Mutual Detectability and System Enlargement of Detection Filters: An Invariant Zero Approach

Yongmin Kim and Jaehong Park

Abstract: In this paper, we discuss the problem of non-mutual detectability using the invariant zero. We propose a representation method for excess spaces by linear equation based on the Rosenbrock system matrix. As an alternative to the system enlargement method proposed by White [1], we propose an appropriate form of an enlarged system to make a set of faults mutually detectable by assigning sufficient geometric multiplicity of invariant zeros. We show the equivalence between the two methods and a necessary condition for the system enlargement in terms of the geometric and algebraic multiplicities of invariant zeros.

Keywords: Fault detection filter, fault diagnosis, invariant zero, mutual detectability.

1. INTRODUCTION

The detection filter is a Luenberger observer that can detect and isolate multiple faults by limiting their responses to the smallest reachable subspace with respect to each fault direction [1,2]. In order to ensure the stability of the residual, an additional condition is required that the closed-loop eigenvalues should be arbitrarily assigned in association with those subspaces.

A set of multiple faults satisfying this requirement is defined to be *mutually detectable*, and this is crucial for designing the detection filters for multiple faults. If this condition is not satisfied, some fault directions combine with each other and create fixed eigenvalues. In addition, if some of these eigenvalues are in the open right half complex plan, the stability cannot be guaranteed [1-4].

One of the remarkable researches on the mutually detectability is that by Massoumnia [3] where he approached this problem in a geometric formulation and presented the condition for a system to be mutually detectable in terms of the invariant zeros. However, he did not present a solution to the problem relating to coping with a non-mutually detectable fault set. White's [1] solution was a system enlargement where he presented an input-output equivalent system by increasing the dimensions of the detection spaces of fault directions related to the fixed eigenvalues. As pointed out in [1], since these eigenvalues are closely related to invariant zeros, it is essential that the multiplicity of zeros be investigated. However, the existing results do not cover this problem sufficiently enough.

In this paper, we analyze the non-mutual detectability and consider the system enlargement proposed by White [1] in terms of the invariant zero. We show that an *excess space* is described by the linear equation that includes more than two fault directions, by which it is possible to present a way to identify the fault directions associated with an excess space. The condition under which this space can be removed is proposed in terms of the geometric multiplicity of invariant zero. Considering this fact, we propose an appropriate form of an enlarged system and show that this approach is equivalent to the one in [1]. Further, we present a necessary condition for the system enlargement by showing a limitation of this approach in association with the geometric and algebraic multiplicities.

The advantage of the result of this paper is that the non-mutually detectability of a detection filter can simply be described with linear equations. Hence, it is possible to set up a numerical algorithm for identifying the applicability of the system enlargement and to construct an enlarged system using the existing reliable algorithms to calculate invariant zeros.

2. PRELIMINARIES

Let us consider the following linear time-invariant system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + F\mu(t), \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

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where $x(t)$ is the $n\times 1$ state vector, A is the open-loop system dynamics matrix, $u(t)$ is the $p\times 1$ known input vector with the corresponding input distribution matrix B. $F\mu(t)$ is used to represent faults acting upon the system, where the column vectors of $F=[f_1,\cdots,f_r]$ is called the *fault directions*. $\mu(t)$ is the $r\times 1$ time varying vector representing the fault signals. C is the measurement matrix and $y(t)$ is the $q\times 1$ output vector. We assume the observability of the pair (A,C).

Detection filters are given in the form of full-order Luenberg observer as follows:

$$\begin{aligned}\dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + D(y(t)-\hat{y}(t)),\\ \hat{y}(t) &= C\hat{x}(t),\end{aligned} \tag{2}$$

where $\hat{x}(t)$ is the $n\times 1$ state estimation vector and $\hat{y}(t)$ is the $q\times 1$ output estimation vector. D is a gain matrix of size $n\times q$. Defining the state estimation error as $e(t)\triangleq x(t)-\hat{x}(t)$, it is governed by the following equation:

$$\begin{aligned}\dot{e}(t) &= (A-DC)e(t)+F\mu(t),\\ \varepsilon(t) &= Ce(t),\end{aligned} \tag{3}$$

where $\varepsilon(t)$ is the *residual* which is the signal used to detect the fault. If $\mu(t)=0$ and D is designed such that all the eigenvalues of $(A-DC)$ are located in the open left half plane, $\lim_{t\to\infty}\varepsilon(t)=0$. The main objective of the detection filters is to find gain D such that a) the residual $\varepsilon(t)$ is restricted to a pre-determined direction for each f_i in the output space and b) all the eigenvalues $(A-DC)$ can be arbitrarily assigned.

To satisfy the first condition, the minimum reachable subspace with respect to each f_i must be defined, which is referred to as the *detection space* of f_i denoted by D_{f_i}. Among the various definitions of the detection space, we utilize the following [5]:

$$D_{f_i} \triangleq \operatorname{span}\{f_i, v_{z1}^i,\cdots,v_{z\nu_i}^i\}, \tag{4}$$

where v_{zj}^i is the invariant zero vector (direction) of (A,f_i,C) associated with the invariant zero z_j^i defined as $\begin{bmatrix} z_j^i I-A & -f_i\\ C & 0\end{bmatrix}\begin{bmatrix} v_{zj}^i\\ w_j^i\end{bmatrix}=0$, where w_j^i is a $r\times 1$ vector. In the same manner, *group detection space* of F, D_F, can also be defined by the invariant zero vectors of (A, F, C).

For the second condition, the detection spaces must satisfy the following: $D_F=\bigcup_i^r D_{f_i}$, then we say F is *mutually detectable*. This condition can be represented using the invariant zero.

$$z_{ex}\triangleq \sigma(A,F,C)-\biguplus_{i=1}^{r}\sigma(A,f_i,C)=\varnothing, \tag{5}$$

where $\sigma(\cdot)$ means the set of the invariant zeros and $\uplus$ means the union with common elements permitted.

The *excess space* is defined as the span of the invariant zero vectors associated with z_{ex}. If some of z_{ex} are located in the open right half plane, the residual will be unstable as a result of the non-zero initial error [1].

For the analysis of the detection space in the next section, we present the definition of the multiplicities of the invariant zero.

Definition 1: For the following *Rosenbrock system matrix* of the triple (A, F, C) [6]:

$$P(s)\triangleq\begin{bmatrix} sI-A & -F\\ C & 0\end{bmatrix}, \tag{6}$$

the rank deficiency of $P(s)$ at the complex value z is called the *geometric multiplicity* of the corresponding zero and is equal to the number of elementary divisors of $P(s)$. The degree of the product of the elementary divisors corresponding to z is called the *algebraic multiplicity* of the complex value z.

For the simplicity of notation, $m_a(z)$ and $m_g(z)$ are used to represent the algebraic multiplicity and the geometric multiplicity of z, respectively. Note that the following inequality holds: $m_a(z)\ge m_g(z)$.

3. SYSTEM ENLARGEMENT

In this section, we present the method for removing the excess space in consideration of the relationship between the geometric multiplicity and the algebraic multiplicity. First, we define the function $\Xi(\cdot)$, which gives the index of column vectors of F related with $z\in\sigma(A, F, C)$ as follows:

$$\Xi(z)\triangleq\left\{ i \,|\, (w_z)_i\neq 0,\ P(z)\begin{bmatrix} v_z\\ w_z\end{bmatrix}=0,\ i=1,\cdots,r\right\}, \tag{7}$$

where $(w_z)_i$ is the i th element of w_z. We will consider $P(z)$ and $\Xi(z)$ to be defined for the triple (A, F, C) hereafter.

Definition 2: Let z be an invariant zero of the triple (A, F, C). If the number of the elements of $\Xi(z)$ is greater than one, z will be referred to as an *intermediate invariant zero*.

The intermediate invariant zero causes the excess space because of the insufficiency of geometric multiplicity of the invariant zero. Therefore, if we add complex values that are equal to that zero as repeated invariant zeros by enlarging the system, the excess space can be removed.

Lemma 1: For an observable triple (A, F, C), let z be one of its multiple invariant zeros. If $m_g(z) = n[\Xi(z)]$, $z \in (A, f_i, C)$ $(i \in \Xi(z))$.

Proof: First, based on the definition of the geometric multiplicity, in the equation $P(z)\begin{bmatrix} V_z \\ W_z \end{bmatrix} = 0$, the rank of $[V_z^T \; W_z^T]^T$ is equal to $m_g(z)$. Now define $\breve{F}$ and $\breve{W}_z$ collecting the column vectors of F using $\Xi(z)$ as follows:

$$\breve{F} \triangleq \{f_i, i \in \Xi(z)\}, \quad \breve{W}_z \triangleq \{(W_z^T)_i, i \in \Xi(z)\}^T, \quad (8)$$

where the subscript i denotes the i th column vector of the matrix. Then the equation for the Rosenbrock system matrix with the triple $(A, \breve{F}, C)$ is given by

$$\begin{bmatrix} zI - A & -\breve{F} \\ C & 0 \end{bmatrix} \begin{bmatrix} V_z \\ \breve{W}_z \end{bmatrix} = 0. \quad (9)$$

$\breve{W}_z$ is the $m_g(z) \times m_g(z)$ invertible matrix. If we assume that $\breve{W}_z$ is not invertible, we can choose non-zero α such that $\breve{W}_z \alpha = 0$. Multiplying it to (9), we get

$$\begin{bmatrix} zI - A & -\breve{F} \\ C & 0 \end{bmatrix} \begin{bmatrix} V_z \\ \breve{W}_z \end{bmatrix} \alpha = \begin{bmatrix} zI - A \\ C \end{bmatrix} V_z \alpha = 0, \quad (10)$$

which is a contradiction to the observability assumption.

Multiplying $\breve{W}_z^{-1}$ to (9) gives

$$\begin{bmatrix} zI - A & -\breve{F} \\ C & 0 \end{bmatrix} \begin{bmatrix} V_z \breve{W}_z^{-1} \\ I_{m_g(z)} \end{bmatrix} = 0, \quad (11)$$

which can be regarded as the combination of the $m_g(z)$ equations for the Rosenbrock system matrices of the triple (A, f_i, C) as follows:

$$\begin{bmatrix} zI - A & -f_i \\ C & 0 \end{bmatrix} \begin{bmatrix} v_{zx}^i \\ 1 \end{bmatrix} = 0, i \in \Xi(z), \quad (12)$$

where v_{zx}^i is the column vector of $V_z \breve{W}_z^{-1}$ associated with f_i. This implies that z is an invariant zero of (A, f_i, C), which completes the proof. □

Lemma 1 can be applied to the system enlargement problem for removing the intermediate invariant zero of the triple (A, F, C) to obtain the following mutually detectable system.

Theorem 1: Define a set Ω_F which consists of the intermediate invariant zeros of the triple (A, F, C).

$$\Omega_F \triangleq \{z \mid z \in \sigma(A, F, C), n[\Xi(z)] \geq 2\}, \quad (13)$$

where the number of elements of Ω_F is equal to ν_e. Then the following enlarged triple $(\tilde{A}, \tilde{F}, \tilde{C})$ is mutually detectable.

$$\tilde{A} = \begin{bmatrix} A & A_{12} \\ 0 & A_{22} \end{bmatrix}, \; \tilde{C} = \begin{bmatrix} C & 0 \end{bmatrix}, \; \tilde{F} = \begin{bmatrix} F \\ 0 \end{bmatrix}, \quad (14)$$

where A_{12} and A_{22} are given by

$$A_{12} = \begin{bmatrix} FW_2^1 & FW_2^2 & \cdots & FW_2^{\nu_e} \end{bmatrix}$$
$$\text{and } A_{22} = \text{diag}\{z_1 I_{\xi_1}, \cdots, z_{\nu_e} I_{\xi_{\nu_e}}\},$$

respectively, where $\xi_i = n[\Xi(z_i)] - m_g(z_i)$, $z_i \in \Omega_F$ and W_2^i is an $n[\Xi(z_i)] \times (n[\Xi(z_i)] - m_g(z_i))$ matrix which makes the matrix $[W_1^i \; W_2^i]$ invertible from the equation $\begin{bmatrix} z_i I - A & -F \\ C & 0 \end{bmatrix} \begin{bmatrix} V_1^i \\ W_1^i \end{bmatrix} = 0.$

Proof: For the simplicity of the proof, we consider one of the ν_e intermediate invariant zeros, say z_i. In the above enlarged system, the Rosenbrock system matrix for $(\bar{A}, \bar{F}, \bar{C})$ and its null space satisfy the following equation:

$$\begin{bmatrix} zI - \bar{A}_i & -\bar{F} \\ \bar{C} & 0 \end{bmatrix} \begin{bmatrix} V_1^i & 0 \\ 0 & I \\ W_1^i & -W_2^i \end{bmatrix} = 0, \quad (15)$$

where

$$\bar{A}_i = \begin{bmatrix} A & FW_2^i \\ 0 & z_i I_{\xi_i} \end{bmatrix}, \; \bar{F} = \begin{bmatrix} F \\ 0 \end{bmatrix}, \; \bar{C} = \begin{bmatrix} C & 0 \end{bmatrix}.$$

Since the rank of the null space is given by $n[\Xi(z_1)]$ from the definition of W_2^i, z_i is not an intermediate invariant zeros by Lemma 1. Applying this result to the remaining elements of Ω_F we obtain a mutually detectable triple $(\tilde{A}, \tilde{F}, \tilde{C})$. □

Remark 1: $(\tilde{A}, \tilde{F}, \tilde{C})$ and (A, F, C) are input-output equivalent since $CA^jF = \tilde{C}\tilde{A}^j\tilde{F}$ for $j \le n$ [7]; therefore, the transfer matrices for the two triples are identical. The enlarged input distribution matrix $\tilde{B}$ is also given by $[B^T\ 0^T]^T$.

Remark 2: The detection order of $(\tilde{F})_i = [f_i^T\ 0^T]^T$ will be increased by the number of the elements of $\{z \mid i \in \Xi(z),\ n[\Xi(z)] \ge 2\}$, and the required additional dimension for the enlarged system is given by

$$n_{ex} = \sum_{z \in \sigma(A,F,C)} (n[\Xi(z)] - m_g(z)). \tag{16}$$

Remark 3: Note that the relative arguments (angles) among the directions of the fault response in the output space are reserved after the enlargement since $CF = \tilde{C}\tilde{F}$.

From the above result, for a repeated invariant zero z, the dimension which can be increased while maintaining the system to be observable is given by $(n[\Xi(z)] - m_g(z))$. Since the enlargement increases the algebraic multiplicity as well as the geometric multiplicitytherefore, the following condition is required.

Theorem 2: Define a set Ω_F following the same way as in (13). The necessary condition to obtain a mutually detectable system as in Theorem 1 is given by

$$m_a(z) = m_g(z),\ z \in \Omega_F. \tag{17}$$

Proof: For the simplicity of the proof, we consider one of the elements of Ω_F, such as z_i as in (15). Then, from (15), the invariant zeros of the triple $(\bar{A}_i, \bar{F}, \bar{C})$ can be decomposed as follows:

$$\sigma(\bar{A}_i, \bar{F}, \bar{C}) = \underbrace{\{z_i, \cdots, z_i\}}_{\xi_i} \cup \biguplus_{k \in \Xi(z_i)} \sigma(A, f_k, C),$$

where $\xi_i = n[\Xi(z_i)] - m_g(z_i)$. As for the number of elements of these sets, the following equation holds:

$$n[\sigma(\bar{A}_i, \bar{F}, \bar{C})] - (n[\Xi(z_i)] - m_g(z_i)) = m_a(z_i) + \sum_{k \in \Xi(z_i)} n[\sigma(A, f_k, C)].$$

When the system is enlarged with respect to z_i, the number of invariant zeros of $(\bar{A}_i, \bar{f}_k, \bar{C})$ is increased by one. Therefore,

$$n[\sigma(\bar{A}_i, \bar{F}, \bar{C})] - \sum_{k \in \Xi(z_i)} n[\sigma(A_i, \bar{f}_k, C)] = m_a(z_i) - m_g(z_i).$$

For $\bar{F}$ to be mutually detectable, the left-hand side term should be equal to zero, which completes the proof. □

Similar result is observed in [1], where the system is the same as (14) except for A_{12} and A_{22}.

$$A_{12} = [-\sigma_k^1(\lambda_{ek}^1 f_k - \bar{f}_k), \cdots, -\sigma_k^{v_{ek}}(\lambda_{ek}^{v_{ek}} f_k - \bar{f}_k)] \tag{18}$$

$$A_{22} = \mathrm{diag}[\lambda_{ek}^1, \cdots, \lambda_{ek}^{v_{ek}}], \tag{19}$$

where v_{ek} represents the number of eigenvalues of the original excess space which has the output component that lie along the direction Cf_k, λ_{ek}^j are the eigenvalues of $(A - DC)$ associated with the excess space related with f_k, σ_k^j are appropriate nonzero constants and $\bar{f}_k = (A - DC)f_k$ which can be represented with the eigenvectors spanning D_{f_k}.

To show the equivalence between the above result and (14), first we define the inverse function of $\Xi(z)$ as follows:

$$\Xi^{-1}(i) \triangleq \{z \mid i \in \Xi(z),\ z \in \sigma(A, F, C)\}. \tag{20}$$

Using this function, if we define a set $\Xi_{ek}^{-1} \triangleq \{k \mid z \in \Xi^{-1}(k),\ z \in \Omega_F\}$, $v_{ek} = n[\Xi_{ek}^{-1}]$.

White [1] proposed the required additional dimensions to make the fault set F mutually detectable as follows:

$$\tilde{n}_{ex} = \sum_{k=1}^{r} v_{ek} - v_e. \tag{21}$$

In this equation, we get the following result using the definition of Ξ_{ek}^{-1},

$$\sum_{k=1}^{r} v_{ek} = \sum_{k=1}^{r} n[\Xi_{ek}^{-1}] = \sum_{z \in \Omega_F} n[\Xi(z)]$$

and v_e is the number of intermediate invariant zeros, so that the following equation holds.

$$v_e = \sum_{z \in \Omega_F} m_g(z)$$

From the above results, we know that the results in (16) and (21) are identical. In this paper, however, it is shown that the multiplicities of invariant zeros must be taken into account for the system enlargement, which is not sufficiently discussed in [1]. This will be briefly discussed as a necessary condition in Theorem 2.

In addition, n_{ex} and $\tilde{n}_{ex}$ are the minimum number of additional dimension for the enlarged system to be observable.

Lemma 2: Let z be a repeated invariant zero of the triple (A, F, C). If $m_g(z) > n[\Xi(z)]$, (A, C) is the unobservable pair with the $(m_g(z) - n[\Xi(z)])$ unobservable eigenvalues equal to z.

Proof: We can prove this lemma using the proof of Lemma 1. Since the above condition corresponds to the case that the number of column vectors of $\breve{W}_z$ is greater than the number of column vectors of $\breve{F}$ in (9), the unobservability condition in (10) is established. □

Remark 4: We can make $(\tilde{A}', \tilde{F}', \tilde{C}')$ by adding some extra invariant zeros equal to the intermediate invariant zeros to the enlarged system $(\tilde{A}, \tilde{F}, \tilde{C})$ in the similar way to (14); however, the resulting pair $(\tilde{A}', \tilde{C}')$ becomes unobservable. Therefore, $(n\,[\Xi(z)] - m_g(z))$ is the maximum number of invariant zeros that can be assigned in order to make the fault directions, F, mutually detectable.

4. ILLUSTRATIVE EXAMPLE

In this section, we consider the system enlargement problem for two systems; one satisfies the necessary condition in Theorem 2 and the other does not.

First, consider a system of third order as in (1) with the following matrices:

$$A = \begin{bmatrix} 0 & 3 & 4 \\ 1 & 2 & 3 \\ 0 & 2 & 5 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, F = \begin{bmatrix} 3 & 1 \\ 1 & -0.5 \\ 1 & 0.5 \end{bmatrix}, \quad (22)$$

where the input distribution matrix B is omitted.

The invariant zero of the triple (A, F, C) is given by -0.5. Since the following equation holds

$$((-0.5)I - A)\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + F\begin{bmatrix} 0.5 \\ -1 \end{bmatrix} = 0, \quad (23)$$

where $\Xi(-0.5) = \{1,2\}$ and $w_1 = \begin{bmatrix} 0.5 \\ -1 \end{bmatrix}$. The required additional dimension is given by $(n\,[\Xi(-0.5)] - 1) = 2 - 1 = 1$ from Remark 2. Applying the result of Theorem 1, if we take $W_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $[w_1\ W_2]$ is invertible. Then, the enlarged system is given by

$$\tilde{A} = \begin{bmatrix} 0 & 3 & 4 & 1 \\ 1 & 2 & 3 & -0.5 \\ 0 & 2 & 5 & 0.5 \\ 0 & 0 & 0 & -0.5 \end{bmatrix}, \tilde{C} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \tilde{F} = \begin{bmatrix} 3 & 1 \\ 1 & -0.5 \\ 1 & 0.5 \\ 0 & 0 \end{bmatrix}.$$

This triple satisfies the multiplicity condition presented in Lemma 1, that is, $\tilde{m}_g(-0.5) = \tilde{m}_a(-0.5) = n[\Xi(-0.5)] = 2$, and thus the system becomes mutually detectable. The associated invariant zero vectors are given by $[1\,0\,0\,1]^T$ and $[0\,0\,0\,1]^T$, so that the detection spaces of $\tilde{f}_1$ and $\tilde{f}_2$ are as follows:

$$D_{\tilde{f}_1} = \text{span}\left\{\begin{bmatrix} 3 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}\right\}, \; D_{\tilde{f}_2} = \text{span}\left\{\begin{bmatrix} 1 \\ -0.5 \\ 0.5 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}\right\}.$$

The simulation result for this fault detection filter is presented in Fig. 1. We assign the eigenvalues $\{-1,-2\}$ for $D_{\tilde{f}_1}$ and $\{-2,-3\}$ for $D_{\tilde{f}_2}$, respectively. For the detailed design method, refer to [1,2]. The fault $f_1\mu_1$ is activated in 1 second and the fault $f_2\mu_2$ is activated in 5 seconds. We add two zero-mean gaussian noises whose variances are commonly equal to 0.00001 as sensor noises:

$$y(t) = Cx(t) + v_n(t),$$

where $v_n(t)$ denotes the noise. In addition, since $CF = \tilde{C}\tilde{F}$ is not an identity matrix, we multiply $(CF)^\dagger$ to the residual to make the transfer matrix from $\mu(t)$ to the residual a diagonal matrix. As can be seen in the figure, we obtain complete isolation of the two faults with this fault detection filter.

Next, let us consider the triple (A, F, C) where

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}, F = \begin{bmatrix} -2 & -2 \\ 2 & -3 \\ -7 & 4 \\ 6 & -2 \end{bmatrix}.$$

The invariant zero of the triple is given by 2 with $m_a(2) = 2$ and $m_g(2) = 1$. This triple does not satisfy the necessary condition of Theorem 2. Therefore, the difference between the two multiplicities, $(m_a(2) - m_g(2)) = 1$, cannot be eliminated by the system enlargement, making it impossible to obtain the mutual detectable system. In particular, since the intermediate invariant zero is

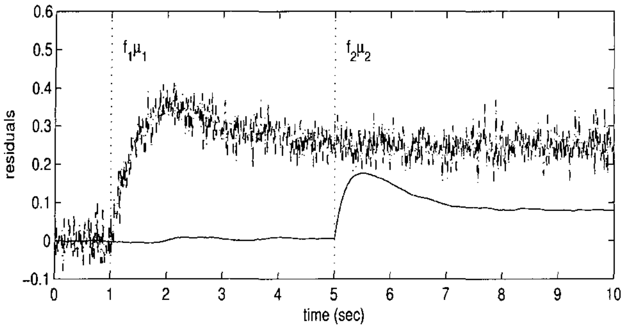

Fig. 1. Simulation result.

located in the open right half plane, a stable detection filter for isolating these two faults simultaneously cannot be designed. In this case, two separated detection filters for the respective faults should be designed.

5. CONCLUSIONS

In this paper, we have analyzed the non-mutual detectability using the invariant zero; we have shown that a set of fault directions is mutually detectable if the system has sufficient geometric multiplicity. With this fact, the system enlargement is reinterpreted as a method to obtain a reducible and input-output equivalent system with appropriate geometric multiplicity by assigning additional invariant zeros. We showed that the proposed method is equivalent to the one in [1], and further we presented a necessary condition for the system enlargement in terms of the geometric and algebraic multiplicities.

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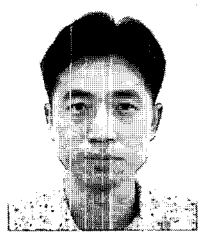

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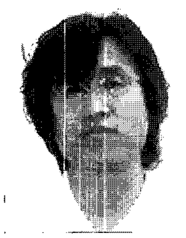

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