H_∞ Filtering for Descriptor Systems

Yue-Peng Chen, Zu-De Zhou, Chun-Nian Zeng, and Qing-Ling Zhang

Abstract: The paper is concerned with H_∞ filtering for descriptor systems. A necessary and sufficient condition is established in terms of linear matrix inequalities (LMIs) for the existence of normal filters such that the error systems are admissible and the transfer function from the disturbance to the filtering error output satisfies a prescribed H_∞-norm bound constraint. When these LMIs are feasible, an explicit parameterization expression of all desired normal filter is given. All these results are yielded without decomposing the original descriptor systems, which makes the filter design procedure simple and direct. Finally, a numerical example is derived to demonstrate the applicability of the proposed approach.

Keywords: Descriptor systems, H_∞ filtering, normal filter, linear matrix inequalities.

1. INTRODUCTION

In the last three decades, we have witnessed significant advances in the celebrated Kalman filtering which seems to be the most effective estimation approach. The Kalman filtering has been used widely [1,2], such as, aviation, spaceflight, industry process control, and so on. Kalman filtering approach is a very popular way, which generally provides an optimal estimation of state variables of nominal systems in the sense that the covariance of the estimation error is minimized [1]. [1] pointed out that the Kalman filtering approach is based on the assumption that the system under consideration has exactly known dynamics described by certain well-posed model, and these disturbances are stationary Gaussian noises with known statistics. In some cases, however, we may not exactly know the noise sources. Also, for a system model with parameter uncertainties, we can not get satisfactory performance sometimes when we employ the standard Kalman filtering algorithm [3]. These limit the scope of application of Kalman filtering technique. In this case, an alternative estimation method based on H_∞ filtering method has been proposed. In the past decade, numerous results based on H_∞ filtering method have been reported in the literature (see, for example, [4,5]). The H_∞ filtering technique is to seek a filter such that the resulting filtering error systems are asymptotically stable and satisfy a prescribed H_∞ performance level from the input disturbance to the filtering error output. It has been shown that the H_∞ filtering technique provides not only a guaranteed noise attenuation level but also robustness against unmodeled dynamics [4]. Scholars prosecute considerable study to H_∞ filtering and put forward many approaches to solve the problem, such as, the convex optimization approach or linear matrix inequality (LMI) approach [6,7], interpolation approach [8], polynomial equation approach [9], algebraic Riccati equation approach [10], and so on.

Recently, descriptor systems (also known as singular systems, generalized state-space systems, implicit systems, semi-state systems, etc.) have received great attention due to extensive application backgrounds to circuits [11], boundary control [12], etc. A lot of contributions have been made to the study of these systems, and a great number of results on the theory of normal systems have been generalized to the area of descriptor systems (see [13-17]). The filtering problems for descriptor systems are one of theirs. For example, in [18], Xu *et al.* study the H_∞ filtering problem for linear continuous descriptor systems. A necessary and sufficient condition for the solvability of this problem is obtained in terms of a set of linear matrix inequalities (LMIs). When these LMIs are feasible, a descriptor filter is obtained. [19] discussed the Kalman filtering problem for descriptor systems, in which a generalization of the shuffle algorithm was used to solve the problem and some unnecessary

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assumptions in earlier works were removed. For H_∞ filtering problem of descriptor systems, there are few results in the literature.

In this paper, we deal with H_∞ filtering problem for descriptor systems. Attention is focused on the design of linear normal filter systems such that the resulting error systems are regular, impulse-free and stable while the closed-loop transfer function from the disturbance to the filtering error output satisfies a prescribed H_∞-norm bound constraint. A necessary and sufficient condition for solvability of this problem is obtained in terms of LMIs, for which the so called interior point algorithms can be resorted to [20].

The outline of the paper is as follows. In Section 2, we formulate the problem to be solved and give some definitions. In Section 3, we give the necessary and sufficient condition that there exists the H_∞ filter, and the design approach is obtained. In Section 4, a numerical example is given to demonstrate the applicability of the proposed approach. Finally, we give the conclusion.

2. PROBLEM FORMULATION AND DEFINITIONS

For unforced descriptor systems

$$E\dot{x}(t) = Ax(t), \tag{1}$$

where $x(t) \in R^n$ is the state, rank E$=r \le n$.

We first give some elementary definitions that will be adopted throughout this paper. These definitions are easily found in some literatures, for example [13,18].

Definition 1: The descriptor systems (1) are regular if $\det(sE-A)$ is not identically zero.

Definition 2: The descriptor systems (1) are impulse-free if $\deg(\det(sE-A)) = rank(E)$.

Definition 3: The descriptor systems (1) are stable if all the roots of $\det(sE-A)=0$ have negative real parts.

Definition 4: The descriptor systems (1) are admissible if systems (1) are regular, stable and impulse-free.

For a matrix $M \in R^{n\times m}$ with rank r, the orthogonal complement $M^\perp$ is defined as a (possibly non-unique) $(n-r)\times n$ matrix such that $M^\perp M = 0$ and $M^\perp M^{\perp T} > 0$.

Now, consider the following linear descriptor systems

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Bw(t),\\ y(t) &= Cx(t) + Dw(t),\\ z(t) &= Lx(t), \end{aligned} \tag{2}$$

where $x(t) \in R^n$ is the state, $y(t) \in R^r$ is the measured output, $w(t) \in R^m$ is the disturbance input and $z(t) \in R^s$ is the vector to be estimated. A, B, C, D, and L are known constant matrices with appropriate dimensions; The matrix $E \in R^{n\times n}$ are singular. Without loss of generality, we assume that the matrix D is of full row rank.

The filter is of the form

$$\begin{aligned} \dot{\hat{x}}(t) &= A_F\hat{x}(t) + B_F y(t),\\ \hat{z}(t) &= C_F\hat{x}(t) + D_F y(t), \end{aligned} \tag{3}$$

where $\hat{x}(t) \in R^n$ and $\hat{z}(t) \in R^s$ are the state and output of the filter, respectively. The matrices A_F, B_F, C_F and D_F are the filter parameter matrices to be designed. Let $\tilde{z}(t) = z(t) - \hat{z}(t)$, $e(t) = [x(t)^T \ \ \hat{x}(t)^T]^T$. Concatenating the filter (3) to the systems (2), we obtain filtering error systems

$$\begin{aligned} \bar{E}\dot{e}(t) &= \bar{A}e(t) + \bar{B}w(t),\\ \tilde{z}(t) &= \bar{L}e(t) + \bar{D}w(t), \end{aligned} \tag{4}$$

where

$$\bar{E} = \begin{bmatrix} E & 0\\ 0 & I \end{bmatrix},\ \bar{A} = \begin{bmatrix} A & 0\\ B_F C & A_F \end{bmatrix},\ \bar{B} = \begin{bmatrix} B\\ B_F D \end{bmatrix}, \tag{5a}$$

$$\bar{L} = \begin{bmatrix} L - D_F C & -C_F \end{bmatrix},\ \bar{D} = -D_F D. \tag{5b}$$

The transfer function from $w(t)$ to $\tilde{z}(t)$ is given as

$$T(s) = \bar{L}(s\bar{E} - \bar{A})^{-1}\bar{B} + \bar{D}. \tag{6}$$

The H_∞ filtering problem to be addressed in this paper is formulated as follows: given the descriptor systems (2) and a prescribed $\gamma > 0$, determine a filter (3) such that the filtering error systems (4) are admissible and $\|T(s)\|_\infty < \gamma$.

Remark 1: Here, we assume that the filter is of the same order as the descriptor systems (2). It has been proven that for optimal reduced-order filter, the error variance can never be better than that of the Kalman filter, which has the same order as the system model. For higher order filters, it is possible to achieve the same error variance as the Kalman filter but can not be better. Detailed discussions can be found in [21].

Remark 2: In terms of (5), the admissibility of the error descriptor systems (4) implies that the descriptor systems (2) are admissible. Hence, we assume that the descriptor systems (2) are admissible in the following discussion.

3. MAIN RESULTS

In this section, an LMI approach will be developed to solve the H_∞ filtering problem formulated in the previous section. First, we present the following lemmas which will be used in the proof of our main result.

Lemma 1 [16,22]**:** Given a symmetric matrix Ξ and two matrices Γ and Π, consider the problem of finding some matrix Θ such that

$$\Xi + \Gamma\Theta\Pi + (\Gamma\Theta\Pi)^T < 0. \tag{7}$$

Then (7) is solvable for Θ if and only if

$$\Gamma^{\perp}\Xi\Gamma^{\perp T} < 0,\ \Pi^{T\perp}\Xi\Pi^{T\perp T} < 0. \tag{8}$$

The following lemma is a counterpart of theorem 1 in [23] (also see the Lemma 1 in [24]).

Lemma 2: The following statements S1) and S2) are equivalent.

S1) The following conditions i), ii) and iii) hold simultaneously.
   i) The descriptor systems (2) with $w(t)$=0 are admissible.
   ii) The transfer function given as

   $$T_1(s) = C(sE - A)^{-1}B + D,$$

   satisfies $\|T_1(s)\|_\infty < \gamma$.
   iii) The matrix D satisfies $\gamma^2 I - D^T D > 0$.

S2) There exists a matrix P satisfying the following LMIs

$$E^T P = P^T E \geq 0, \tag{9a}$$

$$\begin{bmatrix} A^T P + P^T A & P^T B & L^T \\ B^T P & -\gamma^2 I & D^T \\ L & D & -I \end{bmatrix} < 0. \tag{9b}$$

Now, we give our main result on the H_∞ filtering problem for descriptor systems.

Theorem 1: There exists a filter (3) such that the H_∞ filtering problem is solvable if and only if there exist matrices $X \geq 0$ and Y satisfying

$$E^T Y = Y^T E \geq 0, \tag{10}$$

$$\begin{bmatrix} Y^T A + A^T Y & Y^T B \\ B^T Y & -\gamma^2 I \end{bmatrix} < 0, \tag{11}$$

$$\begin{aligned} &W_1(XE+Y)^T A W_1^T + W_1 A^T (XE+Y) W_1^T \\ &+ W_2 B^T (XE+Y) W_1^T + W_1 (XE+Y)^T B W_2^T \\ &-\gamma^2 W_2 W_2^T + W_1 L^T L W_1^T < 0, \end{aligned} \tag{12}$$

where

$$\begin{bmatrix} W_1 & W_2 \end{bmatrix} = \begin{bmatrix} C & D \end{bmatrix}^{T\perp}. \tag{13}$$

In this case, the desired H_∞ filter corresponding to a feasible solution (X, Y) to (10)-(12) are parameterized as follows

$$\begin{aligned} \begin{bmatrix} D_F & C_F \\ B_F & A_F \end{bmatrix} &= -W^{-1}\Psi^T \Lambda \Phi^T (\Phi\Lambda\Phi^T)^{-1} \\ &\quad + W^{-1} S^{1/2} L_1 (\Phi\Lambda\Phi^T)^{-1/2}, \\ S &= W - \Psi^T [\Lambda - \Lambda\Phi^T (\Phi\Lambda\Phi^T)^{-1} \Phi\Lambda]\Psi, \\ \Lambda &= (\Psi W^{-1} \Psi^T - \Omega)^{-1}, \\ \Omega &= \begin{bmatrix} \Psi_{11} & A^T X_{12} & \Psi_{13} & L^T \\ X_{12}^T A & 0 & X_{12}^T B & 0 \\ \Psi_{13}^T & B^T X_{12} & -\gamma^2 I & 0 \\ L & 0 & 0 & -I \end{bmatrix}, \\ \Psi_{11} &= A^T Y + Y^T A + A^T XE + E^T XA, \\ \Psi_{13} &= Y^T B + E^T XB, \\ \Phi &= \begin{bmatrix} C & 0 & D & 0 \\ 0 & I & 0 & 0 \end{bmatrix},\ \Psi = \begin{bmatrix} 0 & E^T X_{12} \\ 0 & X_{22} \\ 0 & 0 \\ -I & 0 \end{bmatrix}, \end{aligned} \tag{14}$$

where L_1 is any matrix satisfying $\|L_1\| < 1$, and $X_{12} \in R^{n\times n}$, $0 < X_{22} = X_{22}^T \in R^{n\times n}$, $W > 0$ satisfying $\Lambda > 0$ and

$$X = X_{12} X_{22}^{-1} X_{12}^T \geq 0. \tag{15}$$

Proof: For (4), we let

$$G = \begin{bmatrix} D_F & C_F \\ B_F & A_F \end{bmatrix},\ \hat{A} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix},\ \hat{B} = \begin{bmatrix} B \\ 0 \end{bmatrix},\ \hat{F} = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}, \tag{16a}$$

$$\hat{H} = \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix},\ \hat{N} = \begin{bmatrix} D \\ 0 \end{bmatrix},\ \hat{C} = \begin{bmatrix} L & 0 \end{bmatrix},\ \hat{S} = \begin{bmatrix} -I & 0 \end{bmatrix}. \tag{16b}$$

Hence, we have

$$\bar{A} = \hat{A} + \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix} G \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix} = \hat{A} + \hat{F}G\hat{H}, \tag{17a}$$

$$\bar{B} = \hat{B} + \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix} G \begin{bmatrix} D \\ 0 \end{bmatrix} = \hat{B} + \hat{F}G\hat{N}, \tag{17b}$$

$$\bar{L} = \hat{C} + \begin{bmatrix} -I & 0 \end{bmatrix} G \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix} = \hat{C} + \hat{S}G\hat{H}, \tag{17c}$$

$$\bar{D} = \hat{S}G\hat{N}. \tag{17d}$$

From Lemma 2, the H_∞ filter problem has a solution if and only if there exists a matrix P such that

$$\overline{E}^T P = P^T \overline{E} \geq 0, \tag{18}$$

$$\Pi_1 = \begin{bmatrix} \overline{A}^T P + P^T \overline{A} & P^T \overline{B} & \overline{L}^T \\ \overline{B}^T P & -\gamma^2 I & \overline{D}^T \\ \overline{L} & \overline{D} & -I \end{bmatrix} < 0. \tag{19}$$

In terms of (19), we have

$$\overline{A}^T P + P^T \overline{A} < 0.$$

It implied that the matrix P is invertible. Now let

$$P = \begin{bmatrix} Z & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix},$$

and the matrix P partition is compatible with the matrix $\overline{A}$. Substituting the matrix P into (18), we have

$$\begin{bmatrix} E^T Z & E^T Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z^T E & Z_{21}^T \\ Z_{12}^T E & Z_{22}^T \end{bmatrix} \geq 0. \tag{20}$$

Hence,

$$E^T Z = Z^T E \geq 0, \tag{21}$$

$$Z_{22} = Z_{22}^T \geq 0, \tag{22}$$

$$Z_{21} = Z_{12}^T E. \tag{23}$$

By (19), we have

$$0 > \overline{A}^T P + P^T \overline{A}. \tag{24}$$

Substituting the matrix P and (5) into (24), and by (22), we yield

$$0 > \overline{A}^T P + P^T \overline{A} = \begin{bmatrix} A^T Z + C^T B_F^T Z_{21} + (A^T Z + C^T B_F^T Z_{21})^T & A^T Z_{12} + C^T B_F^T Z_{22} + (A^T Z_{12} + C^T B_F^T Z_{22})^T \\ A_F^T Z_{21} + Z_{21}^T A_F & A_F^T Z_{22} + Z_{22} A_F \end{bmatrix}. \tag{25}$$

By (25), we obtain

$$A_F^T Z_{22} + Z_{22} A_F < 0. \tag{26}$$

Therefore, the matrix Z_{22} is invertible, and combining with (22), then $Z_{22} > 0$, and A_F is stable. Then

$$P^{-1} = \begin{bmatrix} V & V_{12} \\ V_{21} & V_{22} \end{bmatrix}, \tag{27}$$

where

$$V = (Z - Z_{12} Z_{22}^{-1} Z_{21})^{-1}, \quad V_{12} = -V Z_{12} Z_{22}^{-1},$$
$$V_{21} = -Z_{22}^{-1} Z_{21} V, \quad V_{22} = Z_{22}^{-1} + Z_{22}^{-1} Z_{21} V Z_{12} Z_{22}^{-1}.$$

Utilizing Schur complement formula for (20), together with (27), we have

$$E^T V^{-1} = V^{-T} E \geq 0. \tag{28}$$

Now, we again consider (19). By (17), we derive

$$0 > \Pi_1 = \begin{bmatrix} \Psi_{111} & P^T(\hat{B} + \hat{F}G\hat{N}) & (\hat{C} + \hat{S}G\hat{H})^T \\ (\hat{B} + \hat{F}G\hat{N})^T P & -\gamma^2 I & (\hat{S}G\hat{N})^T \\ (\hat{C} + \hat{S}G\hat{H}) & \hat{S}G\hat{N} & -I \end{bmatrix}$$
$$= \begin{bmatrix} \hat{A}^T P + P^T \hat{A} & P^T \hat{B} & \hat{C}^T \\ \hat{B}^T P & -\gamma^2 I & 0 \\ \hat{C} & 0 & -I \end{bmatrix} + \begin{bmatrix} (\hat{F}G\hat{H})^T P & 0 & (\hat{S}G\hat{H})^T \\ (\hat{F}G\hat{N})^T P & 0 & (\hat{S}G\hat{N})^T \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} P^T \hat{F}G\hat{H} & P^T \hat{F}G\hat{N} & 0 \\ 0 & 0 & 0 \\ \hat{S}G\hat{H} & \hat{S}G\hat{N} & 0 \end{bmatrix},$$

where

$$\Psi_{111} = (\hat{A} + \hat{F}G\hat{H})^T P + P^T(\hat{A} + \hat{F}G\hat{H}).$$

Hence, we obtain

$$0 > \Pi_1 = \begin{bmatrix} \hat{A}^T P + P^T \hat{A} & P^T \hat{B} & \hat{C}^T \\ \hat{B}^T P & -\gamma^2 I & 0 \\ \hat{C} & 0 & -I \end{bmatrix}^T + \begin{bmatrix} P^T \hat{F} \\ 0 \\ \hat{S} \end{bmatrix} G \begin{bmatrix} \hat{H} & \hat{N} & 0 \end{bmatrix} + \left(\begin{bmatrix} P^T \hat{F} \\ 0 \\ \hat{S} \end{bmatrix} G \begin{bmatrix} \hat{H} & \hat{N} & 0 \end{bmatrix} \right).$$

We denote

$$\Phi_1 = \begin{bmatrix} \hat{H} & \hat{N} & 0 \end{bmatrix}, \quad \Psi_1 = \begin{bmatrix} \hat{F}^T P & 0 & \hat{S}^T \end{bmatrix}^T.$$

By (13) and (16), we have

$$\Psi_1^\perp = \begin{bmatrix} [I \quad 0] & 0 & 0 \\ [0 \quad 0] & I & 0 \end{bmatrix} \begin{bmatrix} P^{-T} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix},$$

$$\Phi_1^{T\perp} = \begin{bmatrix} [W_1 \;\; 0] & W_2 & 0 \\ [0 \;\; 0] & 0 & I \end{bmatrix}.$$

Therefore,

$$0 > \Pi_1 = \begin{bmatrix} \hat{A}^T P + P^T \hat{A} & P^T \hat{B} & \hat{C}^T \\ \hat{B}^T P & -\gamma^2 I & 0 \\ \hat{C} & 0 & -I \end{bmatrix} + \Psi_1 G \Phi_1 + (\Psi_1 G \Phi_1)^T. \quad (29)$$

In terms of Lemma 1, (29) holds if and only if the following two inequalities are true simultaneously.

$$\Pi_2 = \Psi_1^{\perp} \begin{bmatrix} \hat{A}^T P + P^T \hat{A} & P^T \hat{B} & \hat{C}^T \\ \hat{B}^T P & -\gamma^2 I & 0 \\ \hat{C} & 0 & -I \end{bmatrix} \Psi_1^{\perp T} < 0,$$

$$\Pi_3 = \Phi_1^{T\perp} \begin{bmatrix} \hat{A}^T P + P^T \hat{A} & P^T \hat{B} & \hat{C}^T \\ \hat{B}^T P & -\gamma^2 I & 0 \\ \hat{C} & 0 & -I \end{bmatrix} \Phi_1^{T\perp T} < 0.$$

Hence

$$\Pi_2 = \begin{bmatrix} AV + V^T A^T & B \\ B^T & -\gamma^2 I \end{bmatrix} < 0, \quad (30)$$

$$\Pi_3 = \begin{bmatrix} \Psi_{211} & W_1 L^T \\ L W_1^T & -I \end{bmatrix} < 0, \quad (31)$$

where

$$\Psi_{211} = W_1 Z^T A W_1^T + W_1 A^T Z W_1^T + W_2 B^T Z W_1^T + W_1 Z^T B W_2^T - \gamma^2 W_2 W_2^T.$$

Therefore, the necessary and sufficient condition for (29) to have a solution G is that (30) and (31) hold simultaneously. Let

$$X = Z_{12} Z_{22}^{-1} Z_{12}^T, \quad Y = V^{-1}. \quad (32)$$

Then, from $Z_{22} > 0$, we obtain $X \geq 0$ and

$$Y = V^{-1} = (Z - Z_{12} Z_{22}^{-1} Z_{21}) = Z - XE,$$

that is

$$Z = XE + Y. \quad (33)$$

From (28) and (32), we can easy deduce (11). Substituting (32) and (33) into (30) and (31), respectively, we obtain

$$\begin{bmatrix} AY^{-1} + Y^{-T} A^T & B \\ B^T & -\gamma^2 I \end{bmatrix} < 0, \quad (34a)$$

$$\begin{bmatrix} W_1 (XE + Y)^T A W_1^T + W_1 A^T (XE + Y) W_1^T \\ + W_2 B^T (XE + Y) W_1^T \\ + W_1 (XE + Y)^T B W_2^T - \gamma^2 W_2 W_2^T & W_1 L^T \\ L W_1^T & -I \end{bmatrix} < 0, \quad (34b)$$

then multiplying (34a) by $T = diag\begin{bmatrix} Y^T & I \end{bmatrix}$ on the left and by T^T on the right, we derive

$$\begin{bmatrix} Y^T A + A^T Y & Y^T B \\ B^T Y & -\gamma^2 I \end{bmatrix} < 0.$$

By Schur complement, (34b) is equivalent to (12).

Finally, when (10)-(12) are true, and taking note of the matrix D is of full row rank, then the parameterization of all H_∞ filters satisfying (29) can be given by using the results in [16,22]. This completes the proof. □

Remark 3: In [18], Xu *et al.* studies H_∞ filter problem for descriptor systems (2), and obtains the descriptor filter. The number of the finite modes of the descriptor filter equals to rank E. In other words, the filter is proper with a McMillan degree no more than the number of the exponential modes of the given plant (2). However, the descriptor filter is more difficult to physically realize than the normal filter (3), and the descriptor filter in [18] is more insensitive to external noise input. These are the disadvantage of [18].

Remark 4: Under the assumption of the detectability, a Luenberger-type filter can be adopted, and the filter gain can be determined through the method given in [13]. It is noted that Luenberger-type filters are different from those considered in the present paper. In this paper, we give an explicit parameterization expression of all desired normal filter and the order of designed normal filter is assumed to be with the same dimension of the descriptor variable vector as the systems (2). However, the order of the normal filter given by (14) is a little higher than necessary. It will be more significant if the order of designed normal filter is the same degree as the finite modes of the descriptor systems (2). This is one of our further research interests.

Remark 5: We can resort to the interior point algorithms and obtain the desired normal filter. No adjusting of parameters is required to solve LMIs in (10)-(12) [20]. It is also worth mentioning that the optimal normal filter in the sense of the minimum H_∞ norm γ can be obtained by solving the following optimization problem

$$\min_{X,Y,\delta} \delta \quad (35)$$

Subject to (i) $\delta = \gamma^2$;

(ii) LMI (10)-(12).

Remark 6: In the case when $E = I$, that is, descriptor systems (2) generates to a normal systems, we have the following H_∞ filtering result:

Corollary 1: Consider the normal systems described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bw(t), \\ y(t) &= Cx(t) + Dw(t), \\ z(t) &= Lx(t). \end{aligned} \tag{36}$$

Then, there exists a filter (3) such that the H_∞ filtering problem is solvable if and only if there exist matrices $X \geq 0$ and $Y > 0$ satisfying

$$\begin{bmatrix} YA + A^T Y & YB \\ B^T Y & -\gamma^2 I \end{bmatrix} < 0, \tag{37}$$

$$\begin{aligned} &W_1(X+Y)AW_1^T + W_1A^T(X+Y)W_1^T \\ &+W_2B^T(X+Y)W_1^T + W_1(X+Y)BW_2^T \\ &-\gamma^2 W_2 W_2^T + W_1 L^T L W_1^T < 0. \end{aligned} \tag{38}$$

In this case, the parameterization of all H_∞ filters can be given by using the results in [16,22].

4. NUMERICAL EXAMPLE

In this section, we give an example to demonstrate the effectiveness of the proposed method.

Consider the descriptor systems (2) with the following parameters

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} -1 & 0 \\ 0.5 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0.5 \\ 0.2 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D = 1, \quad L = \begin{bmatrix} 1 & -1 \end{bmatrix}.$$

In terms of (13), we can obtain

$$W_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad W_2 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}.$$

We aim at designing normal filter systems (3) such that the resulting error systems (4) are admissible and $\|T(s)\|_\infty < \gamma$, where $\gamma > 0$ is specified. In this example, we suppose that $\gamma = 1$. We utilize the Matlab LMI Control Toolbox to solve the LMIs in (10)-(12) and yield the solution

$$X = \begin{bmatrix} 0.9682 & 0.1793 \\ 0.1793 & 2.1828 \end{bmatrix}, \quad Y = \begin{bmatrix} 0.9963 & 0 \\ 0.3774 & -1.3630 \end{bmatrix}.$$

By (15), we can choose $X_{12} = X_{22} = X > 0$. Then by calculating, we obtain

$$\Phi = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix},$$

$$\Psi = \begin{bmatrix} 0 & 0.9682 & 0.1793 \\ 0 & 0 & 0 \\ 0 & 0.9682 & 0.1793 \\ 0 & 0.1793 & 2.1828 \\ 0 & 0 & 0 \\ -1.0000 & 0 & 0 \end{bmatrix},$$

$$\Omega = \begin{bmatrix} -3.3722 & -0.1247 & -0.8785 & 0.9121 & 1.0936 & 1.0000 \\ -0.1247 & -2.7260 & 0.1793 & 2.1828 & -0.2726 & -1.0000 \\ -0.8785 & 0.1793 & 0 & 0 & 0.5200 & 0 \\ 0.9121 & 2.1828 & 0 & 0 & 0.5262 & 0 \\ 1.0936 & -0.2726 & 0.5200 & 0.5262 & -1.0000 & 0 \\ 1.0000 & -1.0000 & 0 & 0 & 0 & -1.0000 \end{bmatrix}_{6\times 6}.$$

Let

$$W = \begin{bmatrix} 0.6667 & 0 & 0 \\ 0 & 0.6667 & 0 \\ 0 & 0 & 0.6667 \end{bmatrix},$$

it is easy to check $\Lambda > 0$. So

$$S = \begin{bmatrix} 0.1720 & 0.0566 & 0.0105 \\ 0.0566 & 0.5431 & -0.0229 \\ 0.0105 & -0.0229 & 0.6624 \end{bmatrix}.$$

Therefore, the all parameter matrices of filter (3) can be obtained

$$\begin{bmatrix} D_F & C_F \\ B_F & A_F \end{bmatrix} = \begin{bmatrix} 0.2564 & -0.2980 & -0.6243 \\ -0.4064 & -0.9024 & -0.1353 \\ -0.0753 & -0.1671 & -3.2495 \end{bmatrix} + \begin{bmatrix} 0.6174 & 0.0742 & 0.0137 \\ 0.0742 & 1.1027 & -0.0226 \\ 0.0137 & -0.0226 & 1.2206 \end{bmatrix} L_1$$
$$* \begin{bmatrix} 0.5255 & 0.3095 & -0.0445 \\ 0.3095 & 0.4823 & 0.1570 \\ -0.0445 & 0.1570 & 2.2485 \end{bmatrix},$$

where the matrix L_1 is any matrix satisfying $\|L_1\|<1$. For example, if we choose

$$L_1=\begin{bmatrix}0.5 & 0 & 0\\ 0 & 0.5 & 0\\ 0 & 0 & 0.5\end{bmatrix},$$

then the corresponding matrices of filter (3) are given by

$$\begin{bmatrix}D_F & C_F\\ B_F & A_F\end{bmatrix}=\left[\begin{array}{c|cc}0.4298 & -0.1835 & -0.6168\\ \hline -0.2158 & -0.6268 & -0.0758\\ -0.1023 & -0.0747 & -1.8793\end{array}\right].$$

That is,

$$\dot{\hat{x}}(t)=\begin{bmatrix}-0.6268 & -0.0758\\ -0.0747 & -1.8793\end{bmatrix}\hat{x}(t)+\begin{bmatrix}-0.2158\\ -0.1023\end{bmatrix}y(t),$$

$$\hat{z}(t)=\begin{bmatrix}-0.1835 & -0.6168\end{bmatrix}\hat{x}(t)+0.4298y(t).$$

Then this normal filter is asymptotically, and the resulting filtering error dynamics (4) is admissible and the transfer function from the disturbance to the error output satisfying $\|T(s)\|_\infty<1$. Furthermore, the optimal H_∞-norm can be calculated to be 0.1.

5. CONCLUSIONS

In this paper, we have studied the design approachof H_∞ filter for descriptor systems. In terms of a series of LMIs, a necessary and sufficient condition for the existence of a linear normal filter has been obtained, which guarantees that the error dynamics is admissible and the transfer function from the disturbance to the error output satisfies a prescribed H_∞-norm bound constraint. An example has been provided to illustrate the effectiveness of the proposed approach.

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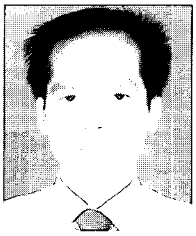

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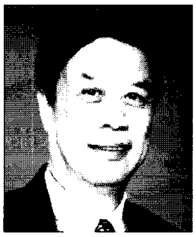

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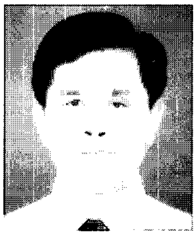

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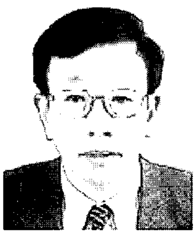

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