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p-STACKS ON SUPRATOPOLOGICAL SPACES

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ABSTRACT. In [1], we introduced the notion of p-stacks. In this paper, by using p-stacks we characterize S^*-continuous functions, separation axioms, supracompactness and some properties on supratopological spaces. We also introduce the notion of p-supracompactness and study some properties.

1. Introduction

In [1], D. C. Kent and the author introduced neighborhood structures and neighborhood spaces which are generalized topological spaces. In order to describe a convergence theory in neighborhood spaces, we introduced "p-stack" [1] which is defined as the following: Given a set X, a collection $\mathbf{C}$ of subsets of X is called a p-stack if (1) $A \in \mathbf{C}$ whenever $B \in \mathbf{C}$ and $B \subset A$ and (2) $A, B \in \mathbf{C}$ implies $A \cap B \neq \emptyset$. And we characterized some properties of neighborhood spaces by using the notion of "p-stack". In 1983, A. S. Mashhour et. al. [2] defined a supratopology on a set X to be a collection of subsets of X which contains X and is closed under arbitrary union. In this paper, by using the notion of p-stacks we will characterize the closure operator, the interior operator, continuity, separation axioms and some properties on supratopological spaces. We also introduce the notion of p-supracompactness by using convergence of ultrapstaks and investigate some properties

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2. Preliminaries

Let X be a nonempty set. A subcollection $\tau \subset 2^X$ is called a *supratopology* [2] on X if $X \in \tau$ and τ is closed under arbitrary union. (X, τ) is called a *supratopological space.* The members of τ are called *supraopen* sets and a set is called *supraclosed* if the complement is a member of τ. Let (X, τ) be a supratopological space, $x \in X$. A set V is called a *supra-neighborhood* of x if there is a supraopen set U such that $x \in U \subset V$. The set of all supratopologies on X is denoted by $\mathbf{ST}(X)$.

Let (X, τ) and (Y, μ) be supratopological spaces. A function $f : X \to Y$ is S^**-continuous* [2] if the inverse image of each supraopen set in Y is a supraopen set in X.

DEFINITION 2.1 [2]. Let (X, τ) be a supratopological space and $A \subset X$.

(1) The suprainterior of A, denoted by $Sint(A)$, is defined by $Sint(A) = \cup \{U \in \tau : U \subset A\}$;
(2) The supraclosure of A, denoted by $Scl(A)$, is defined by $Scl(A) = \cap\{F \subset X : A \subset F \text{ and } X - F \in \tau\}$;
(3) X is $S - T_1$ if for every two distinct points x and y in X, there exist two supraopen sets U and V such that $x \in U, y \notin U$ and $y \in V, x \notin V$;
(4) X is $S - T_2$ if for every two distinct points x and y in X, there exist two disjoint supraopen sets U and V such that $x \in U$ and $y \in V$;
(5) X is *supra-regular* if for each supraclosed set H and $x \notin H$, there exist two disjoint supraopen sets U and V such that $H \subset U$ and $x \in V$;
(6) X is *supracompact* if each cover of X by supraopen sets has a finite subcover.

THEOREM 2.2 [2]. *Let (X, μ) be a supratopological space and $A \subset X$.*

(1) $Sint(X) = X$;
(2) $Sint(A) \subset A$ *for all* $A \subset X$;
(3) $A \subset B \Rightarrow Sint(A) \subset Sint(B)$ *for all* $A, B \subset X$;
(4) $Sint(Sint(A)) = Sint(A)$ *for all* $A \subset X$;
(5) $Sint(A) = X - Scl(X - A)$ *and* $Scl(A) = X - Sint(X - A)$ *for* $A \subset X$.

Given a set X, a collection $\mathbf{C}$ of subsets of X is called a *stack* if $A \in \mathbf{C}$

whenever $B \in \mathbf{C}$ and $B \subset A$. A stack $\mathbf{H}$ on a set X is called a *p-stack* [1] if it satisfies the following condition:

(p) $A, B \in \mathbf{H}$ implies $A \cap B \neq \emptyset$.

Condition (p) is called the *pairwise intersection property* (PIP). A collection $\mathbf{B}$ of subsets of X with the PIP is called a *p-stack base.* For a p-stack base $\mathbf{B}$, we denote by $< \mathbf{B} > = \{A \subset X :$ there exists $F \in \mathbf{B}$ such that $F \subset A\}$ the p-stack generated by $\mathbf{B}$. If $\mathbf{B} = \{B\}$, then $< \mathbf{B} >$ will be denoted by simply $< B >$. In case $x \in X$ and $\mathbf{B} = \{\{x\}\}$, $< \{x\} >$ is usually $\dot{x}$. Let $pS(X)$ denote the collection of all p-stacks on X, partially ordered by inclusion. The maximal elements in $pS(X)$ are called *ultrapstacks.* It is obvious that every ultrafilter is an ultrapstack, and that every p-stack is contained in an ultrapstack. For function $f : X \to Y$ and $\mathbf{H} \in pS(X)$, the image p-stack $f(\mathbf{H})$ in $pS(Y)$ has p-stack base $\{f(H) : H \in \mathbf{H}\}$. Likewise, if $\mathbf{G} \in pS(Y)$, $f^{-1}(\mathbf{G})$ denotes the p-stack on X generated by $\{f^{-1}(G) : G \in \mathbf{G}\}$.

LEMMA 2.3 [1]. *For $\mathbf{H} \in pS(X)$, the following are equivalent;*

(1) *$\mathbf{H}$ is an ultrapstack;*
(2) *If $A \cap H \neq \emptyset$ for all $H \in \mathbf{H}$, then $A \in \mathbf{H}$;*
(3) *$B \notin \mathbf{H}$ implies $X - B \in \mathbf{H}$*

3. Main results

First we introduce the notion of convergence of p-stacks on supratopological spaces in order to give characterizations of the supraclosure operator and the suprainterior operator.

DEFINITION 3.1. Let (X, μ) be a supratopological space.

For each $x \in X$, let $\mathbf{V}_\mu(x) = \{V : V$ is a supra-neighborhood of $x\}$, and $\mathbf{V}_\mu(x)$ is called the *supra-neighborhood stack* at x.

A p-stack $\mathbf{F}$ on X *μ-converges* to x if $\mathbf{V}_\mu(x) \subset \mathbf{F}$.

REMARK. From the above definition, we can get a filter generated by each supra-neighborhood stack on a supratopological space and in [3, 4] we called the filter a *supra-neighborhood filter.*

We get the following theorem from Definition 3.1.

THEOREM 3.2. *Let (X, μ) be a supratopological space.*

(1) *$\dot{x}$ μ-converges to x for all $x \in X$;*
(2) *For $\mathbf{F}$, $\mathbf{G} \in p\mathbf{S}(X)$, if $\mathbb{F}$ μ-converges to x and $\mathbf{F} \subset \mathbf{G}$, then $\mathbf{G}$ μ-converges to x;*

(3) *For* $\mathbf{F}$, $\mathbf{G} \in p\mathbf{S}(X)$, *if both* $\mathbf{F}$ *and* $\mathbf{G}$ *μ-converge to* x, *then* $\mathbf{F} \cap \mathbf{G} = \{F \cup G : F \in \mathbf{F},\ G \in \mathbf{G}\}$ *μ-converges to* x;

DEFINITION 3.3. Let (X, μ) be a supratopological space, $A \subset X$ and let $\mathbf{V}_\mu(x)$ be the supra-neighborhood stack at x.

(1) $I_\mu(A) = \{x \in A : A \in \mathbf{V}_\mu(x)\}$;
(2) $Cl_\mu(A) = \{x \in X : A \cap U \neq \emptyset \text{ for all } U \in \mathbf{V}_\mu(x)\}$.

LEMMA 3.4. *Let* (X, μ) *be a supratopological space,* $A \subset X$. *Then the following are equivalent:*

(1) $Scl(A) = Cl_\mu(A)$;
(2) $Sint(A) = I_\mu(A)$.

THEOREM 3.5. *Let* (X, μ) *be a supratopological space,* $A \subset X$.

(1) $x \in I_\mu(A)$ *iff* $A \in \mathbf{H}$, *for every p-stack* $\mathbf{H}$ *μ-converging to* x;
(2) $x \in Cl_\mu(A)$ *iff there exists* $\mathbf{H} \in p\mathbf{S}(X)$ *such that* $\mathbf{H}$ *μ-converges to* x *and* $A \in \mathbf{H}$.

PROOF. (1) Since the supra-neighborhood stack $\mathbf{V}_\mu(x)$ always μ-converges to x, we get $A \in \mathbf{V}_\mu(x)$ by hypothesis. Thus $x \in I_\mu(A)$.

The converse is obvious by Definition 3.1.

(2) If $x \in Cl_\mu(A)$, let $\mathbf{H} = \mathbf{V}_\mu(x) \cup < A > = < \{F \cap G : F \in \mathbf{V}_\mu(x),\ G \in < A >\} >$. Then a p-stack $\mathbf{H}$ μ-converges to x and $A \in \mathbf{H}$.

Conversely let $x \notin Cl_\mu(A)$. Then there is some $V \in \mathbf{V}_\mu(x)$ such that it has the empty intersection with A, and so no a p-stack containing A can μ-converge to x. □

From Lemma 3.4 and Theorem 3.5 we get the following theorem:

THEOREM 3.6. *Let* (X, μ) *be a supratopological space,* $A \subset X$. *Then the following are equivalent:*

(1) A *is supraclosed iff* $Cl_\mu(A) = A$;
(2) A *is supraopen iff* $I_\mu(A) = A$;
(3) $x \in Scl(A)$ *iff there exists* $\mathbf{H} \in p\mathbf{S}(X)$ *such that* $\mathbf{H}$ *μ-converges to* x *and* $A \in \mathbf{H}$;
(4) $x \in Sint(A)$ *iff* $A \in \mathbf{H}$, *for every p-stack* $\mathbf{H}$ *μ-converging to* x.

Now we characterize the S^*-continuity of a map by using p-stacks as the continuity of a map is expressed by filters.

THEOREM 3.7. *Let (X,μ) and (Y,ν) be supratopological spaces. If $f : (X,\mu) \to (Y,\nu)$ is a function, then the following statements are equivalent:*

(1) *f is S^*-continuous;*
(2) *$\mathbf{V}_\nu(f(x)) \subset f(\mathbf{V}_\mu(x))$ for all $x \in X$;*
(3) *$f^{-1}(I_\nu(A)) \subset I_\mu(f^{-1}(A))$ for all $A \subset Y$;*
(4) *$f(Cl_\mu(B)) \subset Cl_\nu(f(B))$ for all $B \subset X$;*
(5) *If a p-stack $\mathbf{F}$ μ-converges to x, then the image p-stack $f(\mathbf{F})$ ν-converges to $f(x)$.*

PROOF. (1) $\Rightarrow$ (2) Let V be a member of $\mathbf{V}_\nu(f(x))$ in Y. Then there is a supraopen W such that $f(x) \in W \subset V$. Since f is S^*-continuous, there exists a supraopen U of x such that $f(U) \subset W \subset V$ and since $U \in \mathbf{V}_\mu(x)$ we get $V \in f(\mathbf{V}_\mu(x))$.

(1) $\Leftrightarrow$ (3) $\Leftrightarrow$ (4) It is obvious from Lemma 3.5.

(2)$\Rightarrow$ (5) Let $\mathbf{F}$ be a p-stack μ-converging to x. Then $\mathbf{V}_\mu(x) \subset \mathbf{F}$, and so $\mathbf{V}_\nu(f(x)) \subset f(\mathbf{F})$ from (2).

(5) $\Rightarrow$ (1) If f is not S^*-continuous, then for some $x \in X$, there is a supraopen $V \in \mathbf{V}_\nu(f(x))$ such that for all supraopen $U \in \mathbf{V}_\mu(x)$, $f(U)$ does not inclode in V. For all $U \in \mathbf{V}_\mu(x)$, since $f(U) \cap (Y - V) \neq \emptyset$, we get a p-stack $\mathbf{F} = f(\mathbf{V}_\mu(x)) \cup < Y - V >$. And since $U \cap f^{-1}(Y - V) \neq \emptyset$, we also get a p-stack $\mathbf{G} = \mathbf{V}_\mu(x) \cup < f^{-1}(Y - V) >$ which μ-converges to x. But since $f(\mathbf{G})$ is a finer p-stack than $\mathbf{F}$ and $Y - V \in \mathbf{F}$, $f(\mathbf{G})$ can not ν-converge to $f(x)$, contradicting to (5). □

THEOREM 3.8. *Let (X,μ) be a supratopological space and $A \subset X$.*

(1) *$I_\mu(X) = X$;*
(2) *$I_\mu(A) \subset A$ for all $A \subset X$;*
(3) *$A \subset B \Rightarrow I_\mu(A) \subset I_\mu(B)$ for all $A, B \subset X$;*
(4) *$I_\mu(I_\mu(A)) = I_\mu(A)$ for all $A \subset X$.*

PROOF. They are obvious by Lemma 3.4 and Theorem 2.2. □

Let X be a nonempty set. Consider a set function $I : 2^X \to 2^X$ satisfying these axioms:

($\mathbf{i_1}$) $I(X) = X$;
($\mathbf{i_2}$) $I(A) \subset A$ for all $A \subset X$;
($\mathbf{i_3}$) $A \subset B \Rightarrow I(A) \subset I(B)$ for all $A, B \in 2^X$;
($\mathbf{i_4}$) $I(I(A)) = I(A)$ for all $A \subset X$.

Let $\mathbf{I}(X)$ be the set of all these interior operators on X.

LEMMA 3.9. *If $I \in \mathbf{I}(X)$, then the operator I uniquely determines a supratopology on X whose interior operator is I.*

PROOF. Consider $\mu = \{A \subset X : I(A) = A\}$ on X. Then by $(\mathbf{i_1})$ and $(\mathbf{i_2})$, both X and $\emptyset$ are in μ.

Let $A_i \in \mu$ for all $i \in J$. Then $A_i = I(A_i) \subset I(\cup_{i\in J} A_i)$, so $\cup_{i\in J} A_i = I(\cup_{i\in J} A_i)$. Thus μ is a supratopology on X. It is obvious μ is the unique supratopology determined by I whose interior operator is I. □

From Lemma 3.9 and Theorem 3.8, we have the following result.

THEOREM 3.10. *If $\varphi : \mathbf{ST}(X) \to \mathbf{I}(X)$ is defined by $\varphi(\mu) = I_\mu$, where $I_\mu(A) = \{x \in X : A \in \mathbf{V}_\mu(x)\}$ for all $A \subset X$, then φ is a bijection.*

PROOF. If $\mu \in \mathbf{ST}(X)$, then $\varphi(\mu) = I_\mu \in \mathbf{I}(X)$ by Theorem 3.8. If $I \in \mathbf{I}(X)$, let $\psi(I) = \mu_I$, where $\mathbf{V}_{\mu_I}(x) = \{A \subset X : x \in I(A)\}$ for each $x \in X$. Then the map $\psi : \mathbf{I}(X) \to \mathbf{S}T(X)$ is well defined, and by Theorem 3.6, we get $\psi \circ \varphi$ is the identity map on $\mathbf{ST}(X)$.

Now we show that $\varphi \circ \psi$ is the identity map on $\mathbf{I}(X)$. For each $I \in \mathbf{I}(X)$, let $\varphi(\psi(I)) = I'$, where $\psi(I) = \mu_I = \{A : I(A) = A\}$ and $I'(A) = \{x \in A : A \in \mathbf{V}_{\mu I}(x)\}$.

Let $x \in I(A)$ for all $A \subset X$. Then by $(\mathbf{i_4})$ and the notion of μ_I, we can say that $I(A)$ is a supraopen set containing x on a supratopological space (X, μ_I). Then it follows from $I(A) \subset A$ that $A \in \mathbf{V}_{\mu I}(x)$. Thus from the notion of I', we get $x \in I'(A)$.

For the other inclusion, let $x \in I'(A)$. Then A is an element of the supraneighborhood stack $\mathbf{V}_{\mu I}(x)$ at x, and so we can take a supraopen set U satisfying $x \in U \subset A$ in the supratopology μ_I. Then by definition of the supratopology μ_I and $(\mathbf{i_3})$, we get $x \in U = I(U) \subset I(A)$. Consequently, $I = I'$. □

Now we give characterizations of $S-T_1$, $S-T_2$, supra-regularity and supracompactness on a supratopological space by using p-stacks.

THEOREM 3.11. *Let (X, μ) be a supratopological space. Then the following are equivalent:*

(1) *(X, μ) is $S - T_1$;*
(2) *$\cap \mathbf{V}_\mu(x) = \{x\}$ for $x \in X$;*
(3) *If $\dot{x}$ μ-converges to y, then $x = y$.*

PROOF. (1) ⇒ (2) Let y be an element in $\cap \mathbf{V}_\mu(x)$. Then $y \in U$ for each supraopen neighborhood U of x. Since X is $S-T_1, y = x$.

(2) ⇒ (3) Let $\dot{x}$ μ-converge to y. Since $\mathbf{V}_\mu(y) \subset \dot{x}$, x is the element in $\cap \mathbf{V}_\mu(y)$. Thus $x = y$.

(3) ⇒ (1) Suppose that X is not $S - T_1$. Then there are distinct x and y such that every supraopen neighborhood of x contains y. Thus $\mathbf{V}_\mu(x) \subset \dot{y}$ and $\dot{y}$ μ-converges to x, contrary to (3). □

THEOREM 3.12. *Let (X, μ) be a supratopological space. Then the following are equivalent:*

(1) *(X, μ) is $S - T_2$;*
(2) *Every μ-convergent p-stack* $\mathbf{F}$ *on X μ-converges to exactly one point.*
(3) *Every μ-convergent ultrapstack* $\mathbf{F}$ *on X μ-converges to exactly one point.*

PROOF. (1) ⇒ (2) Suppose that X is $S - T_2$ and a p-stack $\mathbf{F}$ μ-converges to x. For any $y \neq x$, there are disjoint supraopen neighborhoods $U(x)$ and $U(y)$. Since $\mathbf{V}_\mu(x) \subset \mathbf{F}$ and $\mathbf{F}$ is a p-stack, both $U(x)$ and $X - U(y)$ are elements of $\mathbf{F}$. Thus $\mathbf{F}$ is not finer than $\mathbf{V}_\mu(y)$, so $\mathbf{F}$ does not μ-converge to y.

(2) ⇒ (3) It is obvious.

(3) ⇒ (1) Suppose that X is not $S - T_2$. Then there must exist x, y such that $U(x) \cap U(y) \neq \emptyset$ for every supraopen neighborhoods $U(x)$ and $U(y)$ of x and y, respectively. Let $\mathbf{F}$ be the ultrapstak finer than a p-stack $\mathbf{V}_\mu(x) \cup \mathbf{V}_\mu(y)$. Then $\mathbf{F}$ is finer than $\mathbf{V}_\mu(x)$ and $\mathbf{V}_\mu(y)$, so the ultrapstack $\mathbf{F}$ μ-converges to both x and y, contrary (2). □

Let (X, μ) be a supratopological space and $\mathbf{F} \in pS(X)$. Then $\mathbf{B} = \{Scl(F) : F \in \mathbf{F}\}$ is a p-stack base on X. The p-stack generated by $\mathbf{B}$ is denoted $Scl(\mathbf{F})$. We call $Scl(\mathbf{F})$ the *supraclosure p-stack* of $\mathbf{F}$.

THEOREM 3.13. *Let (X, μ) be a supratopological space. Then the following are equivalent:*

(1) *(X, μ) is supra-regular;*
(2) *For every x in X, $\mathbf{V}_\mu(x) = Scl(\mathbf{V}_\mu(x))$.*
(3) *If a p-stack* $\mathbf{F}$ *μ-converges to x, then $Scl(\mathbf{F})$ μ-converges to x.*

PROOF. (1) ⇒ (2) Let F be an element in $\mathbf{V}_\mu(x)$. There exists a supraopen neighborhood $U(x)$ such that $U(x) \subset F$. Since X is supra-regular, there is a supraopen neighborhood $W(x)$ such that $W(x) \subset$

$Scl(W(x)) \subset U(x) \subset F$. And since $Scl(W(x)) \in Scl(\mathbf{V}_\mu(x))$ and $Scl(\mathbf{V}_\mu(x))$ is a p-stack, $F \in Scl(\mathbf{V}_\mu(x))$.

(2) $\Rightarrow$ (3) Let a p-stack $\mathbf{F}$ μ-converge to x. Then $Scl(\mathbf{V}_\mu(x)) \subset Scl(\mathbf{F})$. Thus we get that $Scl(\mathbf{F})$ μ-converges to x by (2).

(3) $\Rightarrow$ (1) Let U be a supraopen neighborhood of x for each $x \in X$. Since $\mathbf{V}_\mu(x)$ μ-converges to x, $Scl(\mathbf{V}_\mu(x))$ μ-converges to x by (3), and so $U \in Scl(\mathbf{V}_\mu(x))$. Then by the definition of supraclosure of p-stack, there is a supraopen neighborhood V in $\mathbf{V}_\mu(x)$ such that $V \subset Scl(V) \subset U$.□

For $\mathbf{F} \in pS(X)$, if all finite intersection of the elements of $\mathbf{F}$ are non-empty, we say that $\mathbf{F}$ has the *finite intersection property* (FIP).

THEOREM 3.14. *Let (X, μ) be a supratopological space and $A \subset X$. Then X is supracompact iff for every p-stack $\mathbf{F}$ with the FIP, there exist a finer p-stack $\mathbf{G}$ than $\mathbf{F}$ and an $x \in A$ such that $\mathbf{G}$ μ-converges to x.*

PROOF. Assume X is supracompact and $\mathbf{F}$ is a p-stack with the FIP. Suppose any finer p-stack than $\mathbf{F}$ does not μ-convergent to any point in X. Then we can assert that for each $x \in A$, there exist a supraopen neighborhood $U_x \in \mathbf{V}_\mu(x)$ and $F_x \in \mathbf{F}$ such that $U_x \cap F_x = \emptyset$. Thus we can find the collection $\mathbf{U}$ of the supraopen neighborhoods $U_x \in \mathbf{V}_\mu(x)$ for each $x \in X$ such that $U_x \cap F_x = \emptyset$ for some $F_x \in \mathbf{F}$. This collection $\mathbf{U}$ is a supraopen cover of X and for each $U_x \in \mathbf{U}$, $X - U_x$ is a member of the p-stack $\mathbf{F}$. Since X is supracompact, there is a finite subcover $\{U_{x1}, \dots, U_{xn}\}$ of $\mathbf{U}$. Now we get $\cap(X - U_{xi}) = \emptyset$ for $X - U_{xi} \in \mathbf{F}$, where $i = 1, \dots, n$. This is a contradiction, since $\mathbf{F}$ is a p-stack with the FIP.

Conversely, assume X is not supracompact, and let $\mathbf{U}$ be a supraopen cover of X with no finite subcover. If $\mathbf{W}$ is the set of all finite union of members of $\mathbf{U}$, then $X - W \neq \emptyset$ for all $W \in \mathbf{W}$. Let $\mathbf{B} = \{X - W : W \in \mathbf{W}\}$. Then clearly $\mathbf{B}$ is a p-stack base. Thus we get a p-stack $\mathbf{F_B}$ generated by $\mathbf{B}$ such that $\mathbf{F_B}$ has the finite intersection property. Finally for any finer p-stack $\mathbf{G}$ than $\mathbf{F_B}$, we get that $\mathbf{G}$ does not μ-converge to any element of X, contradicting (2). □

We introduce the notion of p-supracompactness by using convergence of ultrapstaks and investigate some properties.

DEFINITION 3.15. A subset A of a supratopological space (X, μ) is *p-supracompact* if every ultrapstack containing A μ-converges to a point in A. The space (X, μ) is p-supracompact if X is p-supracompact.

EXAMPLE 3.16. Let $X = \{a, b, c\}$. Consider $\tau = \{\emptyset, \{a, b\}, \{b, c\}, X\}$. Then the supratopological space (X, τ) is p-supracompact.

However X is a nonempty finite set, a supratopological space on X may not be p-supracompact as the following example:

EXAMPLE 3.17. Let $X = \{a, b, c\}$ and let $\tau = \{\emptyset, \{a\}, \{a, b\}, \{b, c\}, X\}$ be a supratopology on X. And let $\mathbf{H}$ be an ultrapstack containing a p-stack $\mathbf{F}$ generated by $\{\{a, b\}, \{b, c\}, \{a, c\}\}$. Then it does not τ-converge to a point a in X. Thus the supratopological space (X, τ) is not p-supracompact.

THEOREM 3.18. *Let $f : (X, \mu) \to (Y, \nu)$ be an S^*-continuous and surjective function. If X is p-supracompact, then Y is also p-supracompact.*

PROOF. Let X be p-supracompact, and let $\mathbf{H}$ be an ultrapstack in Y. If $\mathbf{G}$ is an ultrapstack containing the p-stack base $\{f^{-1}(H) : H \in \mathbf{H}\}$, then for some $x \in X$, $\mathbf{G}$ μ-converges to x, and $\mathbf{H} = f(\mathbf{G})$ ν-converges to $f(x)$, thus Y is p-supracompact. □

THEOREM 3.19. *A supratopological space (X, μ) is p-supracompact if and only if each supraopen cover of X has a two-member subcover.*

PROOF. Suppose $\mathbf{H}$ is an ultrapstack in X such that it does not μ-converge to any point in X. Then for each $x \in X$, there is $U_x \in \mathbf{V}_\mu(x)$ such that $U_x \notin \mathbf{H}$. By Lemma 2.3(3), $X - U_x \in \mathbf{H}$ for all $x \in X$. Thus the collection $\mathbf{U} = \{U_x : x \in X\}$ is a supraopen cover of X. But $\mathbf{U}$ has no two-element subcover of X, for if $U, V \in \mathbf{U}$ and $X \subset U \cup V$, then $(X - U) \cap (X - V) = X - (U \cup V) = \emptyset$, contradicting that $\mathbf{H}$ is a p-stack. Conversely, let $\mathbf{U}$ be a supraopen cover of X with no two-element subcover of X. Then $\mathbf{B} = \{X - U : U \in \mathbf{U}\}$ is a p-stack base, and any ultrapstack containing $\mathbf{B}$ can not μ-converge to any point in X. □

REMARK. From Theorem 3.19 and Example 3.17, we can say every supracompact space is p-supracompact but the converse is not always true.

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