compensation is possible.

In evaluating performance of a suspension with regard to vibration isolation, frequency domain characteristics play an important role. Since the closed loop system is linear, the frequency domain transfer matrix defined by the following relation

$$x(j\omega)=G(j\omega)w(j\omega) \quad (19)$$

can be obtained by taking Fourier transforms on both sides of equations (15) and (16). After simple manipulations, this yields

$$G(j\omega)=(j\omega I-A_c)^{-1}[-BR^{-1}B^T \times \\ \int_0^{t_p} e^{A_c^T\sigma}(PD_n+N_n)\begin{bmatrix}0\\ e^{j\omega\sigma}\end{bmatrix}d\sigma+D_n] \quad (20)$$

where I is the identiy matrix. Matrix $G(j\omega)$ is of dimension 5×2. The desired frequency domain characteristics may be obtained by plotting individual entries of $G(j\omega)$ versus frequency. For example, the transfer function between the ground velocity input $w_0=\dot{z}_0$ and body acceleration $\dot{x}_2=\ddot{z}_1$ corresponds to $j\omega G_{2,2}(j\omega)$ where $G_{2,2}(j\omega)$ is an element of the second row and the second column of $G(j\omega)$.

4. RESULTS OF NUMERICAL SIMULATIONS

In this section selected results of simulations for a 2-DOF vehicle model with passive and various active suspensions are presented. The following values of vehicle parameters were used in the simulations:

$$m_2/m_1=0.1\ ,\ k_1/m_1=36\ [N/(m\times kg)],$$
$$k_2/m_1=360\ [N/(m\times kg)],$$
$$b/m_1=3.0\ [Ns/(m\times kg)]. \tag{21}$$

These values correspond to the natural frequencies of the body and the wheel of about 1 Hz and 10 Hz, respectively, for a passive suspension. The following weighting constants in the performance index were used: $\rho_1=5\times10^2$, $\rho_2=10^4$, $\rho_3=5\times10^3$, $\rho_4=0$. They correspond to a suspension that emphasizes ride comfort over road holding. The value of ρ_3, corresponding to the integral of the suspension deflection, was selected by trial and error to yield an improvement of attitude control with small penalty in terms of vibration characteristics, as discussed below.

In Figures 2 and 3, the frequency domain characteristics in terms of amplitude ratios of body acceleration, suspension deflection and tire deflection to ground velocity input are shown. In Figure 2 the characteristics for a passive suspension and two active systems: with and without integral action are shown. In Figure 3 the same characteristics are shown with active systems using preview information about road disturbances. The preview time was t_p=0.3 sec. It can be seen that by including an integral action in the suspension controller, suspension deflections at the very low input frequencies are reduced. Other aspects of suspension performance, however, deteriorate. The degree of deterioration of performance increases as the weight ρ_3 on the integral term is increased. The value of ρ_3 selected in this study ensures that the performance in terms of vibration characteristics of systems with integral action is close to that of the optimal suspensions with proportional feedback. As observed in earlier studies (Hac, 1992; Hac and Youn, 1993), simultaneous improvements in all three characteristics are brought about by preview information.

In particular, suspension deflections at frequencies below the sprung mass natural frequency are reduced to a level below that of passive system. The characteristics for the suspension deflections and tire deflections exhibit deep decreases at and around wheel hop natural frequency. At that frequency the characteristic for body acceleration has an invariant point (Hedrick and Butsuen, 1988). That is, characteristics for all passive and active systems have to pass through that point and tire deflection may be drastically reduced at that frequency without compromising ride comfort.

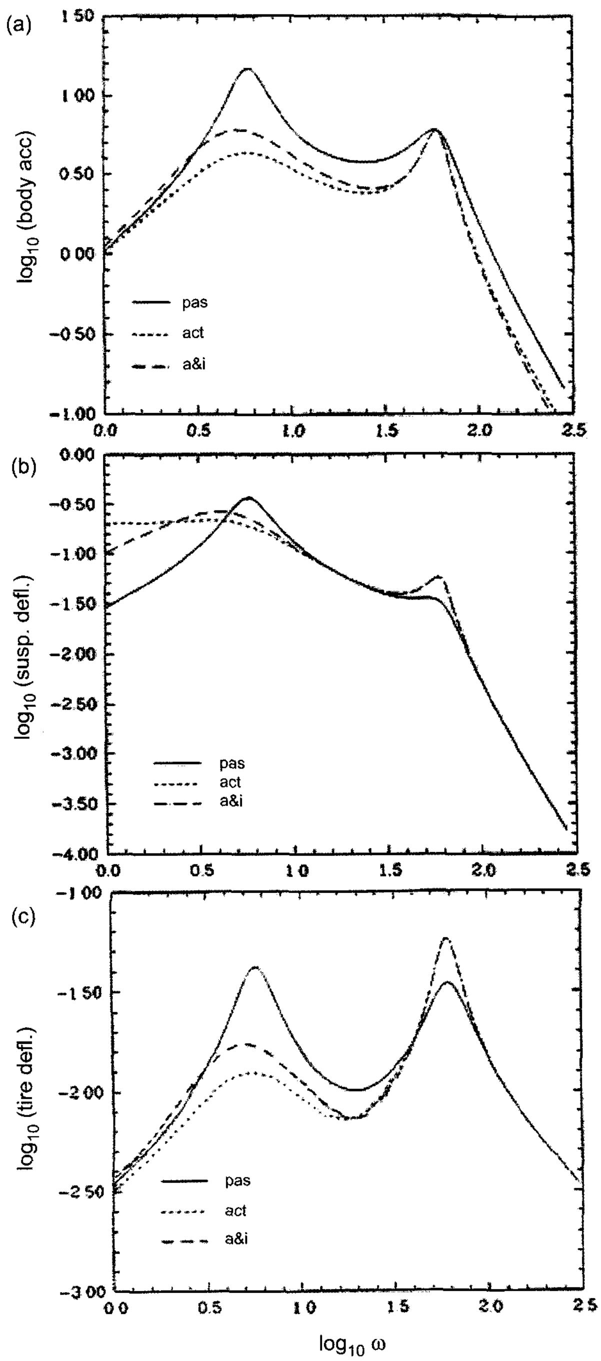

Figure 2. Frequency characteristics of various suspension systems without preview: (a) body acceleration; (b) suspension deflection; (c) tire deflection.

In Figure 4 responses of the vehicle model to a ramp

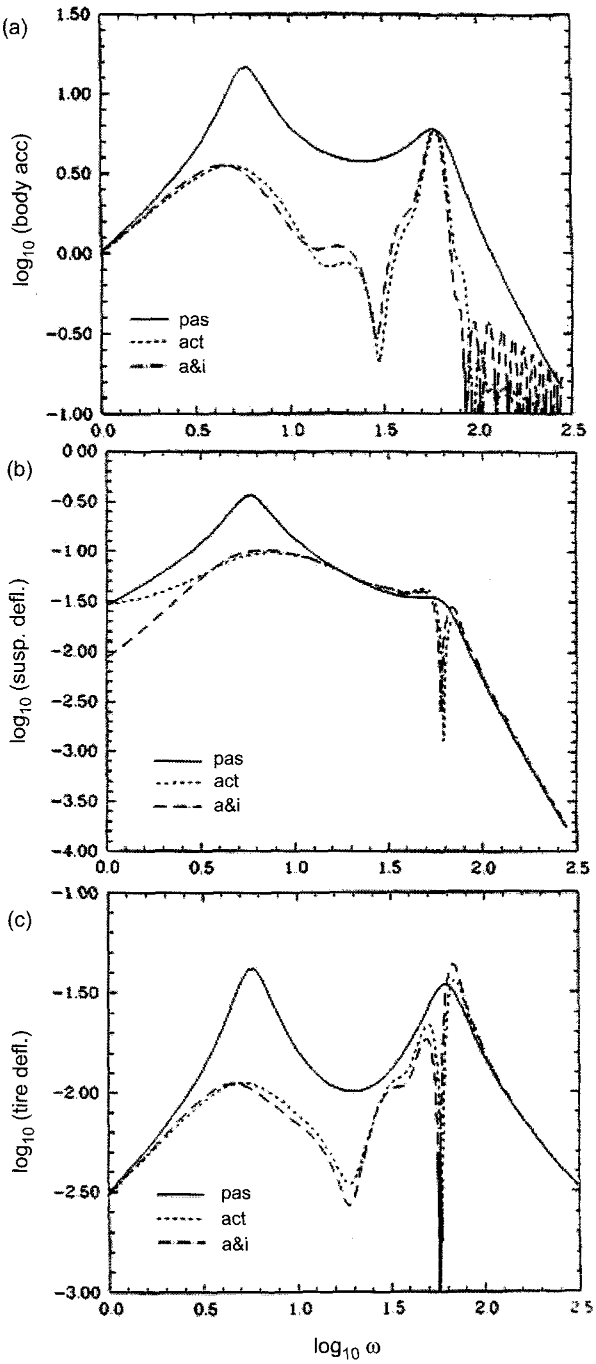

Figure 3. Frequency characteristics of various suspension systems with preview: (a) body acceleration; (b) suspension deflection; (c) tire deflection.

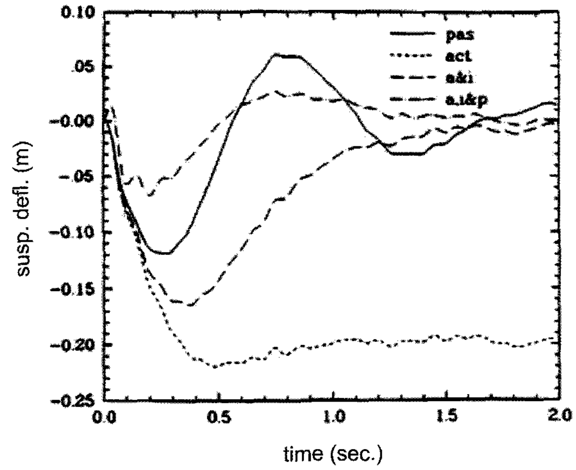

Figure 4. Time responses of the vehicle model with various suspensions to a ramp road input.

road input are shown. The slope of the road was assumed to be 1 *m* per 20 *m*. The road unevenness around the mean value is described by a stationary stochastic process with spectral density

$$S(\omega)=\frac{\sigma^2}{\pi}\frac{a v_0}{\omega^2+(a v_0)^2} \tag{22}$$

where v_0, the forward velocity, was assumed to be 20 m/s, ω-circular frequency, σ the standard deviation of road unevenness and a a constant parameter depending on the type of road surface. The values of a=0.15 m^{-1} and $\sigma^2=9\times10^{-6}$ m^2 were assumed. Using this road input, the responses of the vehicle model in terms of suspension deflection for the passive suspension, active without an integral term, active with integral action, and the active system with preview and integral control were computed. The active suspension without an integral term exhibits considerable steady-state error (offset) in suspension deflection due to a constant ground velocity input on a sloping road. Integral action eliminates offset, while the presence of preview improves the transient performance by reducing maximum overshoot and settling time. The transient response to the ramp input can also be improved by increasing the weighting of the integral term in the performance index. The performance characteristics, however, in terms of vibration isolation of the body, suspension travel, and tire deflection, deteriorate with increasing the weightimg. Hence a compromise is necessary.

In order to examine the attitude control of a vehicle in response to an external body force, a cornering maneuver was simulated. The body force, corresponding to the outer quarter of vehicle is assumed to be described by the following function

$$f_0/m_1=\begin{cases} A\sin 2\pi\tau & if \quad t_0\le t\le t_0+0.5 \\ A & if \quad t_0+0.5<t<t_0+1.5 \\ A\cos 2\pi\tau & if \quad t_0+1.5\le t\le t_0+2 \\ 0 & otherwise \end{cases} \tag{23}$$

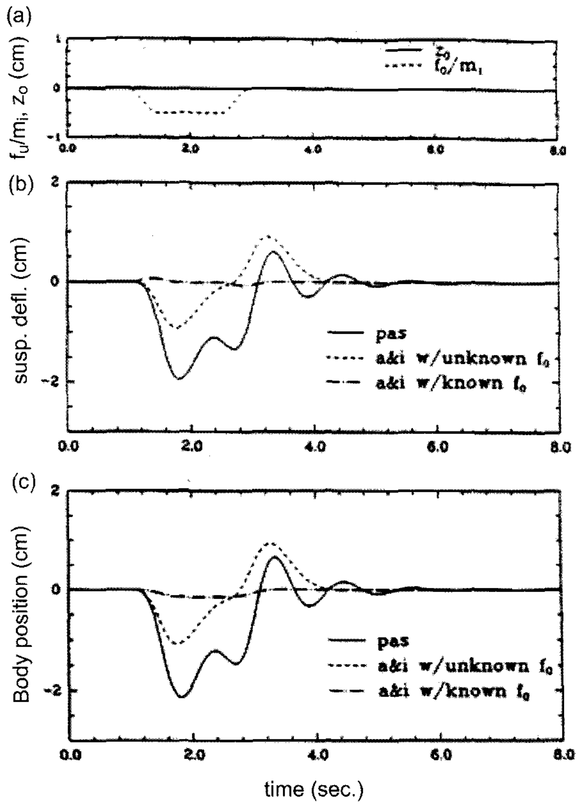

Figure 5. Responses of the vehicle models with various suspensions to body force in a cornering maneuver: (a) body force; (b) suspension deflection; (c) absolute body position.

where $\tau=(t-t_0)/T$. The amplitude of force A=–0.5 was assumed, the total time period of the maneuver was T=2 s and the initial time t_0=1.5 s. The plot of body force is shown in Figure 5a. Responses of the model to this force in terms of suspension deflection and body position are shown in Figure 5b and 5c, respectively for the passive suspension, the active suspension with PI control and unknown body force, and the active suspension with PI control and estimated body force. When the body force disturbance is unknown, the deflection of the active suspension with integral control remains large (about 60% of that of a passive system) during the transient response, and it takes about 1 s to reduce the suspension deflection to the desired level. In addition, the body exhibits lean in the opposite direction after the maneuver is completed. These effects are minimized when the body force can be detected and compensated for by a feedforward term.

5. CONCLUSIONS

In this paper an active suspension system is considered with the purpose of obtaining good vibration isolation properties combined with improved attitude control of the vehicle body under the influence of external body forces and ramp road inputs. The suspension controller was synthesized by applying methods of optimal control theory to a 2-DOF vehicle model subjected to two types of disturbances: road irregularities and external body forces representing changes in payload, aerodymanic forces and/or inertial forces. To assure zero steady-state suspension deflection in response to static body forces or ramp inputs, an integral term of suspension deflection was introduced in the performance index. The proposed controller consists of a traditional proportional preview controller with an additional integral term and a feed-forward term to provide compensation for (measurable) body forces. Results, a steady-state error in suspension deflection can be eliminated when integral action is used, and that presence of preview information about the road input improves transient vehicle performance. Integral action slightly reduces system performance in regard to vibration characteristics but these adverse effects can be minimized when the integral gain is small. Preview, on the other hand, yields large improvements in these characteristics. Simulations of a vehicle performing a cornering maneuver have shown that in order to significantly improve transient behavior, it is essential that the body forces resulting from cornering be detected and compensated for. In that case the proposed controller can counteract even dynamic body forces providing an excellent attitude control.

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