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Engineering Valuation Based on Small Samples

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Box-Cox model and T-factor method have been widely used to measure economic depreciations for industrial property. The Box-Cox model which combines economic efficiency with depreciation pattern is here extended to the reliability function. To do so a Rayleigh distribution which has been used to estimate the reliability of current assets was chosen as an efficiency curve of marginal productivity. Such an approach provides the possibility to classify the efficiency curves into four categories. It is also possible to analyze the types of depreciation curves. Therefore, the power family of a non-linear Box-Cox model could be set at certain constant values, then the model can be transformed into a linear model to estimate the economic depreciation rates by utilizing the reliability function. Estimating the resultant linear regression equation requires minimal number of observations, while at the same time facilitating the test of hypothesis on depreciation rates.

Keywords : Assets valuation, depreciation rates, depreciation period, IOWA curves

1. Introduction

Valuation engineering conventionally follows the procedure : first, estimate the expected years of usage of the machinery by the IOWA curve method, second, determine the economic depreciation rates, finally apply the results to field studies including the equipment replacement analysis.

Since Terborgh's study of 1954 there have been numerous studies on determining the economic depreciation rates in general or in particular[6]. However, due to the lack of data on capital retirement, those tools developed for positive estimation using actual data may not be very often useful in empirical studies. The problem is especially severe in such country as Korea, where data on capital retirement is usually incomplete or completely nonexistent. In this study we utilize concepts from the reliability engineering to modify the Box-Cox model of Hulten-Wykoff[30,31], which is currently the most popular model to estimate the economic depreciation rates, and make it usable even with the minimal amount of data.

2. Control Chart and Correlation

In Hulten-Wykoff model major determinants of the economic depreciation rates are included as explanatory variables. Notably in their "price-oriented" econometric model variables such as the inflation rate was included, but variables reflecting the 'lemon problem' and the survival probability problem, that should have heavy bearings on the economic price of the assets are missing. 'lemon problem' concerns the situation where the valuation of the numerous satisfied users of machines of a certain model may not be properly reflected in the second-hand market price of the same model machines, where mostly bad batches are being traded. The survival probability problem concerns the attempt to adjust the observed price of a particular physical asset according to the probability of failure. For example suppose that a five-year-old equipment trades for $100 in the used equipment market, and the probability of failure of that equipment within five years be 20%, then we would assume that the expected value of the equipment in five years would be $0.2 \times 0 + 0.8 \times 100 = 80$ in dollars.

Hulten-Wykoff estimated the real market transaction price equation of an asset using the Box-Cox transformation method. The market transaction price of asset can be estimated in the following equation :

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$$Q^*_i = \alpha + \beta S^*_i + \gamma T^*_i + \varepsilon_i \quad i = 1 \ldots N \qquad (1)$$

where

$$Q^*_i = \frac{Q_i^{\theta_1} - 1}{\theta_1}, \quad S^*_i = \frac{S_i^{\theta_2} - 1}{\theta_2}, \quad T^*_i = \frac{T_i^{\theta_3} - 1}{\theta_3}$$

and $\varepsilon_i \sim$ iid $N(0, \sigma^2)$.

Q_i = market transaction price of asset

S_i = age of asset

T_i = year

The unknown parameters $\theta = (\theta_1, \theta_2, \theta_3)$ are the power family of the Box-Cox transformation model.

More specifically when θ = (0, 1, 1), with θ_1 converging to 0, Box-Cox model corresponds to the case of the market price being in a semi-log equation form. θ = (1, 1, 1) represents the case of the market price being in a linear form, while θ = (1, 3, 1) represents the case of the market price decaying geometrically.

In the regression equation (1) the age-price relationship is summed up in the coefficient β of S_i, which represents the annual rate of change in the market price due to the age of asset, i.e. the economic rate of depreciation.

3. Suggested Model

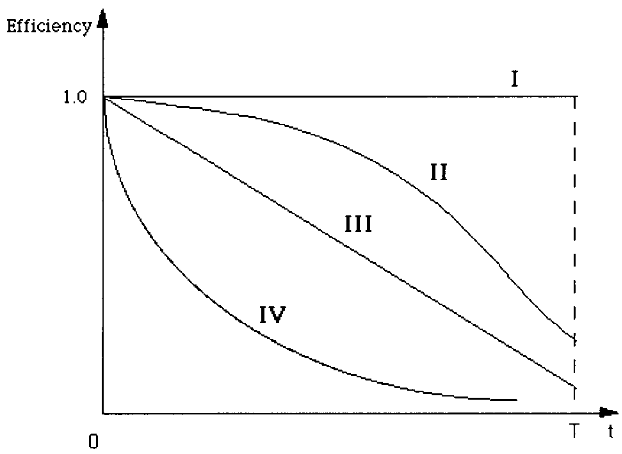
Being a nonlinear estimation method, Box-Cox regression method suffers from difficulties in statistical hypothesis testing. Besides T-factor method[22,24,40] does not easily allow for using standard probability distribution functions, because Iowa curve is based rather on intuitive appraisal (on fitness of the data) than on determining the relevant probability distribution functions.

Here it is suggested that we take a two step estimation procedure : in the first step based on estimates of reliability function, profiles of economic efficiency (the marginal productivity of asset), and age-price profiles, we decide on the values of power family ($\theta_1, \theta_2, \theta_3$) in the Hulten-Wykoff equation, then fix ($\theta_1, \theta_2, \theta_3$) as constants. In the second step we linearize the Hulten-Wykoff equation, estimate the economic depreciation rate and test hypotheses regarding economic depreciation rates. The strength of the suggested method lies in that it allows for simple hypotheses testing regarding the economic depreciation rate(β) (*e.g.* H_0: $\beta = 0$, H_1: $\beta < 0$) without diminishing the importance of estimating the values of the parameters ($\theta_1, \theta_2, \theta_3$) in the Hulten-Wykoff equation.

To avail the engineering data to the estimation of depreciation rates, we first study the relationship between the economic efficiency and reliability, then also the relationship between the economic efficiency and the depreciation rate in the following sections.

3.1 Relationship between Economic Efficiency and Reliability

As they are seen in figure 1 we can identify four types of economic efficiency-time profiles. Here we mean by the economic efficiency the marginal productivity of an asset. The type I curve represents the case where the marginal productivity of an asset stays unaltered in the period of depreciation. Type II curve corresponds to where the marginal productivity decreases at some increasing rate with the period(Rayleigh distribution), while in type III curve it decreases linearly. In type IV curve it decreases fast initially then stays relatively low throughout its life.



<Figure 1> Alternative Efficiency Profiles

We note that initially in the Hulten-Wykoff study type II profile is regrettably missing, especially if we consider that the equipments are becoming increasingly reliable due to stronger quality guarantees becoming effective these days, so that more often the efficiency profiles should be concave.

It is at this point necessary that we clarify the relationship between the economic efficiency and reliability as seen in figure 2 by defining the reliability function, R(t) as follows.[7, 35].

R(t)= 1-F(t) (or F(t)= 1-R(t))

where R(t) : reliability function

F(t) : cumulative failure probability distribution function

f(t) : failure probability distribution function(f(t)=$\frac{d}{dt}$F(t)).

Further r(t) = $\frac{f(t)}{R(t)}$

where r(t) : failure or retirement rate .

If we define marginal productivity to mean one unit of the equipment producing additional unit of output, then 100% reliability of the equipment will mean the productivity of it being 1 unit, and the change in its reliability will coincide with the change in its productivity.

Reliability, R(t) is the probability of the equipment to continue to operate during the time interval [0, t], while availability of the equipment, A(t) under systemic maintenance is the probability of the equipment to be available for use at time t. Therefore, the concept of economic efficiency matches more closely with that of availability, A(t) than with the concept of reliability, R(t).

We also note that R(t)≤A(t), and that R(t) curve generally does not coincide with A(t) curve depending on the maintainability function. However, we should most of the time safely assume that they will have the same functional form with respect to time.[35] Therefore, here we assume that reliability can be identified with economic efficiency at least in determining the overall shape of the time profile and we can follow the rules of thumb below, even though A(t) differs from R(t) to be precise as can be seen in figure 1.

① no failure : type I
② survival probability in Rayleigh distribution : type II
③ survival probability in uniform distribution : type III
④ survival probability in exponential distribution : type IV

3.2 Relationship between Economic Efficiency and Depreciation Rate

First, let us introduce the value of the asset equation (2)[6]. Economic efficiency here is defined as the marginal productivity of the equipment, therefore is a product of the marginal productivity term, c and the depreciation function e^{-rx}. Here the functional form of e^{-rx} is based on Hulten- Wykoff's empirical studies which drew the conclusion that usually assets displayed the exponential depreciation rate patterns[31]. Further we note that this equation assumes constant expected discount rate, therefore the value of the assets is a function of its years of usage.

$$q(s,t)=\int_{t}^{t+T-s} c(s+x,t+x)e^{-rx}dx \quad \cdots\cdots (2)$$

where q = purchase price of the asset
s = years of usage of the asset
t = purchase point of time of the asset
T = lifespan of the asset
c = marginal productivity of the asset or its rental price
r = discount rate
x = time variable

3.2.1 Functional Forms of Efficiency Function

The efficiency function $c\ (s+x,\ t+x)$ in equation (2) should have following functional forms depending on the taxonomy of time-profile types in figure 1.

1) Horizontal time profile (type I)

$$c\ (s+x,\ t+x) = c_1 > 0, \quad \text{if } s+x \le T$$
$$= 0, \quad \text{otherwise.} \quad \cdots\cdots (3)$$

2) Concave time profile (type II)

$$c(s+x,t+x)=c_2 e^{-\frac{1}{2}(\frac{s+x}{\eta})^2}, \quad \text{if } s+x \le T$$
$$= 0, \quad \text{otherwise.} \quad \cdots\cdots (4)$$

where η denotes the scale parameter of Rayleigh distribution. Also note that the residual value in this case is equal to $q_{t+T}=\int_{\infty}^{t+T} C_2 e^{-\frac{1}{2}(\frac{s+x}{\eta})^2} e^{-rx}dx$)

3) Linear time profile (type III)

$$c\ (s+x,t+x)=c_3(1-\frac{s+x}{T}) > 0,$$
$$\text{if } s+x \le T$$
$$= 0, \quad \text{otherwise.} \quad \cdots\cdots (5)$$

4) Exponential time profile (type IV)

$$c\ (s+x, t+x) = c_4 e^{-\delta(s+x)} > 0, \quad \text{if } s + x \leq T$$
$$= 0, \quad \text{otherwise.} \cdots\cdots (6)$$

3.2.2 Functional Forms of Depreciation Function

Now we use above results to derive the asset value depreciation functions for each type of time profiles in figure 1.

1) Horizontal time profile (type I)

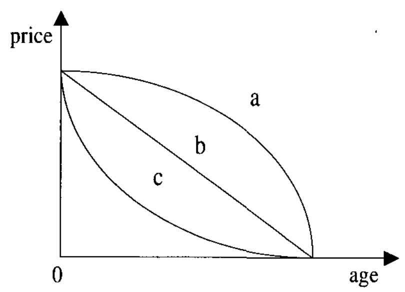
The asset value equation in this case becomes after setting t=0

$$q(s,t) = \int_0^{T-s} c_1 e^{-rx} dx = \frac{c_1}{r}[1 - e^{-r(T-s)}]$$

Also note

$$\frac{\partial q}{\partial s} = -c_1 e^{-r(T-s)} < 0$$
$$\frac{\partial^2 q}{\partial s^2} = -rc_1 e^{-r(T-s)} < 0$$

Which implies that the time profile of the asset value is decreasing with increasing depreciation rate with respect to time, i. e.,. concave to the origin as it is illustrated in figure 2 as curve "a"



<Figure 2> Age-Price Profiles Corresponding to Alternative Efficiency Profiles

2) Concave time profile (type II)

In this case setting again t=0, the asset value function becomes

$$q(s,t) = \int_0^{T-s} c_2 e^{-\frac{1}{2}(\frac{s+x}{\eta})^2} e^{-rx} dx$$

Again note that

$$\frac{\partial q}{\partial s} = -c_2 e^{-rx - s^2/(2\eta)^2} < 0$$
$$\frac{\partial^2 q}{\partial s^2} = \frac{c_2 e^{-rx - s^2/(2\eta)^2}}{\eta^2} > 0$$

Which shows that q is now convex in time, which is now illustrated in figure 2 as curve "c".

3) Linear time profile (type III)

Repeating the same procedure in this case yields

$$q(s,t) = \int_0^{T-s} c_3 (1 - \frac{s+x}{T}) e^{-rx} dx$$
$$\frac{\partial q}{\partial s} = -\int_0^{T-s} \frac{c_3}{T} e^{-rx} dx$$
$$= -\frac{c_3}{rT}[1 - e^{-r(T-s)}] < 0$$
$$\frac{\partial^2 q}{\partial s^2} = \frac{c3}{T} e^{-r(T-s)} > 0$$

Therefore, we infer that given the linear time profile of the physical efficiency depreciation of the asset, the time profile of the economic asset value should be convex to the origin like the curve "c" in figure 2.

4) Exponential time profile (type IV)

In this case the asset price equation would be

$$q(s,t) = \int_0^{T-s} c_4 e^{-\delta(s+x)} e^{-rx} dx + q_{t+T}$$
$$= \int_0^{\infty} c_4 e^{-\delta s} e^{-(\delta+r)x} dx = \frac{c_4 e^{-\delta s}}{(\delta + r)}$$

Therefore,

$$\frac{\partial q}{\partial s} = -\delta \frac{c_4 e^{-\delta s}}{(\delta + r)} < 0$$
$$\frac{\partial^2 q}{\partial s^2} = \delta^2 \frac{c_4 e^{-\delta s}}{\delta + r} > 0$$

Again the asset value depreciation profile will be like curve "c" in figure 2, which is convex to the origin.

3.3 New Method of Estimating Depreciation Rates

In the Box-Cox regression equation (1) we identified the following critical cases in terms of the time profile of the market asset price. When $(\Theta_1, \Theta_2, \Theta_3)=(0, 1, 1)$, with Θ_1 converging to 0, the time price profile will look like curve "c" of figure 2. $(\Theta_1, \Theta_2, \Theta_3)=(1, 1, 1)$ represents the case of the market price being linearly decreasing in time as in curve "b", while $(\Theta_1, \Theta_2, \Theta_3)=(1, 3, 1)$ represents the case of the market price decaying geometrically as in curve "a" in figure 2.

Where different types of depreciation is to be applied to different equipment assets, there should be a general model to measure such depreciations. Box-Cox model lumps together all types of depreciation as a single type without considering that there are different types of depreciation involved. However, since the focus of valuation engineering is on production equipments, the efficiency curve of equipments can be easily found by estimating the reliability curve using intuitive method, and the shape of depreciation curve can be inferred from it. Therefore, we suggest a new method of estimating depreciation rates, a two-step procedure.

In the first step based on estimates of reliability function, profiles of economic efficiency (the marginal productivity of asset), and age-price profiles we decide on the values of power family$(\theta_1, \theta_2, \theta_3)$ in the Hulten-Wykoff equation (1) then fix $(\theta_1, \theta_2, \theta_3)$ as constants. In the second step we linearize the Hulten-Wykoff equation, estimate the economic depreciation rate and test hypotheses. Finally one could also estimate the vintage asset values for additional years of usage. In other words we wish to examine the relationship between economic efficiency (the marginal productivity of asset) as well as the relationship between economic efficiency and the (theoretical?) time profiles of the asset value, compare that with actual time profiles of vintage assets, then linearize the Box-Cox regression equation after fixing the values of the power family$(\theta_1, \theta_2, \theta_3)$ as constants.

4. A Sample Study

We use a vintage dozer sample data (102 observations) to compare the estimates of original Hulten-Wykoff's Box-Cox regression method with those of the two-step estimation procedure set forth in this paper.

4.1 Estimates of Hulten-Wykoff's Box-Cox Regression Method

While using the 1978 price deflator, we can from the Hulten-Wykoff's Box-Cox regression equation (1) eliminate the time variable Ti to obtain

$$\frac{Y_i^{\theta_1}-1}{\theta_1} = \alpha + \beta\frac{X_i^{\theta_2}-1}{\theta_2} + \epsilon_i$$

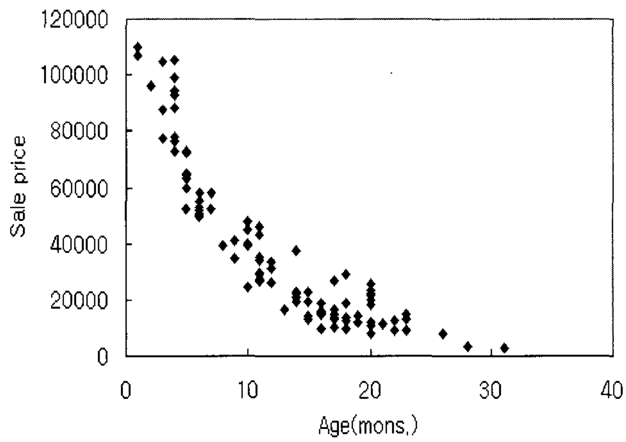
where Yi : the market price of the dozer number i after Xi years of usage, i=1...102

Xi : years of usage of the dozer number i at the time of trade

Using SAS PROC NLIN the following parameter estimates were obtained (disregarding the 'lemon problem' as well as the survival probability problem).

$$\hat{\theta_1} = 0.0050 \quad \hat{\theta_2} = 0.9188 \quad \hat{\alpha} = 12.000 \quad \hat{\beta} = -0.1496$$

We notice that $\hat{\theta}1$ being close to zero, and that $\hat{\theta}2$ being close to one. As we argued that in equation (1) the Box-Cox power family $(\theta_1, \theta_2, \theta_3) = (0, 1, 1)$ represents the semi-log functional form, it shows in figure 3. The economic depreciation rate β was estimated as close to 15%, which we apply in table 1.



<Figure 3> Market Price of Vintage Dozer (1978 constant price ($))

<Table 1> Estimated Prices of Vintage Dozer by Box-Cox Model (1978 year constant price($))

Age	Prices
1	115125.9
2	100402.6
3	88055.3
4	77496.5
5	68375.5
6	60446.4
7	53521.6
8	47453.5
9	42121.7
10	37426.4
11	33284.1
12	29623.8
13	26385.0
14	23515.8
15	20971.1
16	18712.1
17	16705.1
18	14920.4
19	13332.4
20	11918.3

4.2 Estimates of the Suggested Two-Step Estimation Procedure

The reliability function of the sample is said to be in the shape of Rayleigh curve. Therefore, in the second-hand market the price of used dozers will exhibit time profile like curve "c" in figure 2. We note that given constant price data, (θ_1,θ_2) =(0, 1) in (1), which is in the semi-log form :

$$Y_i = \alpha e^{\beta X_i} + \epsilon_i$$

where Yi : second-hand market price of dozer number i after Xi years of usage

Xi : years of usage of dozer number i at the time of trade

and $\varepsilon_i \sim$iid N(0, σ^2)

We now linearize above regression equation, and estimate the linear regression equation to obtain :

$$\ln\ Y_i = \ln\alpha + \beta\ X_i + \epsilon'_i$$
$$\ln \widehat{Y}_i = 11.668 - 0.1125\ X_i$$

with test statistics :

t-value for significance of α estimate = 211.0

t-value for significance of β estimate = -28.7

R^2=0.89

Therefore, we accept $H_1(\alpha \neq 0)$ regarding α as well as $H_1(\beta < 0)$ regarding β both at 5% significance level.

4.3 Analysis of the Sample Study

In table 2 we present the estimate market values, $\widehat{Y}_j$ for $X_j = 1,\ 2,\ldots 20$ The difference of the results from table 1 can be tested for as follows.

$$\begin{cases} H_0:\ \delta = 0 \\ H_1:\ \delta \neq 0 \end{cases}$$

with test statistics

t0= 1.623

P-value=0.1211

Therefore, we accept the hypothesis that there is no difference between the two sets of estimates.

The sample data is from the U.S., with limited validation of assumptions made in this study, however, still we infer following.

As long as there exist second-hand market prices, the application of Box-Cox model and T-factor method jointly can test whether or not firms react to the economic changes neutrally and how the changes in market prices, an external factor to a firm, and the decrease of operating revenues due to the increasing operating/maintenance cost, an internal factor to a firm, are related to each other.

Applying reliability engineering to Box-Cox model, it is possible to identify the asset retirement characteristics of equipments and infer from this result the efficiency curve, which represents marginal productivity, this allow us to perform linear regression on Box-Cox model. This linear regression model allows for simple hypotheses testing regarding the economic depreciation rate(β), (e.g. H0 : β = 0, H1 : β < 0) as well as estimating the depreciation rate parameter, β.

<Table 2> Estimated Prices of Vintage Dozer by Linear Regression

Age	Prices
1	104349.4
2	93246.3
3	83324.7
4	74458.7
5	66536.1
6	59456.5
7	53130.2
8	47477.0
9	42425.3
10	37911.1
11	33877.3
12	30272.7
13	27051.6
14	24173.2
15	21601.1
16	19302.7
17	17248.8
18	15413.5
19	13773.5
20	12307.9

If we can estimate the period of depreciation of the assets based on Iowa curve, we can also forecast the depreciation of assets as follows. That is, the types of depreciation inferred from the application of reliability engineering and efficiency curve can determine the depreciation profile seen in figure 2. We can estimate the depreciation curve which passes through two benchmarks, the acquisition price of a new product and the estimated residual value after the period of depreciation. Here, the premise is that we should be able to estimate the residual value at the end of the period of depreciation. The reasonable estimation of residual value can be done by referring to the one for similar existing assets, or applying Dephi method.

5. Conclusion

In the Hulten-Wykoff's Box-Cox regression equation model economic efficiency is seen to determine the asset price depreciation pattern. Here we enlarge on the model by examining their relationship to reliability function. In addition we added the Rayleigh curve to types of the reliability function to increase the number of the marginal productivity types from three to four. Also each type was matched with different types of depreciation curves. There has been so far no study to discriminate among different types of efficiency curves of equipments, which is being enabled here through the introduction of the reliability function.

In our study through the use of the reliability function after fixing the power family as constants we linearized the Box-Cox model, which not only facilitated the estimation of the depreciation rate, but also enabled us to perform hypothesis testing on the depreciation rate. Furthermore, this newly suggested estimation procedure serves best where scant valuation engineering data has been accumulated as in Korea, since only minimal number of observations is being required for the estimation purpose.

Further researches could be conducted to extend the study to incorporate the aspect of availability under systemic maintenance into the model, since most installations are in maintainable systems. If the shape of the depreciation curve could be identified fairly accurately along with the relevant reliability/availability data within the firm, we would be able to estimate the depreciation curve using the depreciation period obtained from the Iowa curve. Which implies that we could develop the estimation procedure for economic depreciation rate without reference to the market data. This method could also have a significance as an ex-ante study.

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