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The Analysis of Heat Transfer through the Multi-layered Wall of the Insulating Package

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Abstract Thermal insulation is used in a variety of applications to protect temperature sensitive products from thermal damage. Several factors affect the performance of insulation packages. Among these factors, the thermal resistance of the insulating wall is the most important factor to determine the performance of the insulating package. In many cases, insulating wall consists of multi-layered structure and the heat transfer through this structure is a very complex process. In this study, an one-dimensional mathematical model, which includes all of the heat transfer principles covering conduction, convection and radiation in multi-layered structure, were developed. Based on this model, several heat transfer phenomena occurred in the air space between the layer of the insulating wall were investigated. From the simulation results, it was observed that the heat transfer through the air space between the layer were dominated by conduction and radiation and the low emissivity of the surface of each solid layer of the wall can dramatically increase the thermal resistance of the wall. For practical use, an equation was derived for the calculation of the thermal resistance of a multi-layered wall.

Key words Thermal resistance, Insulating Package, Multi-layerd Wall

Introduction

Temperature sensitive products such as biological materials, pharmaceuticals, industrial adhesives, gyroscopes, blood, frozen foods, fresh-products and dairy products should be shipped in temperature controlled containers which keep the temperature constant during transportation and keep products from thermal damage caused by melting, thawing or freezing.

In most cases, thermal insulation is used to protect the products from thermal damage during distribution of products. Thermal insulation is a procedure, which can retard heat transfer and eventually conserve energy by reducing heat loss or gain of the product (ASHRAE, 1993). Due to the complexity of the nature of heat transfer, several factors can affect the insulating ability of the containers.

The geometry of the package can affect the insulating capacity of the container. The addition of aluminum foil can also reduce the infrared radiation, which results substantial improvement of the insulating ability of the package. Sealing the package also plays important role for insulating ability of the package. Air currents can flow in and out through very small openings and can carry enough heat with them to render even the best insulator ineffective. Most of all, material's resistance to heat penetration is the most crucial factor for the insulating ability of the package. It is obvious that thicker walls generate better insulating capability. In many cases, multi-layered materials make better insulators than one thick material with the same thickness. A thin blanket of air trapped between the liner and box produces a significant resistance to heat penetration. This is the same principle behind double and triple pane windows or wearing layered clothes in the winter instead of one thick coat.

Because of the variety of factors affecting the performance of a thermally insulated package, a comprehensive model, which can represent these factors as much as possible, is needed for designing efficient insulating packages. Proper simulation model can reduce time-consuming efforts of preliminary specifications and subsequent validation tests. It also eliminates over-packaging and the resultant unnecessary costs. However, there were very few attempts to predict the ability of insulation packages (Burgess, 1999; Kosituangchai, 2003; Stavish, 1984). Moreover, these models were over simplified or missed very crucial factors of heat transfer through insulating packages.

The first step to accomplish the comprehensive modelis to obtain an accurate thermal resistance of the wall of the insulating package. As mentioned above, material's resistance to heat penetration is the most crucial factor for the insulating ability of the package. In this study, the math-

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ematical model, which can predict the thermal resistance of the multi-layered wall, was developed.

If the wall consists of one material, it is very simple to calculate the thermal resistance of the package. For example, the thermal resistance of pure EPS is the reciprocal of its thermal conductivity, 0.038 W/m·K. However, insulating packages used in many applications employ much more complicated structures. Many insulating packages usea multi-layered structure with combinations of several insulating materials. In many cases, these layered materials are loosely fitted to each other to obtain extra thermal resistance by entrapping air between the insulating materials. A loose-fitting EPS foam jacket inside a corrugated box is a good example of these efforts. In this case, even though thermal conductivities of each of the insulating materials are known, it is still very difficult to estimate the overall thermal resistance of entire multi-layered structure because of the complexity of the heat transfer mechanisms through the 'between the wall'air space.

To explain these phenomena and obtain an overall thermal resistance of the multi-layered insulating materials, all of the basic theories used in the previous part must be combined to yield a comprehensive mathematical model. For these reasons, a mathematical model of the wall by itself, not the entire package, was developed.

Theories

The one-dimensional diagram of a multi-layered wall is shown in Fig. 1. In this case, the area of the surface ofeach layer is the same all through the system. The heat can flow in both direction: from outside to inside or from inside to outside. For convenience of the calculation, however, it is assumed that the heat flows from outside to inside of the container.

In this diagram, the heat transfer through the wall occurs through the solid walls and air spaces between the wall. The conduction heat transfer through i th layer can be described as

$$\frac{Q}{A} = k_i \frac{(T_{2i-1} - T_{2i})}{d_i} \tag{1}$$

Q = heat transfer rate (W)
A = cross-sectional area (m^2)
k_i = conduction heat transfer coefficient (W/m·K)
T_{2i-1}, T_{2i} = temperature of the both side of i th layer (K)
d_i = thickness of the i th layer (m)

The heat transfer through the air space between two walls is more complicated. In many practical problems, heat transfer through the air space can occur by three distinct modes: conduction, convection and radiation. In this case, the total heat transfer where convection, conduction and radiation occur simultaneously can be described as:

$$\frac{Q}{A} = h_i(T_{2i} - T_{2i+1}) \tag{2}$$

$$h_i = h_{r,i} + h_{c,i} \tag{3}$$

h_i = effective convection heat transfer coefficient at i th air space ($W/m^2 \cdot K$)
$h_{r,i}$ = radiation heat transfer coefficient at i th air space ($W/m^2 \cdot K$).
$h_{c,i}$ = pure convection or pure conduction heat transfer coefficient at i th air space ($W/m^2 \cdot K$)

The radiation heat transfer rate between two parallel sur-

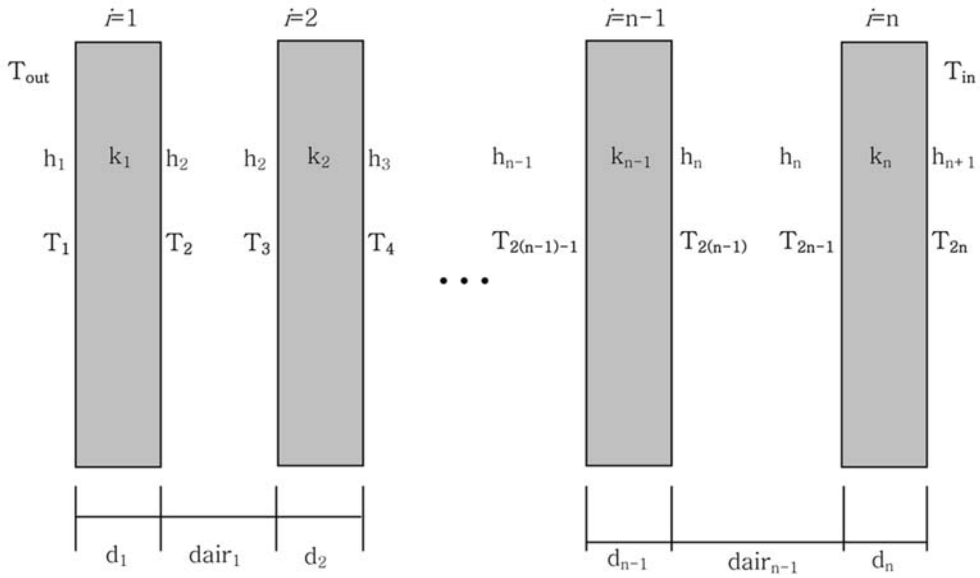

Fig. 1. Diagram of heat transfer through multi-layered wall in contact with inside and outside air

faces with same surface area can be described as the following equation (Kreith, 1973):

$$h_{r,\,i} = \frac{\sigma(T_{2i}^4 - T_{2i+1}^4)/(T_{2i} - T_{2i+1})}{\frac{1}{\varepsilon_{2i}} + \frac{1}{\varepsilon_{2i+1}} - 1} \tag{4}$$

σ = Boltsmann constant, 5.6×10^{-8} W/m^2·K^4

ε_{2i}, ε_{2i+1} = surface emissivity of each side of the *i* th air space

The pure convection or pure conduction heat transfer coefficient is determined by several factors (Holman, 1986): type of fluid (liquid or gas), flow condition (laminar or turbulent), forced or natural convection or phase change, free-stream velocity, surface geometry and roughness, position along the surface and temperature dependence of fluid properties. This study deals with the heat transfer through the multi-wall structure. In this case, the conduction or convection in air space between the layers occurs in a confined air space. In this study, only natural convection in confined air space was considered to represent convection heat transfer.

Natural convection heat transfer in enclosed spaces was investigated in various reports (Jacob, 1949; Kreith, 1973; Holman, 1986). For an isothermal condition, the average natural convection heat transfer coefficient $h_{c,i}$ in an enclosed air space between parallel planes can be expressed by the following empirical formulas:

$$h_{c,\,i} = \frac{k_i}{dair_i} \cdot Nu_\delta \tag{5}$$

k_i = thermal conductivity of air in *i* th air space (W/m·K)

$dair_i$ = thickness of the *i* th air space (m)

Nu = Nusselt number in an enclosed air space.

Nusselt number is a strong function of Grashof number, Gr_δ, which is defined by

$$Gr_\delta = \frac{\rho^2 g\beta\Delta T'\delta^3}{\mu^2} \tag{6}$$

ρ = density of air (Kg/m^3)

γ = acceleration due to gravity (9.8 m/sec^2)

β = volumetric coefficient of expansion of the air in (1/K)

ΔT = positive temperature difference between the wall and the air in (K)

δ = thickness of the air space between the planes on either side (m)

μ = viscosity of air (Kg/m·s)

Grashof number is important because when the Grashof number is below 2000, the heat transfer is dominated by conduction (Nu = 1.0) (Jacob, 1949 Kreith, 1973). When Grashof number is above 2000, the heat transfer is dominated by convection.

Using above heat transfer coefficients, the heat transfer equations through the multi-layered structure can be derived. At steady state, the heat transfer rate through each solid layer is the same as the heat transfer rate through the air space between the solid layers. For an n layered wall with n-1 air spaces between them (See Fig. 1), one can establish a heat transfer balance with following equations assuming that all the area of the layered walls are the same.

for $i = 1$

$$\frac{Q}{A} = h_1(T_{out} - T_1) = \frac{k_1}{d_1}(T_1 - T_2) = h_2(T_2 - T_3) \tag{7}$$

for i= 2 to n-1

$$h_1(T_{2(i-1)} - T_{2i-1}) = \frac{k_i}{d_i}(T_{2i-1} - T_{2i})h_{i+1} + (T_{2i} - T_{2i+1}) \tag{8}$$

for $i = n$

$$h_1(T_{2(n-1)} - T_{2n-1}) = \frac{k_n}{d_n}(T_{2n-1} - T_{2n}) = h_{n+1} + (T_{2n} - T_{in}) \tag{9}$$

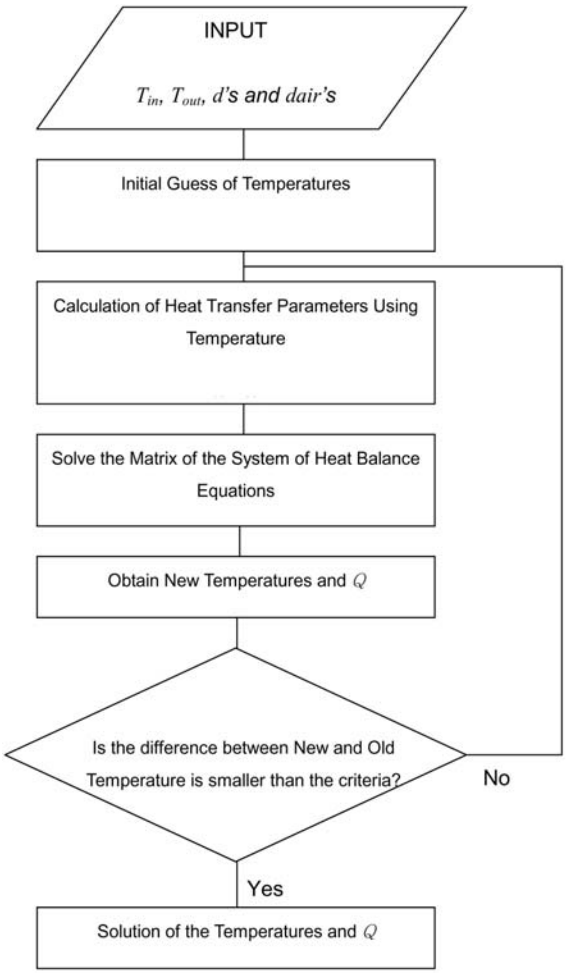

Fig. 2. Iteration procedure to solve non-linear problems.

d_i = thickness of the i th layer of solid material (m)
$dair_i$ = thickness of the i th layer of air space (m)
k_i = conduction heat transfer coefficient of i th layer of solid material (W/m·K)
h_i = effective convection heat transfer coefficient for the i th air space from eq (3) (W/m·K)

In this system of equations, there are $2N+1$ unknowns (Q, T_1, T_2, …, T_{2N-1}, T_{2N}) and $2N+1$ equations. However, this system of equations is highly non-linear. The convection and conduction coefficients, which are essential to solving for the unknown temperatures, are functions of the temperature itself. Moreover, radiation heat transfer coefficients include the terms of temperature powered by four. To solve this non-linearity problem, an iterative procedure should be adopted to solve this problem. The diagram of this procedure is shown in Fig. 2.

Using this algorithm, the system of equations (7), (8) and (9) can be solved in terms of the heat transfer rate, Q, to yield

$$\frac{Q}{A} = \frac{T_1 - T_n}{\sum_{i=1}^{n}\frac{d_i}{k_i} + \sum_{i=1}^{n-1}\frac{1}{h_i}} \quad (10)$$

In this case, the thermal resistance of the entire wall structure, R_{wall}, can be calculated as

$$R_{wall} = \sum_{i=1}^{n}\frac{d_i}{k_i} + \sum_{i=1}^{n-1}\frac{1}{h_i} \quad (11)$$

The unit of the Rwall is $m^2 \cdot K/W$.

Results and Discussion

1. Characteristics of the heat transfer in multi-layered walls

One of the major questions about heat transfer within the airspaces in multi-layered walls is whether heat is conducted by conduction or by convection. In fact, this is determined by the Grashof number, Gr_δ, based on the thickness of the air space. As mentioned above, heat transfer is dominated by conduction if Gr_δ, is below 2000. Otherwise, heat transfer is dominated by convection. According to equation (6), Gr_δ, in confined space is a function of the distance, δ, between the solid layers and the temperature difference between the layers, ΔT. The average temperature between the layers also affects Gr_δ, because $\rho^2 g\beta/\mu^2$ is a function of the average temperature between the layers. Using this equation, the characteristics of heat transfer of 'between-the-wall' air space was simulated.

The BASIC program NGR.BAS was constructed for simulation. Using this program, Gr_δ, was calculated with variety of δ, ΔT and the average temperature of the wall. In most packaging applications, the distance between the two

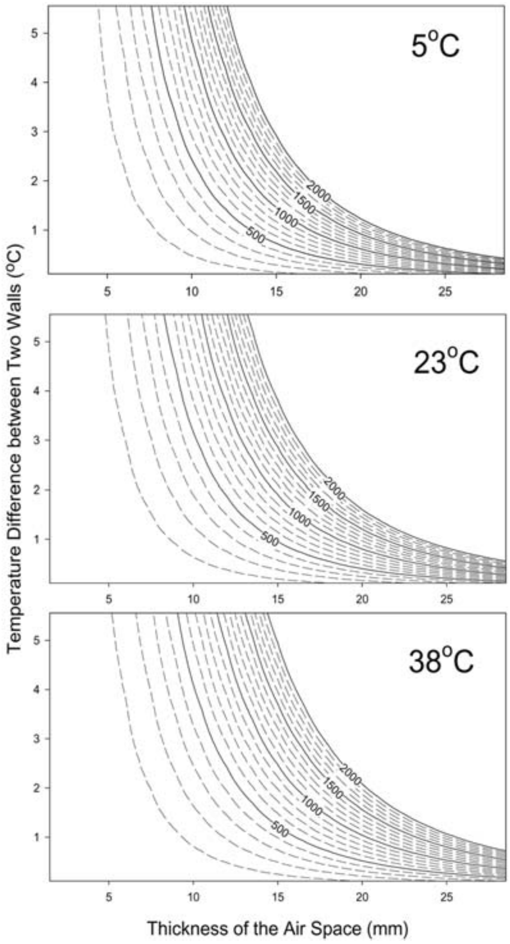

Fig. 3. The Grashof number with the distance between the layers and the temperature difference between the layers

solid layersis less than 20 mm and the temperature difference between this thin air space is less than 5°C. For these reasons, δ and ΔT were restricted 0.2 to 25 mm and 0.1 to 5°C, respectively. Average temperatures of 5°C, 23°C, 38°C were chosen to represent the low temperature, room temperature and high temperature, respectively. The simulation results are shown in Fig. 3 as a contour plot. When Gr_δ, is higher than 2000, which is the upper right part of each graph, heat transfer is dominated by convection. When the distance between the solid walls is smaller than 13 mm (0.5 inch), heat is transferred mainly by conduction regardless of the temperature difference or the average temperature in the simulation range. In most cases, the thickness of the air spaces in multi-layered walls is very thin (< 10 mm). This means that the most of the heat transfer in 'between the wall' air spaces are dominated by conduction combined with radiation.

2. Development of the formula to calculate the *R*-value of multi-layered wall structure

From the simulation result in previous section, one can assume that heat transfer in the air space is dominated by conduction and radiation. In this case, the effective heat transfer coefficient in equation (3) can be rearranged as

$$h_i = h_{c,i} + h_{r,i} = \frac{k_{air}}{dair_i} + h_{r,\,i} \tag{12}$$

Then, thermal resistance of a multi-layered wall in equation (11) can be rearranged as

$$R_{wall} = T_{solid} + R_{air} = \sum_{i=1}^{n} \frac{d_i}{k_i} + \sum_{i=1}^{n-1} \frac{1}{\frac{k_{air}}{dair_i} + h_{r,\,i}}$$

$$= \sum_{i=1}^{n} \frac{d_i}{k_i} + \sum_{i=1}^{n-1} \frac{\frac{1}{h_{r,\,i}} \cdot \frac{dair_i}{k_{air}}}{\frac{1}{h_{r,\,i}} + \frac{dair_i}{k_{air}}} \tag{13}$$

In the equation(13), thermal resistance of the entire wall structure is composed of two main factors: the resistance from the solid material (R_{solid}) and the resistance of air space between the walls (R_{air}).

According to equation (4), the radiation heat transfer coefficient is a function of the emissivities of both surfaces. If one or both surfaces are foiled with aluminum, the radiation heat transfer coefficient, $h_{r,i}$, can be reduced dramatically. In this case, thermal resistance of the air space in equation (13) becomes a linear function of the distance between the walls over the conductivity of the air (*dair/kair*).

For these reasons, the effect of the emissivity on the resistance by the air space was simulated. Three cases were simulated: both sides of the walls foiled ($\varepsilon_1 = 0.05$, $\varepsilon_2 = 0.05$) one side of the wall foiled ($\varepsilon_1 = 0.05$, $\varepsilon_2 = 0.95$), and none of the wall foiled ($\varepsilon_1 = 0.95$, $\varepsilon_2 = 0.95$). The BASIC program, NUSSELT.BAS, was used for simulation. This program is a modified version of NGR.BAS and contained the algorithm using equation (5), (6), (7) and (8), which calculates the heat transfer coefficients and resistance of the air space under various conditions.

In addition to the emissivities, the distance between the two solid layers (< 25 mm), the temperature difference between the layers (< 5^{o}C), the average air temperature (5^{o}C, 23^{o}C, 38^{o}C) were also chosen as variables. Using the simulation results, heat resistances in three cases of the radiation were plotted versus the distance between the walls over the conductivity of the air (*dair/kair*) as an independent variable.

As shown in Fig. 4, the three cases of radiation resulted in three distinctive patterns of resistance according to the independent variable, *dair/kair.* Moreover, the resistances of the air space showed a linear pattern when the emissivity of either walls is small (one or more walls are foiled). The resistances of the air space in these two cases were linearized as functions of *dair/kair.* Linear coefficients for the cases of two-walls- foiled and one-wall-foiled are 0.890and

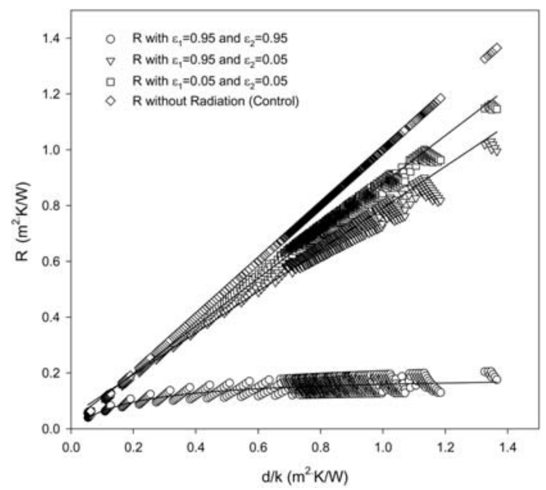

Fig. 4. Thermal resistance of the air space with various emissivities.

0.807 respectively. The coefficients for hyperbolic type heat resistance for no-wall-foiled air space were obtained from the non-linear regression function using Sigmaplot 9.0. 0.192 and 0.200were obtained for the 1/ $h_{r,i}$ of numerator and denominator in the equation (13).

These results were converted into the formulation of system *R*-values. From above results, thermal resistance of the multi-layered structure in equation (13) can be rearranged as

$$R_{wall} = R_{solid} + R_{air1} + R_{air2} + R_{air3}$$

$$\cong \sum_{i=1}^{N} \frac{d_i}{k_i} + 0.890 \sum_{i=1}^{N1} \frac{dair1_i}{kair_i} + 0.807 \sum_{i=1}^{N2} \frac{dair2}{kair_i}$$

$$+ \sum_{i=1}^{N3} \frac{0.192 \frac{dair3_i}{kair_i}}{0.200 + \frac{dair3_i}{kair_i}} \tag{14}$$

R_{solid} = the thermal resistance for the solid material ($m^2 \cdot K/W$)

R_{air1} = the thermal resistance for both-wall-foiled air spaces ($m^2 \cdot K/W$)

R_{air2} = the thermal resistance for one-wall-foiled air spaces ($m^2 \cdot K/W$)

R_{air3} = the thermal resistance for non-foiled air spaces ($m^2 \cdot K/W$)

N = the number of solid layers

N_1 = the number of both-wall-foiled air spaces

N_2 = the number of one-wall-foiled air spaces

N_3 = the number of the airspaces without foiled layers

d_i = the thickness of solid material of *i* th layer (m)

$dair1_i$ = the thickness of *i* th air space which has both-wall-foiled surfaces (m)

$dair2_i$ = the thickness of *i* th air space which has one-wall-foiled surfaces (m)

$dair3_i$ = the thickness of the i th non-foiled air space (m)

Assuming that the conductivity of the solid material, k_i, is 0.038 *W/m·K*, which is usually the case with insulating materials, and the conductivity of the air, $kair_i$, is 0.028 W/m·K, the resistance of the solid material in equation (14) can be described as:

$$R_{wall} = 0.026ths + 0.032tha1 + 0.029tha2 + \sum_{i=1}^{N3} \frac{0.192dair3_i}{5.537 + dair3_i} \quad (15)$$

ths = total thickness of solid layer (mm)

$tha1$ = total thickness of both-wall-foiled air space (mm)

$tha2$ = total thickness of one-wall-foiled air space (mm)

N_3 = number of the airspaces without foiled layer

$dair3_i$ = thickness of the i th non-foiled air space (mm)

3. Evaluation and refinement of the simplified equation using computer program

In the previous section, a model was constructed to represent heat transfer through a multi-layered wall structure. The model was based on heat transfer theories, so it depends on the assumptions that could generate errors. For example, the model assumed that the conductivity of the air was constant at 0.028 W/m·K. In the real world, however, the conductivity of air is a function of its temperature. These kinds of errors should be minor ones, but they might accumulate and create huge disagreements between the model and real situations. For these reasons, evaluation and refinement of the model is necessary. To evaluatethe model, a computer program including all parameters which could affect heat transfer was constructed. BASIC program WALLBG7.BAS was constructed. 96 different cases of multi-layered walls were arbitrary chosen with various constructions (Choi, 2004). Using these data, WALLBG7. BAS created *R*-values, total thickness of solid layers, total thickness of both-wall-foiled layers, total thickness of one-side-foiled layers, the number of the airspaces without foiled layers and the thickness of each non-foiled air space. *R*-values generated by this program were compared with the *R*-values calculated using the simplified equation (15).

The results were shown in Table 2. The errors in the *R*-values between the computer model and the simplified model were less than 20%, which means that the simplified model successfully predicts the system *R*-value. Except for four cases, however, the percentage errors, which were calculated by (*R*-value by computer program-*R*-value by simplified model) / (*R*-value by computer program) × 100, showed positive numbers. This implies that the simplified model generally underestimates the *R*-value. This model was refined by fitting each coefficient in equation (15). First, the coefficients for the linear part of the equation (15) were fitted by multi-variable regression.

The coefficients for total thickness of solid wall (*ths*), total thickness of both-wall-foiled air space (*tha*1), and total thickness of one-side-foiled air space (*tha*2) were obtained by multi-variable linear regression. The BASIC program MVLR.BAS was constructed for the multi-variable linear regression. Among 96 data sets generated by computer model, 36 data sets, which contained foiled layers, were selected for linear regression. Values of 0.027, 0.039 and 0.037 were obtained for the coefficients for total thickness of solid wall (*ths*), total thickness of both-wall-foiled air space (*tha*1), and total thickness of one-side-foiled air space (*tha*2), respectively. Then the non-linear part of the equation was fitted to data for the coefficient of the thickness of each none-foiled air space. MVNLR.BAS, a modified version of MVLR.BAS, which contained an additional loop to solve the non-linear problem, was used for regression. Out of the 96 data sets generated by computer model, 30 data sets, which constructed only with none foiled walls, were used for the regression. For non-linear part of the equation (22), 0.217 and 5.918 were obtained for the coefficients of numerator and denominator, respectively. With these results, equation (23) could be written as follows:

$$R_{wall} = 0.027ths + 0.039tha1 + 0.037tha2 + \sum_{i=1}^{N3} \frac{0.217dair3_i}{5.918 + dair3_i} \quad (16)$$

For the verification of the model, the R-values generated by computer model (See Table 2) were again compared with the *R*-values calculated by equation (16). As shown in Table 3, the percentage error in *R*-value was within 10%. Furthermore, the percentage errors were distributed positive and negative numbers, which implies that the refined model significantly improved the accuracy of the simulation.

The major drawback of this equation is that thermal resistance generated by the solid layer cannot be differentiated between each solid material. Thus, thermal resistances per inch of each solid materials, which is summarized in Table 4, were added to the equation (16) to yield

$$R_{wall} = \sum_{i=1}^{N} R_i ths_i + 0.039tha1 + 0.037tha2 + \sum_{i=1}^{N3} \frac{0.217dair3_i}{5.198 + dair3_i} \quad (17)$$

where R_i is the thermal resistance per milimeter for the solid material of the i th layer and ths_i is the thickness of solid material of i th layer. A short table of conduction heat transfer coefficients for common insulating materials used in packaging is shown in Table 4.

Table 2. Comparison of *R*-values in equation (15) with the computer model

Data Set	R_{wall}	*R* by Eq. (15)		Data Set	R_{wall}	R by Eq. (15)	
		R_{wall}	% Error			R_{wall}	% Error
1	1.039	1.003	3.5	49	1.571	1.585	–0.9
2	2.074	2.005	3.3	50	2.849	2.713	4.8
3	2.163	2.079	3.8	51	1.629	1.585	2.7
4	2.345	2.228	5.0	52	1.686	1.585	6.0
5	3.735	3.526	5.6	53	2.633	2.505	4.9
6	0.787	0.683	13.2	54	2.945	2.787	5.3
7	1.716	1.484	13.5	55	2.239	2.163	3.4
8	1.898	1.558	17.9	56	2.239	2.163	3.4
9	1.822	1.558	14.5	57	0.493	0.465	5.8
10	1.822	1.558	14.4	58	0.879	0.825	6.1
11	1.719	1.410	18.0	59	0.879	0.825	6.1
12	1.535	1.336	12.9	60	0.879	0.825	6.1
13	1.032	1.003	2.8	61	0.879	0.825	6.1
14	2.064	2.005	2.8	62	1.662	1.616	2.8
15	2.164	2.087	3.5	63	2.849	2.713	4.8
16	2.356	2.251	4.4	64	1.701	1.712	–0.7
17	3.740	3.581	4.3	65	2.097	1.974	5.9
18	0.806	0.722	10.4	66	2.633	2.505	4.9
19	1.777	1.585	10.8	67	1.032	1.003	2.8
20	1.968	1.667	15.3	68	2.064	2.005	2.8
21	1.883	1.667	11.5	69	2.132	2.066	3.1
22	1.882	1.667	11.4	70	2.164	2.087	3.5
23	1.782	1.503	15.7	71	3.313	3.202	3.4
24	1.588	1.422	10.5	72	0.613	0.535	12.7
25	1.032	1.003	2.8	73	0.940	0.853	9.3
26	2.064	2.005	2.8	74	1.126	0.997	11.4
27	2.164	2.087	3.5	75	0.940	0.853	9.3
28	2.356	2.251	4.4	76	0.879	0.825	6.1
29	3.740	3.542	5.3	77	1.087	0.952	12.4
30	0.805	0.714	11.2	78	0.839	0.776	7.5
31	1.772	1.570	11.4	79	1.571	1.585	–0.9
32	1.946	1.620	16.7	80	2.909	2.741	5.8
33	1.864	1.636	12.2	81	1.971	1.771	10.1
34	1.850	1.613	12.8	82	1.870	1.676	10.4
35	1.737	1.457	16.1	83	2.880	2.631	8.6
36	1.541	1.359	11.8	84	3.671	3.270	10.9
37	1.032	1.003	2.8	85	2.239	2.163	3.4
38	2.064	2.005	2.8	86	2.801	2.824	–0.8
39	2.132	2.066	3.1	87	0.619	0.558	9.8
40	2.132	2.066	3.1	88	1.131	1.012	10.5
41	3.313	3.202	3.4	89	1.118	0.966	13.6
42	0.493	0.465	5.8	90	0.879	0.825	6.1
43	0.879	0.825	6.1	91	1.138	1.012	11.0
44	0.879	0.825	6.1	92	1.914	1.773	7.4
45	0.879	0.825	6.1	93	3.034	2.819	7.1
46	0.879	0.825	6.1	94	2.238	1.968	12.1
47	0.879	0.825	6.1	95	2.584	2.340	9.4
48	0.809	0.762	5.8	96	2.907	2.629	9.6

Table 3. Comparison of *R*-values in equation (16) with the computer model

Data Set	R_{wall}	*R* by Eq. (16)		Data Set	R_{wall}	R by Eq. (16)	
		R_{wall}	% Error			R_{wall}	% Error
1	1.039	1.032	0.677	49	1.571	1.659	−5.565
2	2.074	2.064	0.464	50	2.849	2.820	1.012
3	2.163	2.157	0.240	51	1.629	1.659	−1.860
4	2.345	2.344	0.040	52	1.686	1.659	1.625
5	3.735	3.750	−0.389	53	2.633	2.606	1.055
6	0.787	0.789	−0.280	54	2.945	2.921	0.809
7	1.716	1.751	−2.000	55	2.239	2.240	−0.016
8	1.898	1.844	2.834	56	2.239	2.240	−0.016
9	1.822	1.844	−1.186	57	0.493	0.487	1.210
10	1.822	1.844	−1.233	58	0.879	0.867	1.331
11	1.719	1.657	3.607	59	0.879	0.867	1.331
12	1.535	1.564	−1.900	60	0.879	0.867	1.331
13	1.032	1.032	0.001	61	0.879	0.867	1.331
14	2.064	2.064	−0.002	62	1.662	1.694	−1.889
15	2.164	2.162	0.092	63	2.849	2.820	1.012
16	2.356	2.358	−0.083	64	1.701	1.803	−5.973
17	3.740	3.781	−1.093	65	2.097	2.074	1.106
18	0.806	0.811	−0.671	66	2.633	2.606	1.055
19	1.777	1.809	−1.773	67	1.032	1.032	0.001
20	1.968	1.907	3.136	68	2.064	2.064	−0.002
21	1.883	1.907	−1.261	69	2.132	2.129	0.142
22	1.882	1.907	−1.288	70	2.164	2.162	0.092
23	1.782	1.711	4.010	71	3.313	3.309	0.125
24	1.588	1.613	−1.576	72	0.613	0.602	1.752
25	1.032	1.032	0.001	73	0.940	0.924	1.762
26	2.064	2.064	−0.002	74	1.126	1.115	0.958
27	2.164	2.162	0.092	75	0.940	0.924	1.762
28	2.356	2.358	−0.083	76	0.879	0.867	1.331
29	3.740	3.759	−0.495	77	1.087	1.069	1.702
30	0.805	0.807	−0.277	78	0.839	0.825	1.636
31	1.772	1.800	−1.574	79	1.571	1.659	−5.565
32	1.946	1.880	3.390	80	2.909	2.877	1.108
33	1.864	1.889	−1.313	81	1.971	1.950	1.074
34	1.850	1.875	−1.390	82	1.870	1.833	1.955
35	1.737	1.684	3.018	83	2.880	2.841	1.350
36	1.541	1.577	−2.352	84	3.671	3.632	1.063
37	1.032	1.032	0.001	85	2.239	2.240	−0.016
38	2.064	2.064	−0.002	86	2.801	3.043	−8.637
39	2.132	2.129	0.142	87	0.619	0.616	0.551
40	2.132	2.129	0.142	88	1.131	1.124	0.598
41	3.313	3.309	0.125	89	1.118	1.097	1.912
42	0.493	0.487	1.210	90	0.879	0.867	1.331
43	0.879	0.867	1.331	91	1.138	1.124	1.257
44	0.879	0.867	1.331	92	1.914	1.940	−1.353
45	0.879	0.867	1.331	93	3.034	3.004	1.015
46	0.879	0.867	1.331	94	2.238	2.209	1.279
47	0.879	0.867	1.331	95	2.584	2.580	0.126
48	0.809	0.797	1.510	96	2.907	2.860	1.614

Table 4. Thermal resistance of various insulation materials per milimeter (Kositruangchai, 2003)

Materials	Thermal resistance per inch at 23C
	R_i (m^2·K/W·mm)
Air	0.0386
Corrugated board	0.0165
EPS foam	0.0262
Polyurethane	0.0321
Vacuum Insulating Panel	0.0694 – 0.2038

Conclusion

One-dimensional heat transfer through the multi-layered insulating packaged were investigated using mathematical models. From Grashof number calculation, it was confirmed that the heat transfer in the air space thinner than 13 mm was dominated by conduction rather than convection. The heat transfer in the air space between the layers was also substantially affected by radiation. The emissivities of the surfaces were very important to characterize the radiation through the air space between the surfaces. If either side of the surface has a low emissivity (one or both surface foiled), thermal resistance of the air space increased dramatically. Otherwise, the air space between the layer provided a low thermal resistance. These factors should be considered to design the multi-layered wall structure. All of these factors were reflected in the equation (17). Using equation (17), one can effectively design the multi-layered structure for the insulating package.

Equation(17) assumed the heat transfer through the insulating package as one-dimensional phenomenon. As mentioned in introduction, more factors involve in the heat transfer through the real insulating package: the three dimensional geometry of the insulating package, the geometry of the product itself, the contact between the product and the insulating package, etc. These factors should be considered to explain the heat transfer through the entire insulating package.

However, this study provided the solution for the thermal resistance of the complex wall structure. In other words, the result of this study can be combined with other factors to generate the comprehensive model to predict the performance of the insulating package. In fact, equation (17) were applied to the model which considered all other factors of the entire insulating package and resultedin a good prediction (4).

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