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실시간 일정계획 문제에 대한 Control 기반의 매개변수 프로그래밍을 이용한 해법의 개발

Development of An On-line Scheduling Framework Based on Control Principles and its Computation Methodology Using Parametric Programming

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Abstract : Scheduling plays an important role in the process management in terms of providing profit-maximizing operation sequence of multiple orders and estimating completion times of them. In order to take its full potential, varying conditions should be properly reflected in computing the schedule. The adjustment of scheduling decisions has to be made frequently in response to the occurrence of variations. It is often challenging because their model has to be adjusted and their solutions have to be computed within short time period. This paper employs Model Predictive Control (MPC) principles for updating the process condition in the scheduling model. The solutions of the resulting problems considering variations are computed using parametric programming techniques. The key advantage of the proposed framework is that repetition of solving similar programming problems with decreasing dimensionis avoided and all potential schedules are obtained before the execution of the actual processes. Therefore, the proposed framework contributes to constructing a robust decision-support tool in the face of varying environment. An example is solved to illustrate the potential of the proposed framework with remarks on potential wide applications.

Keywords : on-line scheduling, model predictive control, parametric programming

I. Introduction

Scheduling plays an important role in the process management in terms of providing profit-maximizing operation sequence of multiple orders and estimating completion times of them. Because the key parameters of the processes such as processing times, resource availabilities and demands are generally subject to variation, an optimal schedule which was obtained based on one set of fixed values turn out to be suboptimal or even infeasible along the actual realization of variations. Reactive scheduling is a way to address uncertainty issues in (typically on-line) scheduling applications in response to uncertainty. The main idea of reactive scheduling is that whenever a variation occurs, deterministic scheduling problems are repeatedly solved : a new schedule is computed and implemented based upon newly realized parameters (for example, from on-line measurements). Types of variation considered in the reactive scheduling problems are processing time(Cott and Macchietto, 1989; Kanakamedala *et al.*, 1994; Huercio *et al.*, 1995); Ishii and Muraki, 1996;) equipment availability (Kanakamedala *et al.*, 1994; Schilling and Pantelides,1997; Vin and Ierapetritou,2000) and rush order (Vin and Ierapetritou, 2000). Studies of reactive scheduling in the literature can be summarized in the following three ways: First, the computational issue is the major obstacle to improving the performance of reactive scheduling. Because of the need for constant re-computations, reactive scheduling approaches may become computationally expensive. Thus, some of the studies in the literature have attempted to accelerate the computational performance of the underlying deterministic scheduling problem by shifting the starting time of jobs in a schedule (Cott and Macchietto, 1988), limiting the search area for a new solution (Kanakamedala *et al.*, 1994), relaxing the constraints (Vin and Ierapetritou, 2000) or resolving the scheduling problem in a hierarchical way (Schilling and Pantelides, 1997). Considering the scale of conventional scheduling problems, the heavy computational load still poses a new challenge for reactive scheduling.

Second, most studies focus on how to minimize the effect of disturbances only mbox textit after their occurrence. Little attention has been given to the more positive approach such as predicting a potential variation and the corresponding optimal schedule in response to the variation. Ishii and Muraki (1997) noticed the importance of predicting the process state in reactive scheduling but their work leaves unanswered the question of how the prediction can be made and realized in the framework of the reactive schedule.

Third, most studies approach the reactive scheduling problem

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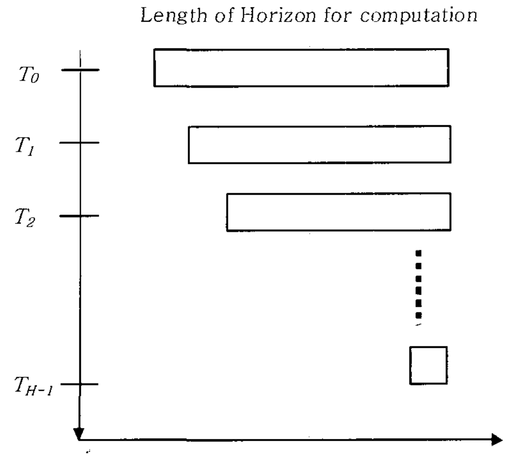

그림 1. Receding horizon 개념에 대한 개념도

Fig. 1. Graphical representation of receding horizon concept.

as a subproblem of plant operation management. The management of plant operations generally consists of (i) production planning (at the highest level), (ii) scheduling of various products (at the intermediate level), and (iii) process control (at the lowest level) (Cott and Macchietto, 1989; Kanakamedala, Reklaitis and Venkatasubramanian, 1994). Research on reactive scheduling has focused on the intermediate level by re-computing new completion times of tasks (Cott and Macchietto, 1989; Ishii and Muraki, 1997). However, in order to respond to uncertainty systematically, it would be necessary to incorporate all three levels, and this has not been fully investigated.

As can be seen in the above described previous research, many researchers have been interested in modifying schedules according to the newly obtained process information, which is mostly undergone through on-line measurement. It is challenging to solve this type of on-line scheduling optimization problems because the new solutions of fairly large-size mathematical scheduling problems should be continuously computed and implemented within the limited available time whenever a variation occurs. Conventionally, such constantly occurring variations and their corresponding adjustment have been addressed in control communities and Model Predictive Control (MPC) principles have been most widely used (refer to Morari and Lee, 1999 for review). It is based on a concept of the receding horizon where a boundary of interest is reduced as the process is operated with the acquirement of new process information (see Fig. 1). The basic concept of receding horizon used in Model Predictive Control (MPC) can be outlined into the following steps:

II. Proposed On-line Scheduling Framework

Step 1: A set of process states covering an entire horizon is forecasted based on information obtained so far.

Step 2: Optimal control actions for the entire horizon are computed based on the forecast and implemented only for the very next move.

Step 3: Process information is updated and the horizon of interest is reduced.

Step 4: Go to step 1 until the end of the horizon.

This paper proposes a decision making framework for on-line scheduling problems based on MPC principles. On-line scheduling problem generally requires a large computational load and there have been few efficient methodologies which forecasts the optimal schedule within the limited computational time. Recently Ryu *et al.* (2005) proposed a proactive scheduling framework by introducing parametric programming techniques.

Parametric programming problems are generally formulated as follows:

$$
\begin{aligned}
,z = \min_{x,y}\ & c^T x + d^T y \\
\text{s.t.}\quad & Ax + Ey = b + F\theta \\
& \theta_{\min} \le \theta \le \theta_{\max} \\
& x \in X ; y \in \{0,1\}
\end{aligned}
\tag{1}
$$

where A, E, F are constant matrix and b is a constant vector, x is constant variables and y is discrete variable. θ is uncertain parameter within its upper and lower bound, θ_{min}, θ_{max}.

For the solution of (1), parametric programming algorithms and software have been recently proposed by Pistikopoulos and co-workers. The aim of parametric programming is to obtain the optimal solution as a function of varying parameters. It provides a complete map of the optimal solution as a function of the varying parameters.

A complete family of optimal schedules are computed as a function of varying parameters before the start of the process by employing parametric programming in solving the scheduling problem,. A new optimal schedule can thus be obtained as a function of varying parameters without the further optimization. This work will incorporate the techniques within the framework of receding horizon in addressing on-line scheduling problems. The proposed on-line scheduling framework consists of the following steps:

Step 1: Generate initial proactive schedules using parametric programming

Step 2: Select a robust schedule from the proactive schedules in step 1

Step 3: Implement some part of the selected robust schedule on the process

Step 4: Update the process information (adjust the boundary of uncertain parameters)

Step 5: Go to step 1 until the last task is operated

Parametric programming techniques provides us a complete family of optimal schedule in response to variations and snapshots of different optimal schedules are connected as the new information is realized in the process operation.

III. A Case Study

A case study is presented to illustrate the potential of the proposed framework. Consider a manufacturing process involving four products in four stages. Processing times at stages are shown in Table 1.

It is known that processing times are subject to variation in their form of two uncertain parameters, θ_1 and θ_2 within the following boundaries:

$$6 \leq \theta_1 \leq 21$$
$$10 \leq \theta_2 \leq 20$$

A scheduling model generally involves two types of constraints, sequencing and assignment. Sequencing constraints typically denote which products are produced in different time instances (slots) and in what sequence, while assignment constraints typically determine completion times of various products at different stages based on the selected sequence.

Based on the notation in Table 2, the deterministic scheduling model employed in this study corresponds to the following MILP problem:

표 1. 예제에 대한 공정 시간 데이터.

Table 1. Processing time data for illustrating example.

Task	*mixing*	*reaction1*	*reaction 2*	*separation*
A	θ_1	15	11	8
B	9	14	17	11
C	19	8	θ_2	15
D	14	11	16	9

표 2. 본 논문 사용 기호 표기.

Table2. Notation.

Indices	
i	time slot (1, cdots, N)
k	product (1, cdots, N)
j	stage (1, cdots, M)
Parameter	
n(j)	number of units in stage j
$P_{k,j}$	processing time of product k in stage j
$\theta 1_{k,j}$	uncertain processing time of product k in stage j
$\theta 2_{i,j}$	uncertain availability parameter of i th time slot in stage j
$\theta 1^L{}_{k,j}$ $\theta 2^L{}_{i,j}$	lower bound
$\theta 1^U{}_{k,j}$ $\theta 2^U{}_{i,j}$	upper bound
A, E, F	constant matrix
b	constant vector
Variable	
$C_{i,j}$	completion time of i th product in stage j
$y_{i,k}$	binary variable; if product *k* is made at *i* th time slot, otherwise 0
$w_{i,k,j}$	auxiliary variable for bilinear term y $_{i,k}$ 1 $_{k,j}$

$$z = \text{Min } C_{N,M}$$

$$\text{s.t.} \sum_{i}^{N} y_{i,k} = 1, \qquad \forall k \qquad (3.1)$$

$$\sum_{k}^{N} y_{i,k} = 1, \qquad \forall i \qquad (3.2)$$

$$C_{i,j} \geq C_{i,j-1} + \sum_{k}^{N} y_{i,k} P_{k,j}, \qquad j>1,: \forall i, \qquad (3.3)$$

$$C_{i,j} \geq C_{i-1,j} + \text{sum k\^{}N y i,k P k,j,} \quad i>n(j); \forall j. \qquad (3.4)$$

(3.1) ensures that each product is assigned only one time slot in a sequence, while (3.2) ensures that only one product is assigned in every time slot. (3.3) and (3.4) indicate that a product in a stage can only be processed if the product and a corresponding unit are available at the same time. The objective of problem (3) is to minimize a makespan, $C_{N,M}$, which is the completion time of the last product in the last stage.

A Multi-parametric Mixed Integer Linear Programming (mp-MILP) model may be formulated as follows:

$$\begin{aligned}
& z = \text{Min } C_{N,M} \\
& \text{s.t.} \sum_{i}^{N} y_{i,k} = 1, && \forall k \\
& \sum_{k}^{N} y_{i,k} = 1, && \forall i \\
& C_{i,j} \geq C_{i,j-1} + \sum_{k}^{N} w_{i,k,j}, \quad j>1, && \forall i \\
& C_{i,j} \geq C_{i-n(j),j} + \sum_{k}^{N} w_{i,k,j}, \quad i>n(j), && \forall j, \\
& \theta 1_{k,j} - \theta 1^U{}_{k,j} (1 - y_{ik}) \leq w_{i,k,j}, \\
& \theta 1_{k,j} - \theta 1^L{}_{k,j} (1 - y_{i,k}) \geq w_{i,k,j}, \\
& y_{i,k} \; \theta 1^L_{k,j} \leq w_{i,k,j} \leq y_{i,k} \; \theta 1^U{}_{k,j}, \\
& \theta 1^L{}_{k,j} \leq \theta 1_{k,j} \leq \theta 1^U{}_{k,j},
\end{aligned} \qquad (4)$$

where $\theta 1_{k,j}$ are the varying processing time parameters of product k at stage j; $\theta 1^L{}_{k,j}$ and $\theta 1^L{}_{k,j}$ are known lower and upper bounds.

Problem (4)can be transformed into the following problem:

$$\begin{aligned}
z(\theta) = \min_{x,y} \quad & c^T x + d^T y \\
\text{s.t.} \quad & Ax + Ey \leq b + F\theta \\
& \theta_{\min} \leq \theta \leq \theta_{\max} \\
& x \in X ; y \in \{0,1\}
\end{aligned}$$

The application of parametric programming techniques yields a family of optimal schedules according to variations of parameters. Refer to Table 3 and Fig. 1 which summarizes the initial proactive schedules.

As can be seen in Table 3 and Fig. 2, two sequences are obtained as optimal and a schedule [A → D → C → B] is

표 3. 초기 매개 변수화된 schedule.

Table3. Initial proactive parametric schedules.

Notation (CR)	Critical region (hr)	Optimal sequence	Makespan
CR1	$6\ \theta_1 \le 18$ $11 \le \theta 2 \le 14$	A → D → C → B	$20\ \theta_1+6\ \theta_2+495$
CR2	$\theta_1 \le 21$ $-\ \theta_2 \le -11$ $-\theta 1+\theta_2 \le -4$ $-17\ \theta_1-\theta_2 \le -321$	D → B → C → A	$3\ \theta_1+5\ \theta_2+816$
CR3	$-\theta_1 \le -18$ $-\theta_1 \le -11$ $-\theta_1+\theta_2 \le -4$ $17\ \theta_1+\theta_2 \le 321$	A → D → C → B	$20\ \theta_1+6\ \theta_2+495$
CR4	$18.0556 \le \theta_1 \le 21$ $\theta_2 \le 20$ $\theta 1-\theta_2 \le 4$	D → B → C → A	$2\ \theta_1+6\ \theta_2+820$
CR5	$6 \le \theta_1 \le 18.0556$ $14 \le \theta_2 \le 20$ $\theta_1-\theta_2 \le 4$	A → D → C → B	$20\ \theta_1+6\ \theta_2+495$
CR6	$6 \le \theta_1 \le 18.2353$ $10 \le \theta_1 \le 11$ $-\theta_1+\theta_2 \le 4$	A → D → C → B	$20\ \theta_1+5\ \theta_2+506$
CR7	$18.2353 \le \theta_1 \le 21$ $10 \le \theta_2 \le 11$ $\theta_1-\theta_2 \le 4$	D → B → C → A	$3\ \theta_1+5\ \theta_2+816$

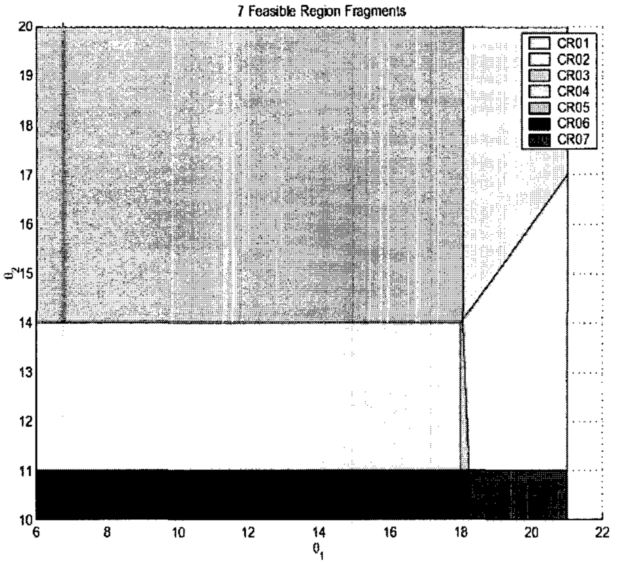

그림 2. 전체 변수 공간에 대한 초기 Proactive 일정에 대한 도식화.

Fig. 2. Graphical representation of initial proactive according to various critical regions.

computed as optimal at the largest critical region including CR1, CR3, CR5, CR6. Therefore [A → D → C → B] is regarded as robust optimal at the initial stage.

표 4. 2단계 매개 변수화된 schedule.

Table4. 2nd proactive parametric schedules.

Notation (CR)	Critical region (hr)	Optimal sequence	Makespan
CR1	$6\ \theta_1 \le 21$ $11 \le \theta_2 \le 20$	A → D → C → B (A is already produced)	$20\ \theta_1+6\ \theta_2+495$
CR2	$6\ \theta_1 \le 21$ $10 \le \theta_2 \le 11$	A → D → C → B (A is already produced)	$20\ \theta_1+5\ \theta_2+406$

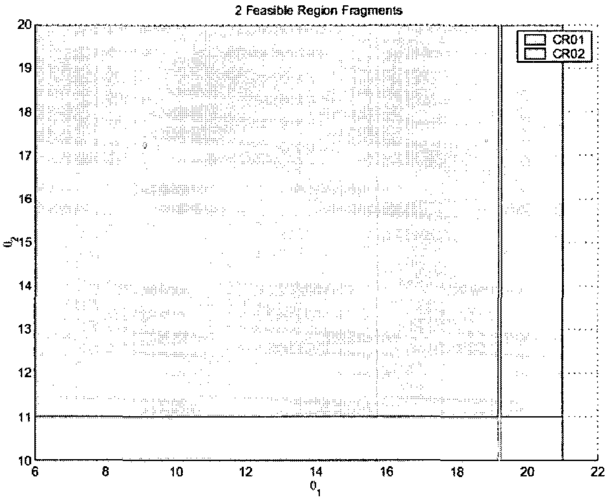

그림 3. 2단계 Proactive schedule에 대한 도식화.

Fig. 3. Graphical representation of 2nd proactive schedules according to various critical regions.

Based on the schedule, the first task, A is operated and a new scheduling problem is formulated and solved. The application of the parametric programming techniques is summarized in Table 4 and Fig. 3.

As can be seen in Fig. 3, the same sequence, [A → D → C → B] is obtained as optimal. Therefore the second task is selected as D according to the computed sequence. The computational on-line reactive scheduling framework for the example is summarized in Fig. 4.

IV. Remarks

There are some issues worthwhile to mention regarding the proposed framework. The first is how to define the robust schedule in a more systematic way. Generally a schedule that minimize the completion time is selected. By utilizing the proposed framework, we can compute a robust schedule before starting a new task. Therefore the process is operation under the more robust management.

In the illustrating example, [A → D → C → B] is the most robust optimal schedule because it has the most wide critical region. However, a more systematic methodology should be researched in order to address unclear cases such as how to choose between two schedules with same critical region space

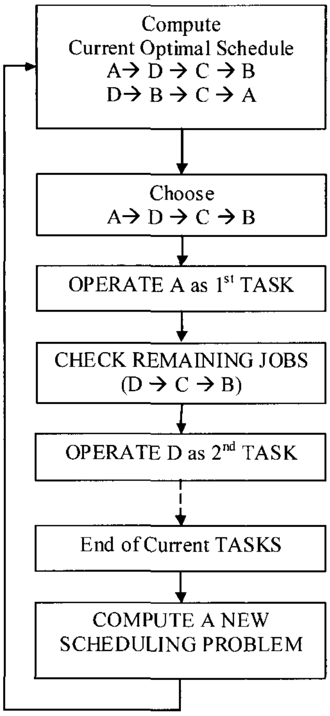

그림 4. 실시간 Scheduling 산출을 위한 공정 흐름도.

Fig. 4. Simplified on-line scheduling flowchart for the example.

respectively.

The second is how the proposed methodology handles cases with a large problem size. The example this paper introduced is for the illustration purpose. The example is intended to show how the proposed framework address the actual problem and demonstrate the computation procedure clearly.

Problems of large size can be also solved using the proposed methodology.

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