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Vibration Filter Using Vector Channel Periodic Lattice

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This paper considered identification of vibration characteristics of flexible structure with vector channel periodic lattice filter. We present an algorithm for AR coefficients for the vector-channel lattice filters, and characteristic equation and transfer function are derived from these coefficients. Vibration lattice filter is then constructed from the vector channel lattice filter, and performance of this vibration filter is tested with a test signal which is a combination of many sine waves to compare the performance of scalar and vector channel lattice. Also it is applied to the cantilever data to identify properties of the system, such as natural frequencies and damping ratios, to show its performance.

Key Words : Vector Channel Lattice Filter, Vibration Filter, Autoregressive (AR) Coefficients, Residual Error, Reflection Coefficient

1. Introduction

The recursive least-squares method is extensively used for adaptive parameter identification. This method is based on a fixed-order model, and has a limitation for identification of large structures. Large flexible structures have many modes of vibration, of which different numbers may be excited at different times. Hence, determination of the effective order of the structure is needed along with identification of parameters. A least-squares lattice filter is an algorithm for least-squares parameter estimation that is recursive in both time and order. The order-recursive property allows the lattice filter to identify the number of substantially excited modes of a structure. The lattice filter is more efficient than the standard least-squares algorithm for large orders, and it is numerically stable.

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The lattice structure is based on two sets of vectors called forward and backward residual errors. The forward residual error vectors are obtained from projection of the most recent regression vectors, which contain measurement histories, on the span of previous regression vectors. The norm of the k^{th} order forward residual error is the minimum value of the objective functional to be minimized by a least-squares estimates of the parameters. The backward errors can be thought of as a set of Gram-Schmidt vectors that span the same space as the regression vectors. The lattice form algorithms are fast, numerically stable, and recursive in both time and system order.

Lee, Morf, and Friedlander first derived the time update equations for the lattice and applied to the signal processing and control problem (Lee et al., 1981). Montgomery and Sundararajan have used lattice filters in adaptive identification and control of a flexible beam (Sundararajan and Montgomery, 1983, 1985). Wiberg introduced a vibration lattice to impose the constraint that all measurements from a single structure should satisfy the same autoregressive model (Wiberg, 1985 ; Wiberg and Gillis, 1985). The information represented by this constraint reduces the bias that sensor noise produces in parameter estimates. An important limitation of the vibration lattice was that it applied only to free-response data. The vector channel lattice is derived for infinite histories of the input and output (Jabbari and Gibson, 1988a, 1988b, 1989); this framework facilitates convergence analysis for approximation theory in applications to infinite dimensional models of flexible structures and other distributed systems.

The concept of periodic forms is due to Pagano, and Jabbari extended the circular or periodic structure to the basic vector channel lattice. The main focus of this paper is the identification of the system parameters. We present an algorithm for determining AR coefficients using vector channel lattice. Then the transfer function can be obtained from these AR coefficients. If the only concern is the vibration parameters of the structure such as eigenvalues and damping constants, we can skip the process of determining AR parameters which is time consuming. Instead we can obtain these parameters directly from the reflection coefficients, which is possible via the vibration filter. We present the vibration lattice filter algorithm from the vector channel periodic lattice filter. The procedure is so simple that this filter can be implemented in on-line identification of the system parameters. This vibration filter is applied to test signal which is a combination of many sine waves to compare the performance of scalar with vector channel lattice. Also it is applied to the cantilever data to identify natural frequencies and damping ratios to show its performance. Its convergence is very fast with acceptable accuracy, which shows the superiority of the vector channel lattice and vibration lattice filter.

Consider the following p-channel nth order AR model

$$y(k)=\sum_{i=1}^{n} y(k-i)A_i \tag{1}$$

where $y(k)$ is an $m\times p$ matrix, A_i is $p\times p$ matrix. Let there be m outputs and one input. $y(k)$ can be expressed as follows

$$y(k)=[y_1(k)\ \ y_2(k)\ \ \cdots\ \ y_p(k)] \tag{2}$$

In vector channel lattice, both control system inputs and outputs can be treated as measurements. If the numbers of input $u(k)$ and output are 1 and m, respectively, the first channel becomes the measurement vector $[y_{11}(k)\ \ y_{21}(k)\ \ \cdots\ \ y_{m1}(k)]^T$, the second channel is m-vector $[u(k)\ \ 0\ \cdots\ 0]^T$, the third one becomes $[0\ \ u(k)\ \ 0\ \cdots\ 0]^T$, and finally $m+1^{\text{th}}$ channel becomes $[0\ \cdots\ 0\ \ u(k)]^T$, and they constitute vector channel lattice. It can be shown that this model can be imbedded into a p channel ($p=m+1$) vector channel AR model.

Let us define an infinite history vector for i^{th} channel as

$$Y_i(k)=[y_i^T(k)\ \ y_i^T(k-1)\ \ \cdots]^T,\ i=1,\cdots,p \tag{3}$$

and these history vectors are assumed to be in the following Hilbert space

$$l_2(R^m,\lambda) = \{\phi=[\phi_1^T,\phi_2^T,\phi_3^T,\cdots]^T : \|\phi\|^2=\langle\phi,\phi\rangle<\infty\} \tag{4}$$

where $\langle,\rangle$ implies inner product, and λ, $0<\lambda\leq 1$, is forgetting factor.

$$\langle\phi,\hat{\phi}\rangle=\sum_{j=1}^{\infty}\lambda^{j-1}\phi_j^T\hat{\phi}_j \tag{5}$$

The periodic lattice finds the projection of the most recent history vectors onto the span of previous vectors. That is, projecting $Y_i(k)$, for example, onto the span of $Y_{i-1}(k), Y_{i-2}(k),\cdots$ To simplify the derivation, we use the following notation.

$$\begin{aligned} s^{-1}Y_i(k)&=Y_{i-1}(k),\ i=2,\cdots,p\\ s^{-1}Y_1(k)&=Y_p(k-1)\end{aligned} \tag{6}$$

2. Vector Channel Periodic Lattice Filter

2.1 Filter algorithm

Consider the following subspace.

$$H_i^{1,n}(k)=span\{s^{-1}Y_i(k),s^{-2}Y_i(k),\cdots,s^{-n}Y_i(k)\} \tag{7}$$

The forward residual error $f_i^n(k)$ and backward residual error $b_i^n(k)$, of $Y_i(k)$ and $s^{-n}Y_i(k)$, onto $H_i^{1,n}(k)$ and $H_i^{0,n-1}(k)$, respectively, can be defined as

$$f_i^n(k)=[I-P_i^{1,n}(k)]\,Y_i(k) \tag{8}$$

$$b_i^n(k)=[I-P_i^{0,n-1}(k)]\,s^{-n}Y_i(k) \tag{9}$$

where $P_i^{1,n}(k)$ is orthogonal projection onto $H_i^{1,n}(k)$. Then, the algorithm for the vector channel periodic lattice filter was given by Jabbari (1989)

$$e_i^{n+1}(k) = e_i^n(k) - R_{f,i}^n(k)\, r_{i-1}^n(k) \quad (10)$$

$$r_i^{n+1}(k) = r_{i-1}^n(k) - R_{b,i}^n(k)\, e_i^n(k) \quad (11)$$

$$L_{f,i}^{n+1}(k) = L_{f,i}^n(k) - R_{f,i}^n(k) K_i^n(k) \quad (12)$$

$$L_{b,i}^{n+1}(k) = L_{b,i-1}^n(k) - R_{b,i}^n(k) K_i^n(k) \quad (13)$$

where

$$R_{f,i}^n(k) = \frac{K_i^n(k)}{L_{b,i-1}^n(k)} \quad (14)$$

$$R_{b,i}^n(k) = \frac{K_i^n(k)}{L_{f,i}^n(k)} \quad (15)$$

$$K_i^n(k) = \langle f_i^n(k),\ b_{i-1}^n(k)\rangle \quad (16)$$

$$L_{f,i}^n(k) = \langle f_i^n(k),\ f_i^n(k)\rangle \quad (17)$$

$$L_{b,i}^n(k) = \langle b_i^n(k),\ b_i^n(k)\rangle \quad (18)$$

The length of forward and backward residual errors are infinitely long. The algorithm, when implemented, uses $e_i^n(k)$ and $r_i^n(k)$ which are the top m-entries of f_i^n and $b_i^n(k)$ as in Eqs. (10) and (11). The algorithm is complete with the following time update equations of $K_i^n(k)$.

$$K_i^n(k) = \lambda K_i^n(k-1) + [e_i^n(k)]^T [G_{i-1}^n(k)]^{-1} r_{i-1}^n(k) \quad (19)$$

$$G_i^{n+1}(k) = G_i^n(k) - \frac{r_i^n(k)}{L_{b,i}^n(k)} [r_i^n(k)]^T \quad (20)$$

For the residual algorithm, the $m \times m$ matrices $G_i^n(k)$ are the only matrices that are inverted. The size of m, the number of sensors in each channel, is often considerably smaller.

2.2 AR Coefficients

$P_i^{1,n}(k)\, Y_i(k)$ and $P_i^{0,n-1}(k)\, s^{-n} Y_i(k)$, $i=1,2,\cdots,p$, can be written as a linear combination of the history vectors that span $H_i^{1,n}(k)$ and $H_i^{0,n-1}(k)$, respectively. This means that

$$f_i^n(k) = Y_i(k) - \sum_{j=1}^{n} s^{-j} Y_i(k)\, A_{n,i}^j(k) \quad (21)$$

$$b_i^n(k) = s^{-n} Y_i(k) - \sum_{j=1}^{n} s^{-j} Y_{i+1}(k)\, B_{n,j}^{\theta}(k) \quad (22)$$

Since $f_i^n(k)$ is the error remaining after orthogonal projection of the data taken through time k onto the history space $H_i^{1,n}(k)$, the coefficient $A_{n,i}^j(k)$ in Eq. (21), for example, is the coefficient for the order n AR model, estimated at time k. It minimizes $l_2(R^m, \lambda)$ norms of $f_i^n(k)$ over all autoregressive models of order n. Substituting Eqs. (21) and (22) into the order update Eqs. (10) and (11) and matching coefficients of the history vectors yields

$$A_{n+1,i}^j(k) = A_{n,i}^j(k) - R_{f,i}^n(k)\, B_{n,i-1}^j(k) \quad (23)$$

$$A_{n+1,i}^{n+1}(k) = R_{f,i}^n(k) \quad (24)$$

$$B_{n+1,i}^{j+1}(k) = B_{n,i-1}^j(k) - R_{b,i}^n(k)\, A_{n,i}^j(k) \quad (25)$$

$$B_{n+1,i}^1(k) = R_{b,i}^n(k) \quad (26)$$

$$B_{n,0}^j(k) = B_{n,p}^j(k-1) \quad (27)$$

where $j=1,\cdots,n$. The AR coefficients $A_{n,i}^j(k)$ and $B_{n,i}^j(k)$ can be generated with Eqs. (23)-(27), with the initialization process deduced from the basic definitions. If the AR algorithm is used at every time step, this does not create any difficulties. However, to reduce computational cost and to provide the ability to increase or decrease the order of the model from one time step to another, it is preferable to calculate $B_{n,i}^j(k)$ at time k directly. In the following two new equations are derived to make this possible.

Define m vectors as follows:

$$\phi_j = [0,\cdots,0,1,0,\cdots]^T,\ j=1,\cdots,m \quad (28)$$

where 1 is at j-th position, and the projection error of ϕ_j onto $H_{i-1}^{0,n-1}(k)$ becomes

$$\hat{\phi}_{j,i-1}^{0,n-1}(k) = [I - P_{i-1}^{0,n-1}(k)]\,\phi_j \quad (29)$$

Considering the following subspace

$$\hat{H}_{i-1}^{0,n}(k) = span\{Y_{i-1}(k), s^{-1}Y_{i-1}(k), \cdots, s^{-n}Y_{i-1}(k), \phi_1, \cdots, \phi_m\} = H_{i-1}^{0,n}(k) \oplus span\{\hat{\phi}_{1,i-1}^{0,n}(k), \cdots, \hat{\phi}_{m,i-1}^{0,n}(k)\} \quad (30)$$

we can get

$$\hat{\phi}_{j,i}^{0,n}(k) = \hat{\phi}_{j,i}^{0,n-1}(k) - \frac{b_i^{n,j}(k)}{L_{b,i}^n(k)} b_i^n(k) \quad (31)$$

Since $\hat{\phi}_{j,i}^{0,n}(k) = [I - P_i^{0,n}(k)]\,\phi_j$ and $P_i^{0,n}(k)\,\phi_j \in H_i^{0,n}(k)$, there exists $C_{j,i}^{n,l}(k)$ such that Eq. (32)

holds

$$\hat{\phi}^{0,n}_{j,i}(k)=\phi_j-\sum_{l=0}^{n}s^{-l}Y_i(k)\,C^{n,l}_{j,i}(k) \qquad (32)$$

From Eqs. (31) and (32), there follows

$$\begin{aligned}
&\phi_j-Y_i(k)\,C^{n,0}_{j,i}(k)\\
&-\sum_{l=1}^{n-1}s^{-l}Y_i(k)\,C^{n,l}_{j,i}(k)-s^{-n}Y_i(k)\,C^{n,n}_{j,i}(k)\\
&=\phi_j-Y_i(k)\,C^{n-1,0}_{j,i}(k)+\frac{b^{n,j}_i(k)}{L^n_{b,i}(k)}Y_i(k)\,B^l_{n,i}(k)\\
&-\sum_{l=1}^{n-1}s^{-l}Y_i(k)\{C^{n,l}_{j,i}(k)-\frac{b^{n,j}_i(k)}{L^n_{b,i}(k)}B^{l+1}_{n,i}(k)\}\\
&-\frac{b^{n,j}_i(k)}{L^n_{b,i}(k)}s^{-n}Y_i(k)
\end{aligned} \qquad (33)$$

and comparing the coefficients of Eq. (33), we get

$$C^{n,l}_{j,i}(k)=C^{n-1,l}_{j,i}(k)-\frac{b^{n,j}_i(k)}{L^n_{b,i}(k)}B^{l+1}_{n,i}(k) \qquad (34)$$

$$C^{n,n}_{j,i}(k)=\frac{b^{n,j}_i(k)}{L^n_{b,i}(k)} \qquad (35)$$

where $l=0,\cdots,n-1$, $j=1,\cdots,m$, $i=1,\cdots,p$.

Let $\hat{b}^n_i(k)$ denote the part of $b^n_i(k)$ below the first m rows. Then from Eq. (22),

$$\hat{b}^n_i(k)=s^{-n}Y_i(k-1)-\sum_{j=1}^{n}s^{-j}Y_{i+1}(k-1)\,B^j_{n,i}(k) \qquad (36)$$

and

$$b^n_i(k)=\begin{bmatrix}0^{(m)}\\ b^n_i(k-1)\end{bmatrix}+[\hat{\phi}^{0,n-1}_i(k)][G^n_i(k)]^{-1}r^n_i(k) \qquad (37)$$

Notice that

$$\begin{aligned}
[\hat{\phi}^{0,n-1}_i(k)]&=[\hat{\phi}^{0,n-1}_{1,i}(k)\ \hat{\phi}^{0,n-1}_{2,i}(k)\ \cdots\ \hat{\phi}^{0,n-1}_{m,i}(k)]\\
&=[\phi_1\cdots\phi_m]\\
&\quad-Y_i(k)\,[C^{n-1,0}_{1,i}(k)\ \cdots\ C^{n-1,0}_{m,i}(k)]\\
&\quad\vdots\\
&\quad-s^{-n+1}Y_i(k)\,[C^{n-1,n-1}_{1,i}(k)\ \cdots\ C^{n-1,n-1}_{m,i}(k)]
\end{aligned} \qquad (38)$$

Hence it follows from Eqs. (37) and (38) that

$$\begin{aligned}
\hat{b}^n_i(k)&=s^{-n}Y_i(k-1)-\sum_{j=1}^{n}s^{-j}Y_{i+1}(k-1)\,B^j_{n,i}(k)\\
&-\sum_{j=1}^{n}s^{-j}Y_{i+1}(k-1)\,[C^{n-1,j-1}_{1,i}(k)\ \cdots\ C^{n-1,j-1}_{m,i}(k)][G^n_i(k)]^{-1}r^n_i(k)
\end{aligned} \qquad (39)$$

Comparing the coefficients of history vectors in Eqs. (36) and (39), and simplifying Eqs. (34), (35), with respect to channel,

$$C^j_{n+1,i}(k)=C^j_{n,i}(k)-\frac{b^{n,i}_p(k)}{L^n_{b,p}(k)}B^{j+1}_{n,p}(k) \qquad (40)$$

$$C^n_{n+1,i}(k)=\frac{r^{n,i}_p(k)}{L^n_{b,p}(k)} \qquad (41)$$

$$\begin{aligned}
&B^j_{n,0}(k-1)\\
&=B^j_{n,p}(k)-[C^{j-1}_{n,1}(k)\ \cdots\ C^{j-1}_{n,m}(k)][G^n_p(k)]^{-1}r^n_b(k)
\end{aligned} \qquad (42)$$

where $j=1,\cdots,n-1$, $i=1,\cdots,m$. Using these equations, at any given time, the AR coefficients of arbitrary order can be generated as long as the order is not larger than the order of the residual filter.

Eq. (21) can be expanded to obtain p channel nth order AR model, as follows;

$$y(k)+\sum_{i=1}^{n}y(k-i)\,A_i=W(k) \qquad (43)$$

where $A_i=\Omega_iU^{-1}$, and $W(k)$ is zero mean finite variance white noise process, and

$$U=\begin{bmatrix}1 & -A^1_{n,2} & -A^2_{n,3} & \cdots & -A^{p-1}_{n,p}\\ 0 & 1 & -A^1_{n,3} & \cdots & -A^{p-2}_{n,p}\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \cdots & -A^1_{n,p}\\ 0 & 0 & 0 & \cdots & 1\end{bmatrix} \qquad (44)$$

$$\Omega_1=\begin{bmatrix}-A^p_{n,1} & -A^{p+1}_{n,2} & \cdots & -A^{2p-1}_{n,p}\\ -A^{p-1}_{n,1} & -A^p_{n,2} & \cdots & -A^{2p-2}_{n,p}\\ \vdots & \vdots & \ddots & \vdots\\ -A^2_{n,1} & -A^3_{n,2} & \cdots & -A^{p+1}_{n,p}\\ -A^1_{n,1} & -A^2_{n,2} & \cdots & -A^p_{n,p}\end{bmatrix} \qquad (45)$$

$$\Omega_2=\begin{bmatrix}-A^{2p}_{n,1} & -A^{2p+1}_{n,2} & \cdots & -A^{3p-1}_{n,p}\\ -A^{2p-1}_{n,1} & -A^{2p}_{n,2} & \cdots & -A^{3p-2}_{n,p}\\ \vdots & \vdots & \ddots & \vdots\\ -A^{p+2}_{n,1} & -A^{p+3}_{n,2} & \cdots & -A^{2p+1}_{n,p}\\ -A^{p+1}_{n,1} & -A^{p+2}_{n,2} & \cdots & -A^{2p}_{n,p}\end{bmatrix} \qquad (46)$$

$$\vdots$$

U matrix is upper triangular matrix, and calculation of its inverse is simple.

3. Vibration Filter

3.1 Eigenvalues, frequencies and damping ratios

From Eq. (43), we can derive equations for $y_{j1}(k)$, $j=1,\cdots,n$, as

$$y_{j1}(k)+y_{j1}(k-1)A_1^{11}+y_{j1}(k-2)A_2^{11}+\cdots+u(k-1)A_1^{j+1,1}+u(k-2)A_2^{j+1,1}+\cdots=W_j(k) \quad (47)$$

Hence, its characteristic polynomial is written as

$$P(z^{-1})=1+A_1^{11}z^{-1}+A_2^{11}z^{-2}+A_3^{11}z^{-3}+\cdots+A_n^{11}z^{-n} \quad (48)$$

The characteristic roots $\lambda_i(i=1,2,\cdots,N)$ can be determined from the following matrix

$$\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -A_n^{11} & -A_{n-1}^{11} & -A_{n-2}^{11} & \cdots & -A_1^{11} \end{bmatrix} \quad (49)$$

and the transfer function is

$$H_{ij}(z^{-1})=\frac{A_1^{j+1,1}z^{-1}+A_2^{j+1,1}z^{-2}+A_3^{j+1,1}z^{-3}+\cdots+A_m^{j+1,1}z^{-m}}{1+A_1^{11}z^{-1}+A_2^{11}z^{-2}+A_3^{11}z^{-3}+\cdots+A_n^{11}z^{-n}} =\sum_{k=1}^{N}\left(\frac{r_{ijk}}{1-\lambda_k z^{-1}}+\frac{r_{ijk}^*}{1-\lambda_k^* z^{-1}}\right) \quad (50)$$

where i is response point, j input point, λ_k and λ_k^* are k-th complex conjugate eigenvalues of discrete characteristic equation, and r_{ijk} is complex residue. Since r_{ijk} represents relative amplitude of i point when j point is excited, mode shape can be determined from r_{ijk}. Natural frequency ω_k and damping ratio ζ_k can be obtained from the characteristic roots, as (Jabbari and Gibson, 1988b)

$$\omega_k=\frac{1}{T}\sqrt{\frac{[\ln(\lambda_k\cdot\lambda_k^*)]^2}{4}+\left(\cos^{-1}\frac{\lambda_k+\lambda_k^*}{2\sqrt{\lambda_k\cdot\lambda_k^*}}\right)^2} \quad (51)$$

$$\zeta_k=\frac{[\ln(\lambda_k\cdot\lambda_k^*)]^2}{\sqrt{[\ln(\lambda_k\cdot\lambda_k^*)]^2+4\left(\cos^{-1}\frac{\lambda_k+\lambda_k^*}{2\sqrt{\lambda_k\cdot\lambda_k^*}}\right)^2}} \quad (52)$$

where T is sampling time. Note that in Eq. (52), the inverse cosine function appears in the denominator. For an alias frequency, therefore, this equation results in a higher damping ratio as compared to the use of the true frequency.

3.2 Vibration lattice filter

The usual lattice computes reflection coefficients of autoregressive (AR) process Eq. (1) directly, but not the A_i coefficients. One method of obtaining the A_i's is to use another lattice-like recursion. Then, the A_i's can be put into block companion form, and some eigenvalue routines can be used to obtain the frequencies of vibration, as shown in previous sections. The problem appears to be that the procedure for estimating the A_i's depends on the reflection coefficients reaching steady-state values. The implication is that the method works better for more lightly damped systems. To reduce numerical errors, it is desirable to calculate the natural frequencies directly from the reflection coefficients. The vibration filter is proposed to solve this problem (Wiberg and Gillis, 1985).

This vibration filter is now generalized to the vector channel case. From Eqs. (10)-(11),

$$\begin{aligned} r_i^0(k) &= y_i(k) \\ r_i^1(k) &= r_{i-1}^0(k)-R_{b,i}^0(k)y_i(k) \\ r_i^2(k) &= R_{b,i}^1(k)R_{f,i}^0(k)r_{i-1}^0(k)+r_{i-1}^1(k) -R_{b,i}^1(k)y_i(k) \\ &\vdots \\ r_i^{n-1}(k) &= R_{b,i}^{n-2}(k)R_{f,i}^0(k)r_{i-1}^0(k) +R_{b,i}^{n-2}(k)R_{f,i}^1(k)r_{i-1}^1(k)+\cdots +R_{b,i}^{n-2}(k)R_{f,i}^{n-3}(k)r_{i-1}^{n-3}(k) +r_{i-1}^{n-2}(k)-R_{b,i}^{n-2}(k)y_i(k) \end{aligned} \quad (53)$$

$$e_i^{n-1}(k)=-R_{f,i}^0(k)r_{i-1}^0(k) -R_{f,i}^1(k)r_{i-1}^1(k)-\cdots -R_{f,i}^{n-2}(k)r_{i-1}^{n-2}(k)+y_i(k) \quad (54)$$

Defining $x_i(k)=[r_{i-1}^0(k)\ r_{i-1}^1(k)\ \cdots\ r_{i-1}^{n-1}(k)]^T$, and $e_i^n(k)$ as lattice output, Eqs. (53) and (54) can be written as

$$x_{i+1}(k)=A(k)x_i(k)+B(k)y_i(k) \quad (55)$$

$$e_i^n(k)=C(k)x_i(k)+y_i(k) \quad (56)$$

where

$$A(k)=\begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & 0 \\ A_{21} & 0 & \cdots & 0 & 0 & 0 \\ A_{31} & A_{32} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nn-2} & A_{nn-1} & 0 \end{bmatrix} \quad (57)$$

$$\begin{aligned} &A_{21}=1,\ A_{31}=R_{b,i}^1(x)R_{f,i}^0(k) \\ &A_{32}=1,\ A_{n1}=R_{b,i}^{n-2}(k)R_{f,i}^0(k) \\ &A_{n2}=R_{b,i}^{n-2}(k)R_{f,i}^1(k) \\ &A_{nn-2}=R_{b,i}^{n-2}(k)R_{f,i}^{n-3}(k) \\ &A_{nn-1}=1 \end{aligned}$$

$$B(k)=\begin{bmatrix} 1 \\ -R_{b,i}^{0}(k) \\ -R_{b,i}^{1}(k) \\ \vdots \\ -R_{b,i}^{n-2}(k) \end{bmatrix} \tag{58}$$

$$C(k)=[-R_{f,i}^{0}(k) \;\; -R_{f,i}^{1}(k) \;\; \cdots \;\; -R_{f,i}^{n-1}(k)] \tag{59}$$

Eqs. (55) and (56) can be transformed to

$$\begin{aligned} x_{i+1}(k) &= [A(k)-B(k)C(k)]x_i(k) \\ &\quad + B(k)e_i^n(k) \end{aligned} \tag{60}$$

$$y_i(k)=e_i^n(k)-C(k)x_i(k) \tag{61}$$

The natural frequencies of vibration can be obtained from the eigenvalues of matrix $[A(k)-B(k)C(k)]$, which is written out in block matrix form as

$$A(k)-B(k)C(k) = \begin{bmatrix} N_{11} & N_{12} & \cdots & N_{1n-1} & N_{1n} \\ N_{21} & N_{22} & \cdots & N_{2n-1} & N_{2n} \\ 0 & N_{32} & \cdots & N_{3n-1} & N_{3n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & N_{nn-1} & N_{nn} \end{bmatrix} \tag{62}$$

where

$N_{11}=R_{f,i}^{0}(x)$, $N_{12}=R_{f,i}^{1}(k)$

$N_{1n-1}=R_{f,i}^{n-2}(k)$, $N_{1n}=R_{f,i}^{n-1}(k)$

$N_{21}=1-R_{b,i}^{0}(k)R_{f,i}^{0}(k)$

$N_{22}=-R_{b,i}^{0}(k)R_{f,i}^{1}(k)$

$N_{2n-1}=-R_{b,i}^{0}(k)R_{f,i}^{n-2}(k)$

$N_{2n}=-R_{b,i}^{0}(k)R_{f,i}^{n-1}(k)$

$N_{32}=1-R_{b,i}^{1}(k)R_{f,i}^{1}(k)$

$N_{3n-1}=-R_{b,i}^{1}(k)R_{f,i}^{n-2}(k)$

$N_{3n}=-R_{b,i}^{1}(k)R_{f,i}^{n-1}(k)$

$N_{nn-1}=1-R_{b,i}^{n-2}(k)R_{f,i}^{n-2}(k)$

$N_{nn}=-R_{b,i}^{n-2}(k)R_{f,i}^{n-1}(k)$

The vibration lattice is numerically superior to, and faster converging than, the usual lattice for the case of free vibration with no noise, especially when many sensors are used. The characteristic equation is computed directly, and no extraneous frequencies of vibration are introduced. Hence, for the case of free vibration, the vibration lattice is believed to be superior to any other method presently available because of its inherent numerical properties and robustness with respect to the addition of a small amount of noise.

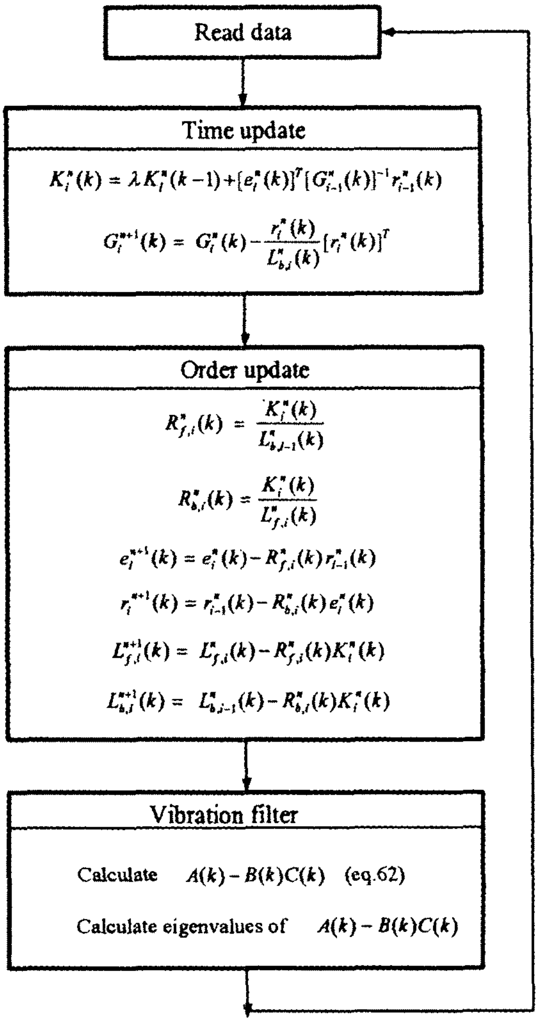

Fig. 1 Flow chart for vibration filter

The flow chart of the vector channel lattice filter with vibration filter is shown in Fig. 1. If AR coefficients are required to determine, then the procedures for AR coefficients, Eqs. (23)-(27), are replaced in the place of vibration filter.

4. Parameter Identification

Both scalar and vector channel lattices have been used to identify the properties of the system, such as natural frequencies and damping ratios. For all runs, the value of the forgetting factor is 0.99.

4.1 Combination of many sine waves

Computer simulations are performed to show the effectiveness of vibration lattice filter. Signals contaminated with noise are generated with MATLAB, and parameters are estimated from these contaminated signals. These signals have 5 frequency components which are shown in Table 1. Signal s_1 has all the five components, and signals s_2 and s_3 are incomplete in the sense that some of their frequency components are extrmely weak. Table 1 shows the relative amplitudes of frequency components of three signals, where randn is white noise with zero mean and unit variance.

Natural frequencies were estimated with scalar and vector channel lattice filter with order 40, and the identification results are shown in Table 2. From the table, it is evident that the filter can estimate the frequencies from the noisy signals. As expected, from the signal s_2 only, two of natural frequencies can not be obtained. Vector channel lattice filter with two incomplete signals s_2 and s_3 can estimate all the frequencies.

Table 1 Amplitudes of each frequency components in signal

Signal / Component	s_1	s_2	s_3
123 Hz	0.4800	0.4800	0.0048
446 Hz	0.5000	0.0050	0.5000
668 Hz	0.4100	0.4100	0.0041
775 Hz	0.2800	0.0028	0.2800
934 Hz	0.3700	0.3700	0.0037
noise	0.1*randn	2.5*randn	2.5*randn

Table 2 Estimated frequencies from the lattice filter and vibration filter

Mode	1	2	3	4	5
Freq. (Hz)	123.0	446.0	668.0	775.0	934.0
s_1	123.0	445..9	667.7	775.1	933.9
s_3	123.0	394.1	668.1	796.6	934.2
s_2 & s_3	123.0	446.0	667.6	775.5	934.1
Vibration Filter (s_2 & s_3)	122.9	446.2	668.0	774.9	934.4

4.2 Natural frequencies of cantilever

To test a performance of the vibration filter in the practical situation, we acquired the vibration signal from the cantilever, and applied the lattice filter algorithm. A flexible beam is cantilevered to the ground, and three accelerometers were used to measure the accelerations of the cantilever. Time history of one output signal is given in Fig. 2. FRF is measured and averaged for 10 measurements. The result is given in Fig. 3, where the natural frequencies of cantilever are shown in the figure. There are four modes of vibration below 900 Hz range. That is 120.4, 311, 626.8, and 892.5 Hz.

Table 3 shows results of estimation of lattice filter from the accelerometer signals. We set order

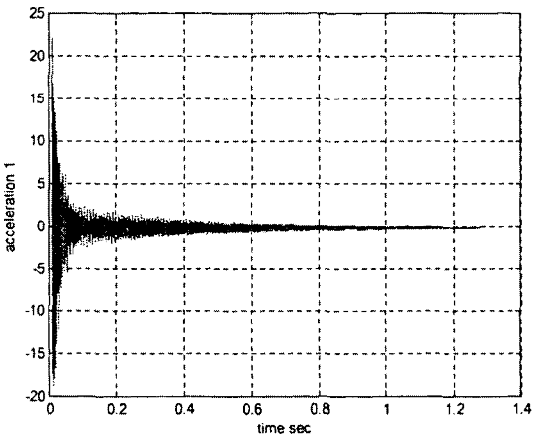

Fig. 2 Acceleration data

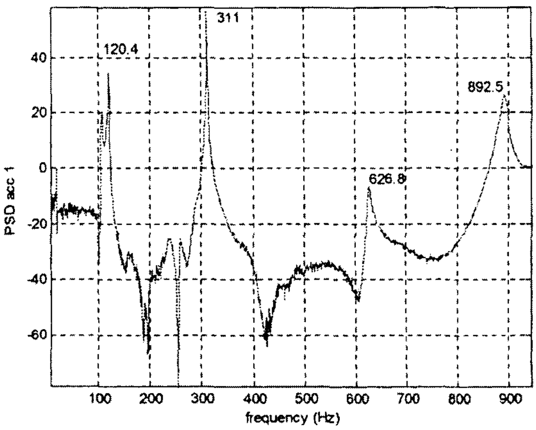

Fig. 3 FRF plot for cantilever

of AR to be 30. It is observed that the estimation results are not satisfactory for scalar channel lattice, except 2^{nd} mode. Notice that the 2^{nd} mode has a sharp peak in Fig. 2. Vector channel vibration filter with three accelerometer signals are successful to estimate all the frequencies with errors less than 1%.

Figure 4 shows the variation of identified eigenvalues with respect to number of iterations, calculated from data of accelerometer 1 using vibration filter. Its convergence is quite fast, but the results have more errors than that of vector channel lattice. Fig. 5 shows the behavior of eigenvalues computed with data from three accelerometers using AR coefficients. It shows that the result converges after 1200 iterations.

Figure 6 shows the convergence behavior of eigenvalues and damping ratios calculated from three accelerometer data using vibration filter.

Table 3 Estimated results with vibration lattice

Mode		1	2	3	4
Freq. (Hz)		120.4	311.0	626.8	892.5
Acc.1		126.0	312.2	646.7	905.2
Acc.2		122.7	312.6	660.0	898.4
Acc.3		113.8	312.5	652.2	910.6
Acc.1, 2 & 3	Freq.	118.2	313.7	628.7	896.0
	Damp' ratio	0.054	0.014	0.039	0.006

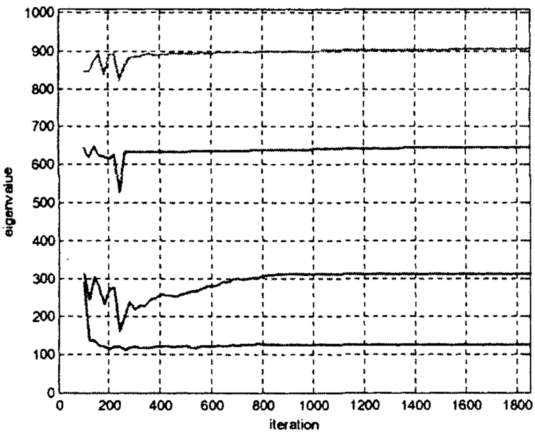

Fig. 4 Eigenvalue plot estimated from scalar channel vibration filter

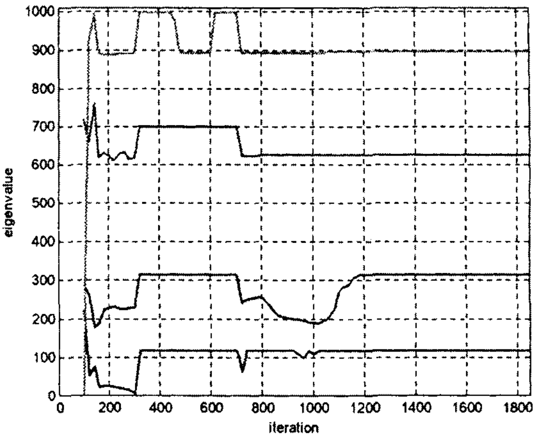

Fig. 5 Trajectories of estimated eigenvalues from vector channel lattice using AR coefficients

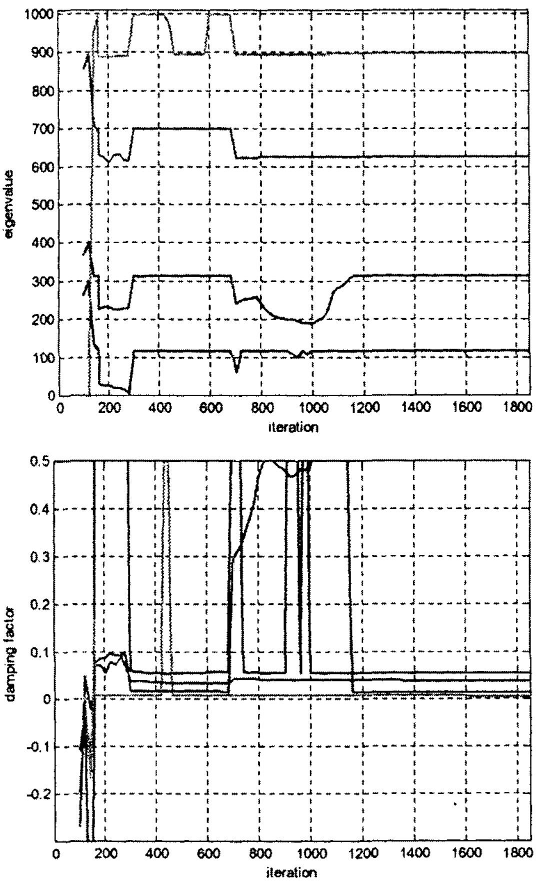

Fig. 6 Eigenvalue and damping ratio plot with vector channel vibration filter usuing three accelerometers

Comparing with Fig. 5 shows that the results from vibration filter are same as the one obtained from AR coefficients, except the early stages of computation below 200 iterations.

The previous results show the comparison of the vector channel with scalar channel lattices, and the superiority of the vector channel lattice and vibration lattice filter.

5. Conclusions

This paper considered vector channel lattice filter, and gave an algorithm for the AR coefficients from vector channel lattice. We also presented the characteristic polynomial and transfer function from the AR coefficients. Then vibration filter using vector channel periodic lattice filter is constructed. This vibration filter is applied to test signal which is a combination of many sine waves to compare the performance of scalar and vector channel lattice. Also it is applied to the cantilever data to identify ARMA coefficients, natural frequencies and damping ratios to show its performance.

The vibration filter yields the system parameters, such as natural frequencies and damping ratios of the structure, without computing AR parameters. This means that even when the ARMA coefficients of the system can not be correctly estimated, some vibration characteristics of the system can be estimated with vibration filter in the presence of unmodeled dynamics. The results also show the computational accuracy of the vector channel lattice, compared with the scalar channel one. Overall, these results suggest that vector channel vibration filter is quite effective for recursive parameter identification of systems, and thus can be used in on-line applications.

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