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손실감도를 이용한 계통손실 최적화에 대하여

(On The Optimal Generation Using The Loss Sensitivities Derived by Angle Reference Transposition)

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요 약

전력계통을 효율적으로 운용하려면 관련량을 정확하고 신속히 계산하는 좋은 알고리즘이 필요하다. 최근 IEEE Transaction on Power System에 위상각 이동을 이용한 손실 최적화 알고리즘이 발표되었다. 동일한 손실최적화 문제를 본 논문에서는 Standard method of Lagrange Multiplier 기법을 적용하여 해석하였으며, 그 결과 저자들은 두 가지 방법이 수학적으로 동일함을 증명하였다.

Abstract

In this article, we apply the standard method of Lagrange multipliers to examine the algorithm in a recent IEEE publication which calculates the optimal generation for minimizing the system loss using loss sensitivities derived by angle reference transposition, and show that the two algorithms are mathematically the same.

Key Words : loss minimization, loss sensitivity, angle reference transposition, method of Lagrange multipliers

1. Introduction

In order to operate a power system in the most efficient and secure manner, it is important to have good algorithms which enable us to calculate the relevant quantities in a correct and fast way.

Recently, the second author has announced in [1] an algorithm of minimizing the system loss using loss sensitivities derived by angle reference transposition. In this article, we show that the same numerical result can be derived by the standard method of Lagrange multipliers and that the two algorithms are in fact mathematically the same, at least in analyzing the sample four-bus system dealt in [1].

For the convenience of the reader, we summarize here some basic facts about power system analysis and the data for the sample four-bus system dealt in [1] : In a power system of n buses, the power flow is governed by the following set of equations :

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$$P_k = V_k \sum_{m=1}^{n} V_m(G_{km}\cos\delta_{km} + B_{km}\sin\delta_{km}), \quad (1)$$
$$Q_k = V_k \sum_{m=1}^{n} V_m(G_{km}\sin\delta_{km} - B_{km}\cos\delta_{km})$$

for $k = 1, \cdots, n$. The system loss is simply the sum of all real powers in the system :

$$P_L := \sum_{i=1}^{n} P_{G_i} - P_D = P_1 + P_2 + \cdots + P_n. \quad (2)$$

Remark : The power flow equations for n buses consist of $4n$ variables and $2n$ equations. We may assume, however, that there are only $4n-1$ variables in the system for the following reason : Let $\varphi_i := \theta_i - \theta_{i+1}$ for $i = 1,2, \cdots, n-1$. Then θ_{km} is a linear combination of φ_i's. For example, $\theta_{24} = \varphi_2 + \varphi_3$ and $\theta_{31} = -\varphi_1 - \varphi_2$. A consequence of this simple observation is that any calculation done only with the power flow equations does not depend upon the reference of the angles, as was pointed out in various articles[2]. That is, only the relative values of the angles matter, not the absolute values. This remark holds in particular for the problem under consideration.

The sample four-bus system is depicted in Figure 1, whose line parameters are as in Table 1. We fix as in Table 2 the active and reactive powers at buses 3 and 4, the voltage magnitudes at buses 1 and 2, and the voltage angle at bus 3.

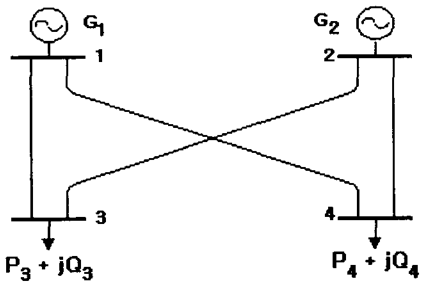

Fig. 1. Single Line Diagram of A Four-Bus System

Table 1. Line Parameters(p.u.)

from	to	R	X	Shunt Y
1	4	.00744	.0372	0.0775
1	3	.01008	.0504	0.1025
2	3	.00744	.0372	0.0775
2	4	.01272	.0636	0.1275

Table 2. Bus Data

Bus	P	Q	V	angle(deg)
1			*1.0*	
2			*1.0*	
3	*-2.20*	*-1.36*		*0*
4	*-2.80*	*-1.73*		

2. Optimization Using Loss Sensitivities

In this section, we summarize the mathematical structure of the optimization algorithm in [1] : Let f be a function of P, Q, V and θ in an arbitrary four-bus system. By the power flow equation (1), we can regard f as a function of V, θ only or, conversely, as a function of P, Q only. The partial derivatives of f with respect to V, θ or with respect to P, Q are related to one another via the following formula :

$$\begin{bmatrix} \partial f/\partial\theta_1 \\ \partial f/\partial\theta_2 \\ \partial f/\partial\theta_3 \\ \partial f/\partial\theta_4 \\ \partial f/\partial V_1 \\ \partial f/\partial V_2 \\ \partial f/\partial V_3 \\ \partial f/\partial V_4 \end{bmatrix} = J^T \begin{bmatrix} \partial f/\partial P_1 \\ \partial f/\partial P_2 \\ \partial f/\partial P_3 \\ \partial f/\partial P_4 \\ \partial f/\partial Q_1 \\ \partial f/\partial Q_2 \\ \partial f/\partial Q_3 \\ \partial f/\partial Q_4 \end{bmatrix} \quad (3)$$

where J^T is the transpose of the Jacobian matrix. For simplicity of notation for later use, we let $J^T[a,b,\cdots,c][d,e,\cdots,f]$ be the matrix obtained from the $a,b, \cdots, c$ rows and $d,e,\cdots,f$ columns of J^T.

Note that the power flow equation (1) with $n=4$ and the data specified in Table 2 is a system of 15 equations and 16 variables. (Here, we are regarding Table 2 as a set of 7 constraint equations.) So, in order to find a set of values for $P_1, P_2, \cdots, \theta_4$ which minimizes the system loss, we need to have one more equation.

The extra equation that is used in [1] is the following :

$$\frac{\partial P_L}{\partial P_1}=\frac{\partial P_L}{\partial P_2} \tag{4}$$

where $\frac{\partial P_L}{\partial P_i}$ for $i=1,2$ are the solutions of following matrix equation :

$$\begin{bmatrix} \partial P_L/\partial\theta_1 \\ \partial P_L/\partial\theta_2 \\ \partial P_L/\partial\theta_4 \\ \partial P_L/\partial V_3 \\ \partial P_L/\partial V_4 \end{bmatrix} = J^T[1,2,4,7,8][1,2,4,7,8] \begin{bmatrix} \partial P_L/\partial P_1 \\ \partial P_L/\partial P_2 \\ \partial P_L/\partial P_4 \\ \partial P_L/\partial Q_3 \\ \partial P_L/\partial Q_4 \end{bmatrix} \tag{5}$$

Since $P_L = P_1 + P_2 + P_3 + P_4$, the vector in the left hand side of the above equation is in fact the sum of the first four columns of the 5×8 matrix $J^T[1,2,4,7,8][1,2,3,4,5,6,7,8]$. $\partial P_L/\partial P_i$ is the solution of (5). By Cramer's rule [3] we have

$$\frac{\partial P_L}{\partial P_1}=\frac{det(J^T[1,2,4,7,8][1+2+3+4,2,4,7,8])}{det(J^T[1,2,4,7,8][1,2,4,7,8])},$$

$$\frac{\partial P_L}{\partial P_2}=\frac{det(J^T[1,2,4,7,8][1,1+2+3+4,4,7,8])}{det(J^T[1,2,4,7,8][1,2,4,7,8])}. \tag{6}$$

Hence, the equation (4) is equivalent to

$$det(J^T[1,2,4,7,8][1+2+3+4,2,4,7,8])$$
$$=det(J^T[1,2,4,7,8][1,1+2+3+4,4,7,8]). \tag{7}$$

The problem of optimal generation which minimizes the system loss is now equivalent to finding $P_1, P_2, \cdots, \theta_4$ which satisfy (1), Table 2, and (7), which consist of 16 equations in 16 variables. Note that (7) is an equation of V, θ at least in an explicit way.

3. Optimal Generation by The Standard Method of Lagrange Multipliers

In this section, we solve the same optimization problem of the previous section from a different point of view. Under the bus condition specified as in Table 2, we can reduce many variables for the problem of optimal generation under consideration and can rephrase the problem as follows :

Minimize

$$P_L=\sum_{k=1}^{4} V_k \sum_{m=1}^{4} V_m (G_{km}\cos\delta_{km}+B_{km}\sin\delta_{km})$$

subject to

$$-2.20 = V_3 \sum_{m=1}^{4} V_m (G_{3m}\cos\delta_{3m}+B_{3m}\sin\delta_{3m}),$$
$$-2.80 = V_4 \sum_{m=1}^{4} V_m (G_{4m}\cos\delta_{4m}+B_{4m}\sin\delta_{4m}),$$
$$-1.36 = V_3 \sum_{m=1}^{4} V_m (G_{3m}\sin\delta_{3m}-B_{3m}\cos\delta_{3m})$$
$$-1.73 = V_4 \sum_{m=1}^{4} V_m (G_{4m}\sin\delta_{4m}-B_{4m}\cos\delta_{4m}) \tag{8}$$

where $V_1 = V_2 = 1.0$ and $\theta_3 = 0$. Note that we do not consider at all the equations for Q_1 and Q_2. We can do so since the system loss P_L has no relevance at all to the reactive powers. Note also that there are 5 variables $V_3, V_4, \theta_1, \theta_2, \theta_4$ and 5 constraint equations. Once this form is known, we can apply the standard method of Lagrange multipliers, and now the problem is reduced to finding $V_3, V_4, \theta_1, \theta_2, \theta_4$ which satisfy the equations (8) and

$$\vec{0} = \vec{\nabla} P_L - \lambda_3 \vec{\nabla} P_3 - \lambda_4 \vec{\nabla} P_4 - \mu_3 \vec{\nabla} Q_3 - \mu_4 \vec{\nabla} Q_4 \qquad (9)$$

for some $\lambda_3, \lambda_4, \mu_3, \mu_4$ where

$$\vec{\nabla} = (\partial_{\theta_1}, \partial_{\theta_2}, \partial_{\theta_4}, \partial_{V_3}, \partial_{V_4}).$$

Note that (8) and (9) constitute a system of 9 variables and 9 equations. We used *Mathematica*® to solve this system of equations, and the solution is described in Table 3, which is in exact accordance with the results of [1].

Table 3. Optimization Result by The Standard Method of Lagrange Multipliers

Bus	P	Q	V	angle(rad)
1	2.7488	1.73027		0.04332
2	2.3369	1.42992		0.04332
3			0.960737	
4			0.943265	-0.01821

4. Comparison of The Two Methods

Now we provide a mathematical reasoning why we obtain the same numerical results by the two procedures. For this purpose, We need to recall two basic facts about the determinant function of square matrices [3,4] :

(i) Suppose that three square matrices A, B and C differ only in a single column, say r-th column, and that the r-th column of C is the sum of the r-th columns of A and B. Then

$$\det C = \det A + \det B. \qquad (10)$$

(ii) Suppose that B is obtained from A by switching two consecutive columns of A. Then

$$\det B = -\det A. \qquad (11)$$

Using (i) and (ii), we see that the equation (7) is equivalent to

$$\begin{aligned} & det(J^T[1,2,4,7,8][1+3,2,4,7,8]) \\ = \ & det(J^T[1,2,4,7,8][1,2+3,4,7,8]) \end{aligned} \qquad (12)$$

which immediately implies by (i) and (ii) again that

$$\begin{aligned} & det(J^T[1,2,4,7,8][1+2,3,4,7,8]) \\ = \ & det(J^T[1,2,4,7,8][1+2+3+4,3,4,7,8]) \qquad (13) \\ = \ & 0 \end{aligned}$$

This means $J^T[1,2,4,7,8][1+2+3+4,3,4,7,8]$ has rank less than 5, hence there is a linear combination of the five columns which is equal to $\vec{0}$ [4]. The implication is that the first column is a linear combination of the remaining four columns, assuming that the coefficient of the first column in the linear combination is not 0 which we believe is true in general. But this is simply the meaning of the equation (9). That is, the extra equations we use in each method are actually the same.

5. Conclusions

The authors examined the algorithm in a recent IEEE publication which calculates the optimal generation for minimizing the system loss using loss sensitivities derived by angle reference transposition, and also applied the standard method of Lagrange multipliers to the same optimization problem.

For the sample four-bus system under consideration, the two algorithms in obtaining the optimal power generation which minimizes the system loss are mathematically equivalent. For larger systems with different set of specified bus data, further analysis will be provided in another occasion.

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