Phase Offset Enumeration Method with Error Detection and Its Application to Synchronization of PN Sequences

Young-Joon Song

Abstract

It is important to know phase offsets of PN(Pseudo Noise) sequences in spread spectrum communications since the acquisition is equivalent to making a phase offset between a receiving PN sequence and a PN sequence of local PN generator be identical. In this paper, a phase offset enumeration method for PN sequences with error detection, and its application to the synchronization are proposed. The phase offset enumeration for an n-tuple PN sequence and its error detection are performed when one period of the sequence is received. Once the phase offset of the receiving sequence is calculated, we can easily accomplish the synchronization by initializing shift registers of a local PN generator according to the phase offset value. The mean acquisition time performance of the proposed scheme was derived analytically. Since this synchronization scheme can be realized by using simple circuit and acquires very rapid acquisition in high SNR but shows performance degradation in low SNR, it can be especially useful in indoor and office environments.

Key words : PN Sequence, Phase Offset Enumeration, Error Detection, Synchronization.

Ⅰ. Introduction

It is important to know phase offsets of PN(Pseudo Noise) sequences in spread spectrum communications since the acquisition is equivalent to making a phase offset between a receiving PN sequence and a PN sequence of local PN generator in the receiver be identical[1]. The conventional synchronization method using serial search[1] shows very stable performance and can be easily realized. Thus the method is the most widely being used in spread spectrum systems. However this method requires long acquisition time even in high SNR environment. In this paper, a phase offset enumeration method for PN sequences with error detection, and its application to the synchronization are proposed to reduce the mean acquisition time especially in high SNR environment.

This work extends our previous analysis for the phase offset enumeration to the code acquisition of PN code [2]. The phase offset enumeration for an n-tuple PN sequence and its error detection are performed when one period of the sequence is received. Once the phase offset of the receiving sequence is calculated, we can easily accomplish the synchronization by initializing shift registers of a local PN generator according to the phase offset value. The mean acquisition time of the proposed synchronization scheme is derived analytically, and we see that the method acquires very fast acquisition in the high SNR(Signal-to-Noise Ratio) environment.

This paper is organized as follows. Section Ⅱ gives some definitions concerning phase offset enumeration. And phase offset enumeration method with error detection for PN sequences are presented in Ⅲ. A synchronization scheme using phase offsets of n-tuple PN sequences is discussed in Ⅳ. Section Ⅴ concludes our study with summary.

Ⅱ. Definitions

Let $\boldsymbol{S}$ be the set of all n-tuple binary sequences. We define a cyclic shift to the right operator T : $\boldsymbol{S} \to \boldsymbol{S}$ by

$$\begin{aligned} \boldsymbol{C} &= (C_0, C_1, \ldots, C_{n-2}, C_{n-1}) \\ T\boldsymbol{C} &= (C_{n-1}, C_0, \ldots, C_{n-3}, C_{n-2}) \\ &\vdots \\ T^i\boldsymbol{C} &= (C_{n-i}, C_{n-i+1}, \ldots, C_{n-i-2}, C_{n-i-1}) \\ &\vdots \\ T^{n-1}\boldsymbol{C} &= (C_1, C_2, \ldots, C_{n-1}, C_0) \end{aligned} \tag{1}$$

for every sequence $\boldsymbol{C} = (C_0, C_1, \cdots, C_{n-1}) \in \boldsymbol{S}$. Here we define $T^0\boldsymbol{C} = \boldsymbol{C}$. We construct the following polynomial $C(x)$ corresponding to a sequence $T^j\boldsymbol{C} \in \boldsymbol{S}$.

$$C(x) = C_{n-j} + C_{n-j+1}x + \cdots + C_{n-j-1}x^{n-1} \tag{2}$$

We define a phase offset evaluation function A^l : $\boldsymbol{S} \to$

Manuscript received December 14, 2004 ; revised March 11, 2005. (ID No. 20041214-037J)
Department of Electronic Engineering at Kumoh National Institute of Technology, Gumi, Gyungbuk, Korea.

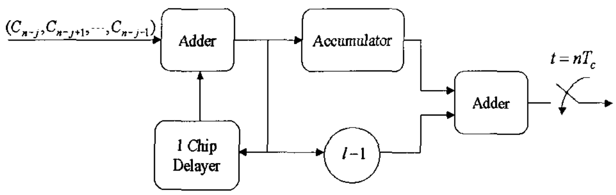

Fig. 1. Circuit to calculate offsets of binary sequences using the phase offset evaluation function with weight l.

$\boldsymbol{Z}$ by

$$A^l(T^jC)=\frac{d}{dx}x^lC(x)\Big|_{x=1} \tag{3}$$

for all $T^j\,C\in S$, where Z is the set of all integer numbers and l is an integer. Here we call the integer l the weight of the phase offset evaluation function which can be constructed by using Fig. 1. The sequence T^jC is said to be a reference sequence of the phase offset evaluation function with weight l if it satisfies $A^l(T^iC)\equiv 0(\text{mod } n)$. The reference sequence has a value of $0(\text{mod } n)$ of $A^l(T^iC)$.

Ⅲ. Phase Offset Enumeration with Error Detection

Let $f(x)=f_0+f_1x+\cdots+f_mx^m$ be a primitive polynomial of degree m over GF(q) and $\{C_i\}$ be the binary sequence generated by $f(x)$. The period of the sequence is the smallest positive integer n for which $f(x)$ divides $1-x^n$. Thus the period of the maximal legnth sequence generated by $f(x)$ is $n=2^n-1$ and under the conditions stated[3],[4] we have

$$\begin{aligned}\frac{1}{f(x)}&=\sum_{i=0}^{\infty}C_ix^i\\&=(C_0+C_1x+\cdots+C_{n-1}x^{n-1})\\&\quad+x^{n-1}(C_0+C_1x+\cdots+C_{n-1}x^{n-1})\\&\quad+x^{2(n-1)}(C_0+C_1x+\cdots+C_{n-1}x^{n-1})\\&\quad+\cdots\\&=(C_0+C_1x+\cdots+C_{n-1}x^{n-1})(1+x^n+x^{2n}+\cdots)\\&=C(x)\frac{1}{1-x^n}\end{aligned} \tag{4}$$

and

$$\begin{aligned}C(x)f(x)&=(x^n-1)\\&\equiv 0\,(\text{mod } x^n-1)\end{aligned} \tag{5}$$

Note that $C(x)f(x)$ is zero cyclic code polynomial in GF(q)[x]/(x^n-1). Thus by the definition of a cyclic code [5]~[7], for $C(x)f(x)$ to be a cyclic code polynomial, $C(x)$ or $f(x)$ should also be a cyclic code polynomial.

Now let $g(x)$ be a generator polynomial of cyclic code in GF(2)[x]/(x^n-1) whose degree is less than n. If $f(x)$ is not a code polynomial of the cyclic code, then $C(x)$ is a cyclic code of length n generated by $g(x)$. If $f(x)$ is a code polynomial of the cyclic code, then $C(x)$ is a cyclic code polynomial or not.

It is necessary to note that if the Hamming weight of $\mathbf{f}=(f_0, f_1, \cdots, f_m)$, $w(\mathbf{f})$, is less than the minimum distance of the cyclic code of length n generated by $g(x)$, then $C(x)$ is a code polynomial of the cyclic code. Thus choosing such a generator polynomial always make an n-tuple PN sequence be a cyclic code of code length n. Hence, we can detect errors occurred in the PN sequence by simply checking the syndrome with no redundancy since the n-tuple PN sequence itself is the cyclic code of length n generated by $g(x)$.

Exmaple 1 : Consider the PN sequence generated by $f(x)=x^5+x^2+1$. Then the n-tuple PN sequence $\boldsymbol{C}$ generated by $f(x)$ with initial shift register states all ones becomes

$$\boldsymbol{C}=(1,1,1,0,0,0,1,1,0,1,1,1,0,1,0,1,0,0,0,0,1,0,0,1,0,1,1,0,0,1,1) \tag{6}$$

where $N=2^5-1=31$. Now consider the cyclic code generated by the following generator polynomial.

$$\begin{aligned}g(x)&=x^{25}+x^{24}+x^{21}+x^{19}+x^{18}+x^{16}+x^{15}+x^{14}\\&\quad+x^{13}+x^{11}+x^9+x^5+x^2+x+1\end{aligned} \tag{7}$$

Then, the minimum distance of the code is d_{nim}=15, and $A_0=1$, $A_{15}=31$, $A_{16}=31$, $A_{31}=1$ where A_i, $0\le i\le n$, are called the weight distribution of the code. Since $w(\mathbf{f})$=3, we know that $\mathbf{f}$ is not a code word and $\boldsymbol{C}$ should be a cyclic code word for $f(x)C(x)$ to be zero code polynomial. By the definition of the cyclic code, we see that $T^i\boldsymbol{C}$, $0\le i\le 30$ are also code words of the cyclic code.

Example 2 : Now consider another PN sequence generated by $f(x)=x^4+x+1$. Then the n-tuple PN sequence generated by $f(x)$ with initial shift register states all ones becomes C=(1,1,1,0,0,0,1,0,0,1,1,0,1,0,1), where $n=2^4-1=15$. If we choose the following generator polynomial.

$$g(x)=x^{10}+x^8+x^5+x^4+x^2+x+1 \tag{8}$$

Then d_{nim}= 7 and $A_0=1$, $A_7=15$, $A_8=15$, $A_{15}=1$. Since $w(\mathbf{f})$=3, $\boldsymbol{C}$ is a cyclic code with Hamming weight "8".

If the PN sequence is used for BSC(Binary Symmetric Channel), the probability of undetectable error becomes[5]

$$P_u(E)=\sum_{i=d_{min}}^{n}A_ip^i(1-p)^{n-i} \tag{9}$$

where p is the transition probability of the BSC.

The phase offset of the PN sequence is given by [2]

$$2\cdot[\mathrm{A}^l(T^{i+j}C)-\mathrm{A}^l(T^iC)]\equiv j(\mathrm{mod}\,n). \tag{10}$$

Since the phase offset evaluation function of (3) can be implemented by the following circuit[2], we can easily obtain the phase offset of the n-tuple PN sequence by using (10), where T_c is a chip time duration of the PN sequence.

Ⅳ. Application to Code Acquisition

From Ⅲ, we know that the PN sequence generated by the characteristic polynomial $f(x)$ becomes a cyclic code of length n generated by $g(x)$ whose degree is less than n if the Hamming weight of $\mathbf{f}=(f_0, f_1, \cdots, f_m)$ is less than the minimum distance of the cyclic code. Thus, after choosing the proper $f(x)$ and $g(x)$, we can enumerate the phase offset of the sequence with error detection.

Once the phase offset value is given, shift registers of the PN generator in the receiver is initialized by using the value. This process is equivalent to the code acquisition. After this process, for the exact synchronization, the receiver goes to the code tracking mode. In the code tracking mode, if the phase offset value is not correct, the receiver fails synchronization and go back to the initial state. This time loss is given by penalty time by the false alarm. Fig. 2 denotes the synchronization algorithm using the phase offset calculation with error detection, and Fig. 3 shows the block diagram implementing the algorithm. In Fig. 3, $\{\hat{C}_i\}$ denotes the received PN sequence after demodulation, and "Sync" signal indicates the success or fail of synchronization.

One of the most important parameters to evaluate the performance of synchronization schemes is the mean acquisition time. Assume that the signal of a PN sequence is transmitted through the BSC, and let p be the transition probability. Then the probability of receiving one period of the PN sequence with no error is $(1-p)^n$. Now let t be the number of receiving n-tuple PN sequences. Then the probability of receiving an n-tuple PN sequence with no error is given by

$$P(t)=(1-p)^n(1-(1-p)^n)^{t-1} \tag{11}$$

and the average of t becomes

$$\begin{aligned}\bar{t}&=\sum_{t=1}^{\infty}t\cdot P(t)\\&=\sum_{t=1}^{\infty}t\cdot(1-p)^n\cdot(1-(1-p)^n)^{t-1}\cdot P(t).\\&=\frac{1}{(1-p)^n}\end{aligned} \tag{12}$$

The mean time to determine whether there are errors in the received n-tuple PN sequence is expressed as

$$\bar{T}_e=P_u(E)\cdot KnT_c \tag{13}$$

where K is the penalty due to undetectable error pattern by the cyclic decoder. Therefore, the mean acquisition time can be expressed as

$$\begin{aligned}\bar{T}_{acq}&=\bar{t}\cdot(nT_c+\bar{T}_e)\\&=\bar{t}\cdot nT_c\cdot(1+KP_u(E))\end{aligned} \tag{14}$$

The minimum mean acquisition time is obtained when p=0 and $P_u(E)$=0 and thus (14) is lower bounded by

$$\bar{T}_{acq}\geq nT_c \tag{15}$$

For the fair comparison, we consider the synchronization method using serial search since the method shows the almost same complexity as the proposed one.

The mean acquisition time of serial search method is given by

$$\bar{T}_{SE,acq}\approx\frac{(2-P_D)(1+KP_{FA})}{2P_D}(q\tau_D) \tag{16}$$

where

P_D : probability of detection
P_{FA} : probability of false alarm
K : penalty for a false alarm
τ_D : dwell time
q : number of cells to be searched.

When P_D=1 and P_{FA}=0 the performance of the method is bounded by (17) with q=n and τ_D=nT_C.

$$\bar{T}_{SE,acq}\geq\frac{q\tau_D}{2}=\frac{n}{2}\cdot(nT_c) \tag{17}$$

From (15) and (17), the proposed method can reduce the mean acquisition time by up to $n/2$ times than the serial search method.

When the generator polynomial (7) is used for error detection of the PN sequence generated by $f(x)$= x^5+x^2+1, the mean acquisition time performance over AWGN and Rayleigh fading channel are depicted in Fig. 4, where n=31, K=10, and BPSK with coherent detection are assumed. From Fig. 4, we see that it acquires very rapid acquisition especially in high SNR environment but shows performance degradation in low SNR environment. Since the complexity of the proposed method is almost same as that of serial search method, it would be beneficial to use the proposed method in

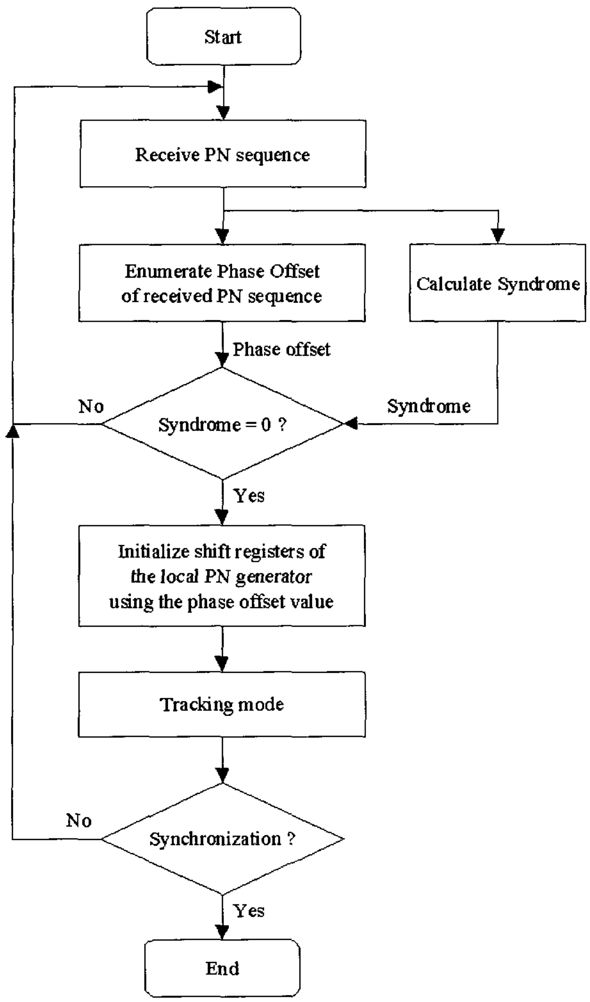

Fig. 2. Synchronization algorithm using phase offset with error detection.

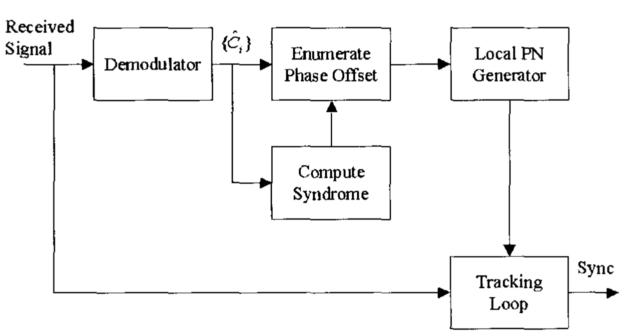

Fig. 3. Synchronization block diagram implementing the synchronization algorithm using phase offsets of PN sequences with error detection.

high SNR environment such as indoor and office environment.

V. Conclusion

In this paper, we proposed a synchronization scheme

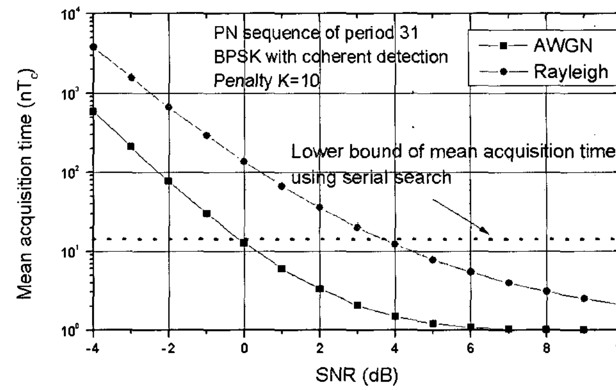

Fig. 4. Mean acquisition time for PN sequence of period 31 over AWGN and Rayleigh fading channel with the coherent BPSK when the generator polynomial by (7) is used for error detection.

using phase offsets of PN sequences with error detection. An error detection of phase offsets for PN sequences using the property of cyclic code, and its application to synchronization were discussed. Once the phase offset of a PN sequence is calculated, we can easily accomplish the synchronization by initializing shift registers of a local PN generator according to the phase offset value. The mean acquisition time performance of the proposed scheme was derived analytically and we found that the proposed method can reduce the mean acquisition time by up to $n/2$ times than the serial search method, where n is the period of the PN sequence. Since this synchronization scheme can be realized by using simple circuit and acquires very rapid acquisition in high SNR, it can be useful in indoor and office environments. Based on this research results we will further extend our work to complement the performance degradation in low SNR.

This paper was supported by Research Fund, Kumoh National Institute of Technology.

Reference

[1] J. K. Holmes, C. C. Chen, "Acquisition time performance of PN spread-spectrum system", *IEEE Trans. on Comm.*, vol. Com-25, no. 8, pp. 778-783, Aug. 1977.

[2] Y. Y. Han, Y. J. Song, "Phase offset of binary code and its application to the CDMA mobile communications", *IEICE Trans. Fundamentals*, vol. E81-A, no. 6, pp. 1145-1151, Jun. 1998.

[3] N. Zieler, "Linear recurring sequence", *J. Soc. Appl. Math.*, pp. 31-48, Jul. 1959.

[4] S. W. Golomb, *Shift Register Sequences*, Holden Day, San Francisco, 1967.
[5] S. Lin, D. J. Costello, *Error Control Coding*, 2nd Edition, Prentice-Hall, 2004.
[6] E. R. Berlekamp, *Algebraic Coding Theory*, Mc Graw-Hill, 1968.
[7] S. B. Wicker, *Error Control Systems for Digital Co mmunication and Storage*, Prentice Hall, 1995.

Young-Joon Song

He received the B.S., M.S., and Ph.D. degrees in electronic communication engineering from Hanyang University, Seoul, Korea in 1987, 1994, and 1999, respectively. Since 2002 he has been teaching in the Department of Electronic Engineering at Kumoh National Institute of Technology. From 1994 to 2002, he served as a principle engineer at LG Electronics. His research interest includes wireless mobile communication systems, synchronization, sequence design, and coding theory.