Journal of Control, Automation and Systems Engineering Vol. 12, No. 12, December 2006

가변 조향링크 옵셋을 갖는 캐스터 바퀴 이동로봇의 등방성 분석

Isotropy Analysis of Caster Wheeled Mobile Robot with Variable Steering Link Offset

김 성 복*, 문 병 권
(Sungbok Kim, Byungkwon Moon)

Abstract : Previous isotropy analysis of a caster wheeled omnidirectional mobile robot (COMR) has been made under the assumption that the steering link offset is equal to the caster wheel radius. Nevertheless, many practical COMR's in use take advantage of the steering link offset different from the wheel radius, mainly because of improved stability. This paper presents the isotropy analysis of a fully actuated COMR with variable steering link offset, which can be considered as the generalization of the previous analysis. First, the kinematic model of a COMR under full actuation is obtained based on the orthogonal decomposition of the wheel velocities. Second, the necessary and sufficient conditions for the isotropy of a COMR are derived and examined to categorize three different groups, each of which can be dealt with in a similar way. Third, for each group, the isotropy conditions are further explored so as to identify all possible isotropic configurations completely.

Keywords : caster wheeled mobile robot, variable steering link offset, isotropy analysis, isotropic configuration

I. Introduction

Typically, personal robots are requested to have omnidirectional mobility to navigate at human walking speed in daily life environment that is restricted in space and cluttered with stationary and moving obstacles. Several omnidirectional wheel mechanisms have been proposed, including universal wheels, Swedish wheels, orthogonal wheels, and ball wheels. Recently, caster wheels were employed to develop an omnidirectional mobile robot at Stanford University [1], which was commercialized by Nomadic Technologies as XR4000. Since caster wheels operate without additional peripheral rollers or support structure, a caster wheeled omnidirectional mobile robot (COMR) can maintain good performance even through payload or ground condition changes.

There have been several works on the kinematic issues of a COMR. For a general form of wheeled mobile robots, a systematic procedure for kinematic modeling was developed [2,3]. Regarding the minimal admissible actuation, it was shown that at least four joints out of two caster wheels should be actuated to avoid the singularity [4]. For some specific actuation sets, the global isotropic characteristics over the entire wheel configurations was considered to determine the optimal design parameters [5]. For representative actuation sets, the algebraic conditions for the isotropy were derived to identify the isotropic configurations completely [6]. On the other hand, for an omnidirectional mobile robot employing Swedish wheels, the isotropy analysis was made but the results are incomplete and need further elaboration [7].

With nonzero steering link offset, a COMR always has the omnidirectional mobility independently of a given wheel configuration. In contrast to this, a COMR can fall into the singularity or the isotropy depending on a given wheel configuration. At singular configurations, a COMR becomes instantaneously movable even with all actuated joints locked [8]. On the other hand, at isotropic configurations, the joint velocities of a COMR required for a unity task velocity in all directions become uniform in magnitude [9]. Obviously, it is desirable for robust motion control to keep a COMR away from the singularity but close to the isotropy, as much as possible [7].

Previous isotropy analysis of a COMR has been made under the assumption that the steering link offset is equal to the wheel radius [5,6]. It was found that such a restriction is necessary to have globally optimal isotropic characteristics of a COMR. Nevertheless, many practical COMR's in use take advantage of the steering link offset which is different from the wheel radius, mainly because of improved stability [10]. The purpose of this paper is to analyze the isotropy of a fully actuated COMR with variable steering link offset, which can be considered as the generalization of the previous analysis. To this end, the ratio of the steering offset to the wheel radius is deliberately incorporated into the isotropy analysis of a COMR.

---

* 책임저자(Corresponding Author)
논문접수 : 2006. 5. 19., 채택확정 : 2006. 9. 21.
김성복, 문병권 : 한국외국어대학교 전자정보공학부
(sbkim@hufs.ac.kr/kwon-e@hufs.ac.kr)
※ This work was supported by Hankuk University of Foreign Studies Research Fund of 2006.

This paper is organized as follows. With the characteristic length introduced [7], Section 2 develops the kinematic model based on the orthogonal decomposition of the wheel velocities. Sections 3 derives and examines the necessary and sufficient conditions for the isotropy to categorize three different groups, each of which can be treated in a similar way. For each group, Section 4 explores the isotropy conditions so as to identify all possible isotropic configurations completely. Finally, the conclusion and some discussions are made in Section 5.

II. Kinematic Model

Consider a COMR with three caster wheels attached to a regular triangular platform moving on the xy-plane, as shown in Fig. 1. Let l be the side length of the platform with the center denoted by O_b, and the vertices denoted by O_i, $i=1,2,3$. For the i^{th} caster wheel with the center denoted by P_i, $i=1,2,3$, we define the following. Let d_i and r_i be the steering link offset and the wheel radius, respectively. Let φ_i and θ_i be the steering and the rotating angles of the caster wheel, respectively. In what follows, three caster wheels are assumed to have identical structure, that is, $d_1=d_2=d_3=d\ (\geq 0)$ and $r_1=r_2=r_3=r\ (>0)$.

Let u_i and v_i, $i=1,2,3$, be two orthogonal unit vectors along the steering link and the wheel axis, respectively, such that

$$\mathrm{u}_i=\begin{bmatrix}-\cos\varphi_i\\-\sin\varphi_i\end{bmatrix},\quad \mathrm{v}_i=\begin{bmatrix}-\sin\varphi_i\\\cos\varphi_i\end{bmatrix} \tag{1}$$

Note that

$$\mathrm{u}_i\,\mathrm{u}_i^t+\mathrm{v}_i\,\mathrm{v}_i^t=\mathrm{I}_2 \tag{2}$$

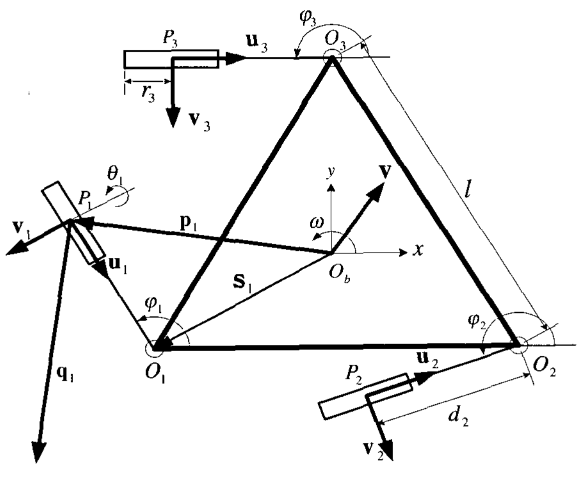


그림 1. 캐스터 바퀴를 갖는 전방향성 이동 로봇.

Fig. 1. A caster wheeled omnidirectional mobile robot.

$$\sum_1^3\mathrm{u}_i=0\ \Leftrightarrow\ \sum_1^3\mathrm{v}_i=0 \tag{3}$$

where I_2 is the 2x2 identity matrix and 0 is the 2x1 zero vector. Let s_i be the vector from O_b to O_i:

$$\mathrm{s}_1=\frac{l}{\sqrt{3}}\begin{bmatrix}-\frac{\sqrt{3}}{2}\\-\frac{1}{2}\end{bmatrix},\ \mathrm{s}_2=\frac{l}{\sqrt{3}}\begin{bmatrix}\frac{\sqrt{3}}{2}\\-\frac{1}{2}\end{bmatrix},\ \mathrm{s}_3=\frac{l}{\sqrt{3}}\begin{bmatrix}0\\1\end{bmatrix} \tag{4}$$

Note that

$$\sum_1^3\mathrm{s}_i=0 \tag{5}$$

Let p_i be the vector from O_b to P_i, and q_i be the rotation of $\mathbf{p}_i$ by 90° counterclockwise. Note that

$$\mathrm{p}_i=\mathrm{s}_i-d\,\mathrm{u}_i \tag{6}$$

$$\sum_1^3\mathrm{p}_i=0\ \Leftrightarrow\ \sum_1^3\mathrm{q}_i=0\ \Leftrightarrow\ \sum_1^3\mathrm{u}_i=0 \tag{7}$$

Let v and ω be the linear and the angular velocities at O_b of the platform, respectively. For the i^{th} caster wheel, $i=1,2,3$, the linear velocity at the point of contact with the ground can be expressed by

$$\mathrm{v}+\omega\,\mathrm{q}_i=r\,\dot{\theta}_i\mathrm{u}_i+d\,\dot{\varphi}_i\,\mathrm{v}_i \tag{8}$$

Premultiplied by u_i^t and v_i^t, respectively, we have

$$\mathrm{u}_i^t\mathrm{v}+\mathrm{u}_i^t\mathrm{q}_i\,\omega=r\,\dot{\theta}_i \tag{9}$$

$$\mathrm{v}_i^t\mathrm{v}+\mathrm{v}_i^t\mathrm{q}_i\,\omega=d\,\dot{\varphi}_i \tag{10}$$

Notice that the instantaneous motion of the wheel is decomposed into two orthogonal components of the rotating and the steering joints. Using (6), the expressions of $\mathrm{u}_i^t\mathrm{q}_i$ and $\mathrm{v}_i^t\mathrm{q}_i$, $i=1,2,3$, can be written as follows:

$$\begin{aligned}\mathrm{u}_i^t\mathrm{q}_i&=\mathrm{v}_i^t\mathrm{p}_i\\&=\mathrm{v}_i^t(\mathrm{s}_i-d\,\mathrm{u}_i)=\mathrm{v}_i^t\mathrm{s}_i\end{aligned} \tag{11}$$

$$\begin{aligned}\mathrm{v}_i^t\mathrm{q}_i&=-\mathrm{u}_i^t\mathrm{p}_i\\&=-\mathrm{u}_i^t(\mathrm{s}_i-d\,\mathrm{u}_i)=-\mathrm{u}_i^t\mathrm{s}_i+d\end{aligned} \tag{12}$$

With the introduction of the characteristic length, $L\ (>0)$ [7], the kinematics of a COMR under full actuation can be written as

$$\mathrm{A}\,\dot{\mathrm{x}}=\mathrm{B}\,\dot{\Theta} \tag{13}$$

where $\dot{\mathrm{x}}=[\,\mathrm{v}\ \ L\omega\,]^t\in\mathrm{R}^{3\times 1}$ is the task velocity vector, and $\dot{\Theta}=[\,\dot{\theta}_1\ \dot{\theta}_2\ \dot{\theta}_3\ \dot{\varphi}_1\ \dot{\varphi}_2\ \dot{\varphi}_3\,]^t\in\mathrm{R}^{6\times 1}$ is the joint velocity vector, and

$$A = \begin{bmatrix} u_1^t & \frac{1}{L} & u_1^t q_1 \\ u_2^t & \frac{1}{L} & u_2^t q_2 \\ u_3^t & \frac{1}{L} & u_3^t q_3 \\ v_1^t & \frac{1}{L} & v_1^t q_1 \\ v_2^t & \frac{1}{L} & v_2^t q_2 \\ v_3^t & \frac{1}{L} & v_3^t q_3 \end{bmatrix} \in R^{6\times 3} \quad (14)$$

$$B = \begin{bmatrix} r\ I_3 & 0_3 \\ 0_3 & d\ I_3 \end{bmatrix} \in R^{6\times 6} \quad (15)$$

are the Jacobian matrices. It should be mentioned that the introduction of the characteristic length L makes all three columns of A to be consistent in physical unit.

III. Isotropy Conditions

1. Three isotropy conditions

The kinematics of a COMR, (13), can be written as

$$Z\ \dot{x} = \Theta \quad (16)$$

where

$$Z = B^{-1} A \quad (17)$$

Based on (16), the necessary and sufficient condition for the isotropy of a COMR can be expressed as

$$Z^t\ Z \propto I_3 \quad (18)$$

Plugging (14) and (15) into (18), we have (19) shown at the bottom of this page. Since

$$\sigma = \frac{1}{3}\left[\ \sum_{i=1}^{3}\left(\frac{1}{r^2}+\frac{1}{d^2}\right)+\sigma\right] \quad (20)$$

it follows that

$$\sigma = \frac{1}{2}\sum_{i=1}^{3}\left(\frac{1}{r^2}+\frac{1}{d^2}\right) \quad (21)$$

From (19), the following three isotropy conditions of a COMR can be obtained:

$$\text{C1: } \sum_{i=1}^{3}\left[\frac{1}{r^2}(u_i\ u_i^t) + \frac{1}{d^2}(v_i\ v_i^t)\right] = \sigma\ I_2 \quad (22)$$

$$\text{C2: } \frac{1}{L}\sum_{i=1}^{3}\left[\frac{1}{r^2}(u_i^t\ q_i)\ u_i + \frac{1}{d^2}(v_i^t\ q_i)\ v_i\right] = 0 \quad (23)$$

$$\text{C3: } \frac{1}{L^2}\sum_{i=1}^{3}\left[\frac{1}{r^2}(u_i^t\ dq_i)^2 + \frac{1}{d^2}(v_i^t\ q_i)^2\right] = \sigma \quad (24)$$

In general, C1 and C2 are functions of the steering joint angles, $(\varphi_1, \varphi_2, \varphi_3)$, from which the isotropic configurations can be identified. With $(\varphi_1, \varphi_2, \varphi_3)$ known, C3

표 1. ($\hat{\varphi}_2$, $\hat{\varphi}_3$)에 대한 3개 그룹의 해.

Table 1. Three groups of ($\hat{\varphi}_2$, $\hat{\varphi}_3$).

Group 1	Group 2	Group 3
$\mu=1$	$(-\frac{\pi}{3}, \frac{\pi}{3})$	$(\frac{\pi}{3}, -\frac{\pi}{3})$
	$(-\frac{\pi}{3}, -\frac{2\pi}{3})$	$(-\frac{2\pi}{3}, -\frac{\pi}{3})$
	$(\frac{2\pi}{3}, \frac{\pi}{3})$	$(\frac{\pi}{3}, \frac{2\pi}{3})$
	$(\frac{2\pi}{3}, -\frac{2\pi}{3})$	$(-\frac{2\pi}{3}, \frac{2\pi}{3})$

determines the value of L which is required for the isotropy, called the isotropic characteristic length.

2. First isotropy condition

From (21) and (22), the first isotropy condition C1 can be written as

$$\sum_{i=1}^{3}[\mu(u_i\ u_i^t) + (v_i\ v_i^t)] = \frac{3}{2}(\mu+1) \quad (25)$$

with

$$\mu = \left(\frac{d}{r}\right)^2 \ (>0) \quad (26)$$

Note that μ represents the square of the ratio of the steering link offset to the wheel radius. Plugging (1) into (25), we have

$$\begin{aligned} &\mu(c_1^2+c_2^2+c_3^2) + s_1^2+s_2^2+s_3^2 = \frac{3}{2}(\mu+1) \\ &\mu(c_1s_1+c_2s_2+c_3s_3) - (c_1s_1+c_2s_2+c_3s_3) = 0 \end{aligned} \quad (27)$$

where $c_i = \cos(\varphi_i)$ and $s_i = \sin(\varphi_i)$, $i=1,2,3$. Using trigonometric function formulas, (27) can be rewritten as

$$\begin{aligned} &(\mu-1)\left(1+\cos 2\hat{\varphi}_2+\cos 2\hat{\varphi}_3\right) = 0 \\ &(\mu-1)\left(\sin 2\hat{\varphi}_2+\sin 2\hat{\varphi}_3\right) = 0 \end{aligned} \quad (28)$$

where $\hat{\varphi}_2 = \varphi_2 - \varphi_1$ and $\hat{\varphi}_3 = \varphi_3 - \varphi_1$. There are three different groups of the solutions to (28), including $\mu=1$ and two groups of ($\hat{\varphi}_2$, $\hat{\varphi}_3$) listed in Table 1.

3. Second isotropy condition

From (21) and (23), the second isotropy condition C2 can be written as

$$\sum_{i=1}^{3}[\mu(u_i^t\ q_i)\ u_i + (v_i^2\ q_i)\ v_i] = 0 \quad (29)$$

Using (11) and (12), (29) can be expressed as

$$\sum_{i=1}^{3}[\mu(v_i^t\ s_i)\ v_i + (u_i^t\ s_i)\ u_i - d\ u_i] = 0 \quad (30)$$

$$\begin{bmatrix} \sum_{i=1}^{3}\left[\frac{1}{r^2}(u_i\ u_i^t)+\frac{1}{d^2}(v_i\ v_i^t)\right] & \frac{1}{L}\sum_{i=1}^{3}\left[\frac{1}{r^2}(u_i^t\ q_i)\ u_i+\frac{1}{d^2}(v_i^t\ q_i)\ v_i\right] \\ \frac{1}{L}\sum_{i=1}^{3}\left[\frac{1}{r^2}(u_i^t\ q_i)\ u_i^t+\frac{1}{d^2}(v_i^t\ q_i)\ v_i^t\right] & \frac{1}{L^2}\sum_{i=1}^{3}\left[\frac{1}{r^2}(u_i^t\ q_i)^2+\frac{1}{d^2}(v_i^t\ q_i)^2\right] \end{bmatrix} = \sigma\ \mathbf{I}_3 \quad (19)$$

Since

$$\sum_{i=1}^{3} s_i = \sum_{i=1}^{3} u_i^t s_i + \sum_{i=1}^{3} v_i^t s_i = 0 \quad (31)$$

(30) can be rewritten as

$$(\mu-1)\sum_{i=1}^{3}(v_i^t s_i) v_i - d\sum_{i=1}^{3} u_i = 0 \quad (32)$$

For three different groups satisfying (28), (32) will be further explored to identify all possible isotropic configurations of a COMR.

IV. Isotropy Configurations

1. Isotropy configurations for Group 1

With $\mu=1$, (32) reduces to

$$\sum_{i=0}^{3} u_i = 0 \quad (33)$$

which yields

$$\varphi_2 = \varphi_1 \pm \frac{2\pi}{3}, \quad \varphi_3 = \varphi_1 \mp \frac{2\pi}{3} \quad (34)$$

There are two sets of infinitely many isotropic configurations: $(\varphi_1, \varphi_1+\frac{2\pi}{3}, \varphi_1-\frac{2\pi}{3})$ and $(\varphi_1, \varphi_1-\frac{2\pi}{3}, \varphi_1+\frac{2\pi}{3})$, as illustrated in Fig. 2(a) and (b). Note that $\mu=1$ corresponds to the case of the steering link offset equal to the wheel radius [6].

2. Isotropy configurations for Group 2

Let us consider the case of $(\hat{\varphi}_2, \hat{\varphi}_3) = (-\frac{\pi}{3}, \frac{\pi}{3})$, for which

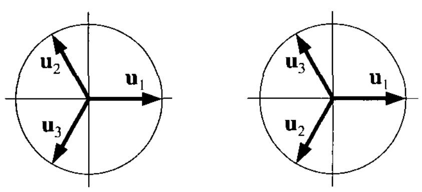
$$v_2^t s_2 = v_3^t s_3 = -v_1^t s_1 \quad (35)$$

so that

$$\sum_{i=1}^{3}(v_i^t s_i) v_1 = v_1^t s_1(v_1 - v_2 - v_3) = 0 \quad (36)$$

Plugging (36) into (32), we have

$$\sum_{i=1}^{3} u_i = 0 \quad (37)$$



(a) $(\varphi_1, \varphi_1+\frac{2\pi}{3}, \varphi_1-\frac{2\pi}{3})$ (b) $(\varphi_1, \varphi_1-\frac{2\pi}{3}, \varphi_1+\frac{2\pi}{3})$

그림 2. 등방성 형상.

Fig. 2. Isotropic configurations.

which cannot be satisfied. This implies that there does not exist the isotropic configuration when $(\hat{\varphi}_2, \hat{\varphi}_3) = (-\frac{\pi}{3}, \frac{\pi}{3})$. Similar analysis to the above can be made for the cases of $(\hat{\varphi}_2, \hat{\varphi}_3) = (-\frac{\pi}{3}, -\frac{2\pi}{3})$ and $(\frac{2\pi}{3}, \frac{\pi}{3})$. For both cases, there exists no isotropic configuration.

Finally, let us consider the case of $(\hat{\varphi}_2, \hat{\varphi}_3) = (\frac{2\pi}{3}, -\frac{2\pi}{3})$, for which

$$v_1^t s_1 = v_2^t s_2 = v_3^t s_3 \quad (37)$$

and

$$\sum_{i=1}^{3} u_i = \sum_{i=1}^{3} v_i = 0 \quad (38)$$

With (37) and (38) held, (32) is satisfied independently of the steering link offset d and the ratio μ. This implies that there are a single set of infinitely many isotropic configurations, $(\varphi_1, \varphi_2+\frac{2\pi}{3}, \varphi_3-\frac{2\pi}{3})$, illustrated in Fig. 2(a). It should be noticed that such isotropic configurations can be found regardless of the values of d and μ.

3. Isotropy configurations for Group 3

Let us consider the case of $(\hat{\varphi}_2, \hat{\varphi}_3) = (\frac{\pi}{3}, -\frac{\pi}{3})$, for which

$$\begin{aligned} v_1^t s_1 &= -\frac{1}{2\sqrt{3}} c_1 + \frac{1}{2} s_1 \\ v_2^t s_2 &= -\frac{1}{\sqrt{3}} c_1 \\ v_3^t s_3 &= \frac{1}{2\sqrt{3}} c_1 + \frac{1}{2} s_1 \end{aligned} \quad (39)$$

Plugging (1) and (39) into (32), we obtain

$$(\mu-1)\begin{bmatrix} \frac{3}{4}c_1^2 + \frac{\sqrt{3}}{2}c_1 s_1 - \frac{3}{4}s_1^2 \\ -\frac{\sqrt{3}}{4}c_1^2 + \frac{3}{2}c_1 s_1 + \frac{\sqrt{3}}{4}s_1^2 \end{bmatrix} - d\begin{bmatrix} -2c_1 \\ -2s_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (40)$$

which can be rewritten in the form of

$$\begin{aligned} A(\mu-1) + Bd &= 0 \\ C(\mu-1) + Dd &= 0 \end{aligned} \quad (41)$$

where

$$\begin{aligned} A &= \frac{3}{4}c_1^2 + \frac{\sqrt{3}}{2}c_1 s_1 - \frac{3}{4}s_1^2, \quad & B &= 2c_1 \\ C &= -\frac{\sqrt{3}}{4}c_1^2 + \frac{3}{2}c_1 s_1 + \frac{\sqrt{3}}{4}s_1^2, \quad & D &= 2s_1 \end{aligned} \quad (42)$$

For the existence of the solution to (41), it should hold that

$$AD - BC = 0 \quad (43)$$

which is

$$c_1 - \sqrt{3}s_1 = 0 \tag{44}$$

hence

$$\varphi_1 = \frac{\pi}{6},\ -\frac{5\pi}{6} \tag{45}$$

Plugging (44) into (40), we obtain

$$(\mu-1)c_1^2 + 2c_1 d = 0 \tag{46}$$

which yields

$$d = \frac{1}{2}(1-\mu)c_1 \tag{47}$$

From (45), (46) and (47), it follows that

$$\begin{aligned} &\varphi_1 = \frac{\pi}{6}, \quad d = \frac{\sqrt{3}}{4}(1-\mu), \quad \text{if } 0<\mu<1 \\ &\varphi_1 = -\frac{5\pi}{6}, \quad d = \frac{\sqrt{3}}{4}(\mu-1), \quad \text{if } \mu>1 \end{aligned} \tag{48}$$

In the case of $(\hat{\varphi}_2, \hat{\varphi}_3) = (\frac{\pi}{3}, -\frac{\pi}{3})$, there is a single isotropic configuration depending on the value of the ratio μ, which is illustrated in Fig. 3(a) and (b). Furthermore, such an isotropic configuration can be found only for a COMR having the specific value of the steering link offset d.

Similar analysis to the above can be made for the cases of $(\hat{\varphi}_2, \hat{\varphi}_3) = (-\frac{2\pi}{3}, -\frac{\pi}{3})$ and $(\frac{\pi}{3}, \frac{2\pi}{3})$. In the case of $(\hat{\varphi}_2, \hat{\varphi}_3) = (-\frac{2\pi}{3}, -\frac{\pi}{3})$, we obtain

$$\begin{aligned} &\varphi_1 = -\frac{\pi}{6}, \quad d = \frac{\sqrt{3}}{4}(1-\mu), \quad \text{if } 0<\mu<1 \\ &\varphi_1 = \frac{5\pi}{6}, \quad d = \frac{\sqrt{3}}{4}(\mu-1), \quad \text{if } \mu>1 \end{aligned} \tag{49}$$

which is illustrated in Fig. 4(a) and (b). And, in the case of $(\hat{\varphi}_2, \hat{\varphi}_3) = (\frac{\pi}{3}, \frac{2\pi}{3})$, we obtain

$$\begin{aligned} &\varphi_1 = \frac{\pi}{2}, \quad d = \frac{\sqrt{3}}{4}(1-\mu), \quad \text{if } 0<\mu<1 \\ &\varphi_1 = -\frac{\pi}{2}, \quad d = \frac{\sqrt{3}}{4}(\mu-1), \quad \text{if } \mu>1 \end{aligned} \tag{50}$$

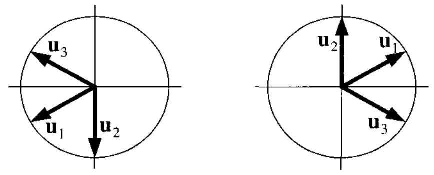
which is illustrated in Fig. 5(a) and (b).

Finally, let us consider the case of $(\hat{\varphi}_2, \hat{\varphi}_3) = (-\frac{2\pi}{3}, \frac{2\pi}{3})$, for which

$$\begin{aligned} v_1^t s_1 &= -\frac{1}{2\sqrt{3}}c_1 + \frac{1}{2}s_1 \\ v_2^t s_2 &= \frac{1}{\sqrt{3}}c_1 \\ v_3^t s_3 &= -\frac{1}{2\sqrt{3}}c_1 - \frac{1}{2}s_1 \end{aligned} \tag{51}$$

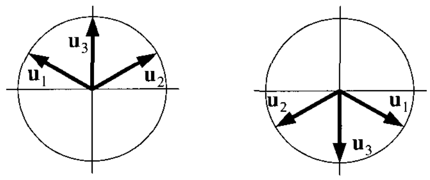
and

$$\sum_{i=1}^{3} u_i = 0 \tag{52}$$



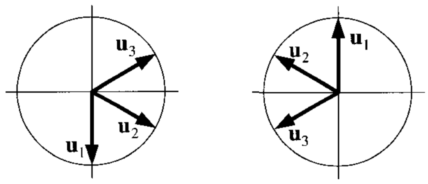
(a) $(\frac{\pi}{6}, \frac{\pi}{2}, -\frac{\pi}{6})$ (b) $(-\frac{5\pi}{6}, -\frac{\pi}{2}, \frac{5\pi}{6})$

그림 3. 등방성 형상.
Fig. 3. Isotropic configurations.



(a) $(-\frac{\pi}{6}, -\frac{5\pi}{6}, -\frac{\pi}{2})$ (b) $(\frac{5\pi}{6}, \frac{\pi}{6}, \frac{\pi}{2})$

그림 4. 등방성 형상.
Fig. 4. Isotropic configurations.



(a) $(\frac{\pi}{2}, \frac{5\pi}{6}, -\frac{5\pi}{6})$ (b) $(-\frac{\pi}{2}, -\frac{\pi}{6}, \frac{\pi}{6})$

그림 5. 등방성 형상.
Fig. 5. Isotropic configurations.

Using (1), (51), and (52), (32) can be written as

$$(\mu-1)\begin{bmatrix} \frac{3}{4}c_1^2 + \frac{\sqrt{3}}{2}c_1 s_1 - \frac{3}{4}s_1^2 \\ -\frac{\sqrt{3}}{4}c_1^2 + \frac{3}{2}c_1 s_1 + \frac{\sqrt{3}}{4}s_1^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{53}$$

Unless $\mu = 1$, there does not exist φ_1 satisfying (53), which implies that there exists no isotropic configuration.

V. Conclusion and Some Discussions

In this paper, we presented the isotropy analysis of a fully actuated caster wheeled omnidirectional mobile robot with the steering link offset different from the wheel radius. First, the kinematic model of a COMR was obtained based on the orthogonal decomposition of the wheel velocities. Second, the necessary and sufficient isotropy conditions of a COMR were derived and examined to find three different groups, each of which can be handled in a similar way.

Third, for each group, the isotropy conditions were further explored so as to identify all possible isotropic configurations completely.

As shown in this paper, it is best for the local isotropy of a COMR to have the steering link offset equal to the wheel radius. Such a design is also found to be optimal for the global isotropic characteristics over the entire wheel configurations [5]. Nevertheless, why do many practical COMR's in use are built to have the steering link offset less than the wheel radius? One of major reasons will be to keep the stability of a COMR to a certain level, especially when a COMR makes a rapid turn while carrying a heavy load. The ratio of the steering link offset to the wheel radius needs to be determined based on a composite measure comprizing the isotropy, the stability, the force transmission ratio, and so on.

We hope that the results of this paper help for the optimal design and control of a COMR with variable steering link relative to wheel radius. For example, given a task trajectory of a COMR over some time interval, the prior knowledge of the isotropic wheel configurations can be quite effective to generate the optimal wheel trajectory that is close to the isotropy and away from the singularity as much as possible [11].

References

[1] R. Holmberg, "Design and development for powered-caster holonomic mobile robot," *Ph.D. Thesis*, Dept. of Mechanical Eng. Stanford University, 2000.

[2] P. F. Muir and C. P. Neuman, "Kinematic modeling of wheeled mobile robots," *Jour. of Robotic Systems*, vol. 4, no. 2, pp. 281-340, 1987.

[3] W. Kim, B. Yi, and D. Lim, "Kinematic modeling of mobile robots by transfer method of augmented generalized coordinates," *Jour. of Robotic Systems*, vol. 21, no. 6, pp. 301-322, 2004.

[4] G. Campion, G. Bastin, and B. D'Andrea Novel, "Structural properties and classification of kinematic and dynamic models of wheeled mobile robots," *IEEE Trans. on Robotics and Automation*, vol. 12, no. 1, pp. 47-62, 1996.

[5] T. Park, J. Lee, B. Yi, W. Kim, B. You, and S. Oh, "Optimal design and actuator sizing of redundantly actuated omni-directional mobile robots," *Proc. of IEEE Int. Conf. on Robotics and Automation*, pp. 732-737, 2002.

[6] S. Kim and H. Kim, "Isotropy analysis of caster wheeled omnidirectional mobile robot," *Proc. of IEEE Int. Conf. on Robotics and Automation*, pp. 3093-3098, 2004.

[7] S. K. Saha, J. Angeles, and J. Darcovich, "The design of kinematically isotropic rolling robots with omnidirectional wheels," *Mechanism and Machine Theory*, vol. 30, no. 8, pp. 1127-1137, 1995.

[8] C. Gosselin and J. Angeles, "Singularity analysis of closed-loop kinematic chains," *IEEE Trans. on Robotics and Automation*, vol. 6, no. 3, pp. 281-290, 1996.

[9] T. Yoshikawa, "Analysis and control of robot manipulators with redundancy," *Robotics Research*, pp. 735-747, MIT Press, 1984.

[10] E. G. Papadopoulos and D. A. Rey, "A new measure of tipover stability margin for mobile manipulators," *Proc. of IEEE Int. Conf. on Robotics and Automation*, pp. 3111-3116, 1996.

[11] S. Kim and B. Moon, "Complete identification of isotropic configurations of a caster wheeled mobile robot with nonredundant/redundant actuation," *Int. Jour. of Control, Automation, and Systems*, vol. 4, no. 4, pp. 486-494, 2006.

Sungbok Kim

received the B.S. degree in Electronics Engineering from Seoul National University, Korea, in 1980, the M.S. degree in Electrical and Electronics Engineering from KAIST, Korea, in 1982, and the Ph.D. degree in Electrical Engineering from the University of Southern California, USA, in 1993. Since 1994, he has been with the School of Electronics and Information Engineering, Hankuk University of Foreign Studies, Korea, where he is currently a Professor. His recent research interests include mobile robots and multiple sensor fusion.

Byungkwon Moon

received the B.S. degrees in Digital and Information Engineering from Hankuk University of Foreign Studies, Korea, in 2005. He is now working toward the M.S. degree in Electronics and Information Engineering at Hankuk University of Foreign Studies, Korea. His research interests include the design and control of omnidirectional mobile robots.