논문번호 05-01-10

A Highly Robust Integral Optimal Variable Structure System

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Abstract

In this paper, a design of an integral augmented optimal variable structure system(IOVSS) is presented for the prescribed output control of uncertain SISO systems under persistent disturbances. This algorithm aims at removing the problems of the reaching phase by incorporating advanced optimal control theory. By means of an integral sliding surface, the reaching phase is completely removed, and the integral sliding surface can be defined from a given initial state to origin without any reaching phase. The ideal sliding dynamics of the integral sliding surface is obtained in the form of the state equation and is designed in an optimal sense by targeting the design of the integral sliding surface and equivalent control input. The corresponding control input is selected in order to generate the sliding mode on the predetermined integral sliding surface. As a result, the whole sliding output from a given initial state to origin is completely guaranteed against persistent disturbances. Moreover the prediction /predetermination of output is enabled, which helps in improving the performance over previously implemented VSS's. Through an illustrative example, the usefulness of the algorithm is shown.

요 약

본 연구는 불확실성이 존재하는 다이나믹 시스템을 고 강인성 및 성능 사전 결정 제어를 하기 위하여 적분 최적 가변구조 시스템을 설계한다. 제안된 제어기에서는 적분 슬라이딩 면을 이용 리칭 문제를 완전히 제거하여 시스템이 초기 값에서부터 바로 슬라이딩하여 외란과 불확실성에 무관하게 사전에 결정된 슬라이딩 면을 슬라이딩 모드 상태로 추종하므로 고 강인성 제어가 이루어진다. 적분 슬라이딩 면이 정의하는 이상 슬라이딩 동특성을 상태방정식 형으로 얻고, 고급 최적 제어 이론을 통하여 최적 의미로 설계한다. 이는 바로 슬라이딩 면과 등가 제어입력의 설계가 된다. 사전에 선정된 슬라이딩 면 위에 슬라이딩 모드를 발생할 제어입력을 설계하였다. 그 결과 외란과 불확정성에도 불구하고 주어진 초기 값에서부터 원점까지 전체 슬라이딩 출력이 완전하게 보장받는다. 더구나 기존의 최적 VSS에서는 설계 성능의 강인성을 보장받기 어려운데 반하여, 제안된 IOVSS에서는 실제 출력의 예측과 사전 결정이 가능하다. 예제를 들어 제안된 알고리듬의 성능을 검진한다.

Keywords - variable structure system, sliding mode control, robust control

I. Introduction

The problem of controlling uncertain dynamical systems under parameter variations and extraneous disturbances has been studied for a long time[1]-[5]. The theory of the variable structure system(VSS) or sliding mode control(SMC) can provide the effective means to solve this problem[1][2]. One of its essential advantages is the capacity of the controlled system to counteract

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接受日:2005年 5月 10日, 修正完了日:2005年 12月 19日

variations of parameters and external disturbances in the sliding mode on the sliding surface $s(t)=0$. The properly designed sliding surface can match desired output dynamics and performances. Many design algorithms including the linear(optimal control[6][7], eigenstructure assignment[8][9], geometric approach[10], differential geometric approach[11]), and nonlinear techniques have been reported. Moreover, an integral action also has augmented by the two groups[7][12]-[14]. The first strategy suggests improving the steady state performance[7][12][13] against the external disturbances in the digital implementation of the VSS, and the other one aims at reducing the chattering problems by filtering the discontinuous input[14]. Futhermore, with respect to the optimal VSS, other augmentation strategies include the optimal design of the sliding surface[6][7][9] and the optimization of the discontinuous VSS itself[1][16].

Unfortunately, most of the VSS's have the reaching phase which called the transient period which is the period till the representative point first touches the sliding surface from the beginning. During this reaching phase, the controlled systems may be sensitive to the parameter variations and external disturbances because the sliding mode is not realized[17]. And it is difficult to improve the performance in the sliding surface for the real output based on the above mentioned strategies. Moreover, introducing the integral action to the VSS without removing the reaching phase can inevitably cause the overshoot problems.

In the context of already established research work on VSS, very few studies deal with the problem of reaching phase. One mitigation strategy is the use of the high-gain feedback[18]. But, it has also some drawbacks for example sensitivity to the unmodelled dynamics and actuator saturation[17]. The adaptive rotating or shifting of the sliding surface is suggested to reduce the reaching phase problems in [2][19], and the sliding surface connected in segments to the origin from a given initial condition is also suggested[20]. But these different techniques and segmented sliding surface are applicable to only second order systems and their outputs are not predictable. In [21], the exponential term is added to the conventional linear sliding surface is order to make $s(t)=0$ at $t=0$. But, its resultant sliding dynamics becomes nonlinear.

In this paper, an integral augmented optimal variables structure system(IOVSS) is suggested for the control of uncertain n-th order SISO systems with the predetermination/prediction of output. The reaching phase is completely removed by the augmentation of an integral with special non zero initial value to the conventional sliding surface. Using the advanced optimal technique by minimizing the time-weighted performance index[25], the stationary linear sliding dynamics are obtained in the form of the state equation that also ensures the predetermination of output. A corresponding control is selected to completely guarantee the sliding mode on the every point of the predetermined integral sliding surface for protecting the designed optimal output from parameter variations and disturbances. By removing reaching phase and optimizing the VSS based on the advanced optimal theory, the advantages of the algorithm are discussed including the predetermination of output. Finally an example is presented to show the effectiveness of the algorithm in comparison to typical VSS having the conventional linear sliding surface.

II. Integral Optimal Variable Structure Systems

2.1. System Description and Backgrounds

The integral-augmented canonical system with $n+1$-th order is considered as

$$\dot{x}_i = x_{i+1}, \qquad i=1,2,\ldots,n-1 \tag{1a}$$

$$\dot{x}_n = -\sum_{i=1}^{n} a_i(t)\cdot x_i + b(t)\cdot u(t) + d(t) \quad x^0 = x(0) \tag{1b}$$

$$\dot{z} = x_1 \qquad z^0 = z(0) \tag{1c}$$

$$y = h\cdot x \qquad h = [1\ 0\ \ldots\ 0] \tag{1d}$$

where $x\in R^n$ is the state variable, and x_1 and z are the output and its integral, respectively, $a_i(t)$ and $b(t)$ are the system parameters, $d(t)$ is disturbance, and $u(t)$ is the control to be determined. To completely describe the system

(1a)-(1d), the assumption on the boundedness of the parameter variations and external disturbances is introduced as follows:

Assumption 1: *The system parameters and external disturbances are bounded as*

$$a_i(t) \in [a_i^- \quad a_i^+] \text{ for } i=1,2,\ldots,n \tag{2a}$$

$$b(t) \in [b^- \quad b^+] \quad b^- > 0 \tag{2b}$$

$$d(t) \in [d^- \quad d^+] \tag{2c}$$

where l^- and l^+, $l=a_i$, b and d are the below and upper bounds of each parameter.

Let $l^0 \in [l^- \quad l^+]$ imply each nominal value possibly estimated during the modelling process for controller design. From Assumption 1, the nominal system of (1) can be expressed as

$$\dot{x} = \Lambda \cdot x + \Gamma \cdot \nu \quad x^0 \tag{3a}$$

$$\dot{z} = x_1 \quad z^0 \tag{3b}$$

$$y = h \cdot x \tag{3c}$$

where ν is the continuous feedback input having the performance data as the component of $u(t)$, and the matrices $\Lambda \in R^{n\times n}$ and $\Gamma \in R^{n\times 1}$ $\Gamma \in R^{n\times 1}$ are as follows:

$$\Lambda = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \cdot & \cdot & \cdot & \ldots & \cdot \\ \cdot & \cdot & \cdot & \ldots & \cdot \\ \cdot & \cdot & \cdot & \ldots & \cdot \\ -a_1^0 & -a_2^0 & -a_3^0 & \ldots & -a_n^0 \end{bmatrix},$$

$$\Gamma = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ b^0 \end{bmatrix} \tag{4}$$

Only the information on the nominal system together with the bounds of the parameter variations in (2) is used in the design of the IOVSS controller. Using (3), the original system can be re-expressed in a compact form as

$$\dot{x} = \Lambda \cdot x + \Gamma \cdot [u(t) + e(x,t)] x^0 \tag{5a}$$

$$\dot{z} = x_1 \quad z^0 \tag{5b}$$

$$y = h \cdot x \tag{5c}$$

where $e(x,t)$ represents for the persistent disturbances or lumped uncertainties as

$$e(x,t) = \sum_{i=1}^{n} \Delta a_i(t) \cdot x_i + \Delta b(t) \cdot u(t) + \Delta d(t) \tag{6}$$

where

$$\Delta a_i(t) = (a_i(t) - a_i^0)/b^0$$

$$\Delta b(t) = (b(t) - b^0)/b^0$$

$$\Delta(t) = d(t)/b^0 \tag{7}$$

and is bounded due to Assumption 1. This term affects robustness and may cause instability and difficulty in designing the controller. So, these factors should be considered into account while designing a controller for practical purposes.

The control $u(t)$ of the proposed IOVSS is designed for the system (5a) with input composing of

$$u(t) = \nu(t) + \Delta\nu(t) \tag{8}$$

where $\nu(t)$ and $\Delta\nu(t)$ are designed with aid of advanced optimal technique and VSS theory, respectively. The objective of this design is to stabilize the system (5a) ensuring the prescribed optimal performance of $\nu(t)$ against all lumped uncertainties. The sense of the optimal is defined in the following

Definition 1: Optimality

For a time invariant linear system (3a), a feedback control $\nu = \phi(t)$ $\nu = \phi(t)$ is optimal in the sense of minimizing the time varying performance index $J: R^n \times R \times R^+ \rightarrow R$:

$$J(x,\nu,t) = \int_0^\infty L(x(t), \nu(t), t)\, dt \tag{9}$$

In order to optimize the control with respect to the time varying performance function of (9), we describe a Lemma 1 shortly. For the following results, define Hamiltonian for $p \in R^n$ as

$$H(x,p,\nu,t) = L(x,\nu,t) + p^T \cdot (\Lambda \cdot x + \Gamma \cdot \nu) \tag{10}$$

Let ∇ denote the derivative with respect to x

Lemma 1: *Consider (3a) with a performance functional (9). It is assumed that a function $V \in C^2: R^n \times R^+ \rightarrow R$ and $\phi: R^n \rightarrow R$ $\phi: R^n \rightarrow R$ exist such that*

$V(0,t)=0$ (11a)

$V(x,t)>0, \quad x\in R^n \quad \text{for} \quad x\neq 0$ (11b)

$\phi(0)=0$ (11c)

$\frac{\partial V(x,t)}{\partial t}+\nabla V(x,t)\cdot(\Lambda x+\Gamma\phi(x))<0$

for $x\in R^n, \quad x\neq 0$

and it is uniformly continuous (11d)

$$\frac{\partial V(x,t)}{\partial t}+\min_{\nu} H\left(x(t),\frac{\partial V(x,t)}{\partial t},\phi(x),t\right)=0 \quad (11e)$$

$$H\left(x(t),\frac{\partial V(x,t)}{\partial t},\phi(x),t\right)\geq 0 \quad \text{for} \quad \in R^n \quad (11f)$$

Then, with the feedback control $\nu(x)=\phi(x)$, *the solution* $x(t), \quad t\geq 0$ *of the closed loop system* $\dot{x}=\Lambda\cdot x+\Gamma\cdot\phi(x)$ *for* x^0 *is asymptotically stable. Furthermore, the feedback control* $\nu(x)=\phi(x)$ *minimize* $J(x,v,t)$ *in the sense that*

$$J(x,\phi,t)=\min_{\nu} J(x,\nu,t) \quad (12)$$

where $\nu\in U(x)\equiv\{\nu(\cdot)|\nu(\cdot)\ \textit{stabilizes}\ x(\cdot)\ \textit{given by}\ (11a)\ \textit{such that}\ \lim_{t\to\infty}V(x,t)=0\}$ (13)

and

$$J(x,\phi,t)=V(x^0,0) \quad (14)$$

Proof: See Appendix

It should be particularly noted that (11e) is familiar with the Hamilton-Jacobi-Belman(HJB) equation charactering the optimal control for time-varying systems on both finite and infinite interval.

Returning the main problem under discussion, the previous VSS's including the optimal VSS do not provide the means of guarantying the performance pre-designed in the sliding surface because of the reaching phase. The reason why the reaching phase exists will be briefly discussed for the most popular conventional sliding surface $S: R^n\to R$ in form of the linear combination of the full states as follows[1][2]:

$$s(x)=\sum_{i=1}^{n}c_i\cdot x_i \quad (15)$$

Since the sliding surface of (15) is geometrically defined at the fixed state satisfying $s(x)=0$ in the state space, it can not vary according to the initial conditions. Hence for the initial condition such that , reaching phase inevitably appears. The steps required for removing the reaching phase are as follows: The sliding surface should be defined from any given initial condition in the state space, and the control should be enabled to establish the sliding mode on every point on the predetermined sliding surface.

As the first design step, an integral-augmented sliding surface is suggested and the advanced optimal design technique is presented

2.2. Integral-Augmented Sliding Surface and its Optimal Design

Assuming the initial condition of x^0 is known, an integral-augmented sliding surface $s: R^{n+1}\to R$ to solve the reaching phase problems is defined as follows:

$$s(x,z)=C^T\cdot x(t)+c_0 z(t) = \sum_{i=1}^{n}c_i\cdot x_i(t)+c_0\cdot z(t) \quad (16)$$

where $z(t)=\int_0^t x_1(\tau)d\tau+z^0 \quad z^0=-c_0^{-1}\sum_{i=1}^{n}c_i\cdot x_i^0$ (17)

$c_n=1 \qquad c_i=constant$

which is modified from [7][12] by further considering the initial condition for the integral state in order to cope with the problems of the reaching phase. In [7][12], since the initial condition for the integral state is not taken into account, i.e., $z^0=0$, the reaching phase problems still exist and an inevitable overshoot problem may occur in outputs because the accumulated integral in the integral sliding surface has to be re-regulated to zero in steady state. However, the suggested integral sliding surface of (16) obviously satisfies

$$s(x^0,z^0)=0 \quad at \quad t=0 \quad (18)$$

for any given initial condition because the integral sliding surface (16) is function of a given initial condition explicitly. Hence, there is no reaching phase, only the controlled system can slide from a given initial condition. Using $\dot{s}(x,z)=0$ and (3a), the equivalent control as the candidate for $\nu(t)$ can be obtained as

$$\nu_{eq}(t)=-\sum_{i=1}^{n}(c_{i-1}-a_i^0)/b^0\cdot x_i = \overline{C}^T(c_i,c_0)\cdot x \tag{19}$$

where $$\overline{C}^T=\left[(c_0-a_1^0)/b^0 \quad (c_1-a_2^0)/b^0 \ldots \quad (c_{n-1}-a_n^0)/b^0\right] \tag{20}$$

and n-th order full sliding dynamics of (16) defined from a given initial state to origin can be expressed as

$$\dot{x}_i=x_{i+1},\ i=1,2,\ldots,n-1 \tag{21a}$$

$$\dot{x}_n=-\sum_{i=1}^{n}c_{i-1}\cdot x_i \qquad x^0 \tag{21b}$$

or in a compact form as

$$\dot{x}_n=\Phi(c_i,c_0)\cdot x \qquad x^0 \tag{22}$$

where

$$\Phi(c_i,c_0)=\begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \cdot & \cdot & \cdot & \ldots & \cdot \\ \cdot & \cdot & \cdot & \ldots & \cdot \\ \cdot & \cdot & \cdot & \ldots & \cdot \\ -c_0^0 & -c_1^0 & -c_2^0 & \ldots & -c_{n-1}^0 \end{bmatrix} \tag{23}$$

which is the reduced n-th order dynamical interpretation of the integral-augmented sliding surface, i.e. ideal sliding dynamics, therefore its solution of (22) becomes the set of the state of the integral-augmented sliding surface, so called integral manifold[24], and the stability of the integral sliding surface directly depends on the stability of the sliding dynamics (22). Because of the invariance property of the sliding mode against the parameter variations and disturbances in the VSS, theoretically, the output can be predetermined and predicted by using the solution of (22) for any given initial condition if the sliding mode is guaranteed for the whole trajectory.

Since the pole assignment to (22) is identical to the determination of the desired coefficient $c_i,\ i=0,1,\ldots,n-1$ in (16), the optimal technique of [25] is introduced for the stable choice of c_i by rearranging (22) into

$$\dot{x}=\Lambda\cdot x+\Gamma\cdot\nu_{eq}(\overline{C},x) \qquad x^0 \tag{24}$$

where Λ has the relationship of

$$\Lambda=\Phi-\Gamma\cdot\overline{C}(c_i,c_0) \tag{25}$$

or

$$a_{i+1}^0=c_i+b^0\cdot\overline{c}+i+1,\quad i=0,1,\ldots,n-1 \tag{26}$$

Since (24) is equal to (3a), the choice of the feedback gain in (24) implies the performance design for the nominal system. then, the time multiplied quadratic performance index $J(x,\nu_{eq})$ is chosen as follows[25]:

$$J=\int_0^\infty (t^N x^T Qx+\gamma\nu^2)dt \tag{27}$$

where $Q=Q^T\in R^{n\times n}\geq 0$ is the weighting matrix, $\nu\in R$ is the scalar weighting factor for inputs, and N is a non negative integer as a time-weighting factor. The weighting matrix Q can be chosen as

$$Q=W^TW \tag{28}$$

where the pair (Λ,W) is observable. Kalman suggests that (27) could lead to a constant control law[26]. As one increases N in (27), the constant optimal gain minimizing the time-multiplied performance index can improve the transient dynamics and is determined by Theorem 1

Theorem 1: The optimal gain matrix $\overline{C}(c_i,c_0)$ for (16) with respect to (27) is given by

$$\overline{C}=\frac{1}{\gamma}\Gamma^T\sum_{i=1}^{N+1}(P_i\cdot L_i)\cdot L_{N+1}^{-1} \tag{29}$$

where P_i and L_i are the solution of the extended algebraic matrix Riccati equations:

$$\Phi^TP_1+P_1\Phi^T+N!Q=0 \tag{30a}$$

$$\Phi^TP_{i+1}+P_{i+1}\Phi^T+P_i=0,\ i=1,2,\ldots,N-1 \tag{30b}$$

$$\Phi^TP_{N+1}+P_{N+1}\Phi^T+\gamma\overline{C}^T\overline{C}=0 \tag{30c}$$

$$\Phi^TL_i+L_i\Phi^T+L_{i+1}=0,\ i=1,2,\ldots,N \tag{30d}$$

$$\Phi^TL_{N+1}+L_{N+1}\Phi^T+x^0x^{0^T}=0 \tag{30e}$$

and the final cost becomes

$$J_F=x^{0^T}P_{N+1}x^0=tr[P_{N+1}X^0],\ X^0=x^0x^{0^T} \tag{31}$$

Proof: See [25]

For (30a)-(30e) being the necessary condition for the minimum, the computation algorithms to find the optimal gain is given by Fletcher and Powel[27]. It is noted that if $N=0$, the gain results of Theorem 1 coincides with those of the basic optimal theory [7] or [28]. Finally the optimal coefficients of the integral sliding surface can be determined by using the relation as

$$\Phi(c_i,c_0)=\Lambda+\Gamma\cdot\overline{C}^T(c_i,c_0) \tag{32}$$

or

$$c_i = a^0_{i+1} - b^0 \cdot \overline{c_{i+1}}.\ i=0,1,\ldots,n-1 \quad (33)$$

and also the equivalent control of (19) is selected at the same time. In [7] as one of the optimal VSS's, the whole output may not exhibit the optimal performance initially designed in the sliding surface. However, in the IOVSS, the optimal performance designed in the integral sliding surface with respect to the time-weighted performance index (27) by means of Theorem 1 can be completely guaranteed by solving the reaching phase problems.

As the second design of the stage of IOVSS, a control input stabilizing (5) with the robustness of the optimal performance resolved in the integral sliding surface will be discussed in the next.

2.3. Stabilizing Control Input and Stability Analysis

To establish the sliding mode on the every point of the predetermined integral sliding surface, a following class of the feedback control is employed

$$u(t) = \nu_{eq} + \Delta\nu \quad (34)$$

where ν_{eq} is the equivalent control directly determined according to the design of the integral sliding surface and $\Delta\nu$ is the discontinuous term to cancel out uncertainties and external disturbances in order to maintain the sliding mode on pre-specified surface from a given x^0 to origin. Also, ν_{eq} governs the main sliding dynamics to be optimal for (27) and $\Delta\nu$ is chosen as

$$\Delta\nu = \left\{ \Psi_o \cdot z + \sum_{i=1}^{n} \Psi_i \cdot x_i + \delta \cdot sgn(s) + \varkappa \cdot s \right\} \quad (35)$$

where $$\Psi_0 = \begin{cases} \alpha_0 > 0 & \text{for } s(x,z)\cdot z > 0 \\ \beta_0 > 0 & \text{for } s(x,z)\cdot z < 0 \end{cases} \quad (36a)$$

$$\Psi_i = \begin{cases} \alpha_i > \left(\dfrac{(a_i^+ - a_i^0) + \max\left\{ \begin{matrix} [(c_{i-1} - a_i^0)(b^0 - b^-)/b^0] \\ ,[(c_{i-1} - a_i^0)(b^0 - b^+)/b^0] \end{matrix} \right\}}{b^-} \right) \\ \beta_i < \left(\dfrac{(a_i^- - a_i^0) + \min\left\{ \begin{matrix} [(c_{i-1} - a_i^0)(b^0 - b^-)/b^0] \\ ,[(c_{i-1} - a_i^0)(b^0 - b^+)/b^0] \end{matrix} \right\}}{b^-} \right) \end{cases}$$

$$\begin{matrix} \text{for} & s(x,z)\cdot x_i > 0 \\ \text{for} & s(x,z)\cdot x_i < 0 \end{matrix} \quad i=1,2,\ldots,n \quad (36b)$$

$$\delta = \begin{cases} \zeta > \{ d^-/b^- & \text{for } s(x,z) > 0 \} \\ \xi < \{ -d^+/b^- & \text{for } s(x,z) < 0 \} \end{cases} \quad (36c)$$

$$\varkappa > 0 \quad (36d)$$

The fourth term of the right hand side of (35) can help for the system to approach more closely to the integral sliding surface. The control is designed in two steps, i.e. choice of the integral sliding surface and discontinuous gain selection in (35) by using (36a)-(36d). The former is the performance design and the latter is the robustness design. In order to posses the robustness of the optimal performance against the lumped uncertainties of (6), the control input should satisfy the existence condition of the sliding mode. In general, the well-known existence condition of the sliding mode is

$$\lim_{s \to 0} s \cdot \dot{s} < 0 \quad (37)$$

For the control input such as (34) having the continuous part, the rigorous proof for the existence condition of the sliding mode is not yet developed. In [28], the additional assumption on the lumped uncertainties is introduced. The existence of the sliding mode for (34) is investigated together the stability of the closed loop system through Theorem 2 without any additional assumption on the uncertainties

Theorem 2: *The control strategy (34) stabilizes (5a) with property of the sliding mode on the integral-augmented sliding surface (16) from a given initial condition to origin in (n+1)-th state space provided that (22) is asymptotically stable.*

Proof: At an initial point($t=0$) and the origin ($t=\infty$), the following is satisfied

$$s(x^0, z^0) = 0 \quad \text{and} \quad s(0,0) = 0 \quad (38)$$

The initial point and origin are included to the integral-augmented sliding surface.

Take a Lyapunov candidate function as

$$V(x,z) = 1/2 s^2(x,z) \quad (39)$$

Differentiating (39) with respect to time lead to

$$\dot{V}(x,z) = s(x,z)\cdot\dot{s}(x,z) \tag{40}$$

From (1) and (16), the derivative of $s(x,z)$ becomes

$$\dot{s}(x,z) = \sum_{i=1}^{n} c_{i-1}\cdot x_i - \sum_{i=1}^{n} a_i(t)\cdot x_i + b(t)\cdot u(t) + d(t) \tag{41}$$

From (34) and (35), it follows

$$\begin{aligned}\dot{s}(x,z) =& \sum_{i=1}^{n} c_{i-1}\cdot x_i - \sum_{i=1}^{n} a_i^0\cdot x_i + b^0\cdot \nu_{eq} \\ &- \sum_{i=1}^{n} \Delta a_i(t)\cdot x_i - (\Delta b/b^0)\cdot \sum_{i=1}^{n}(c_{i-1}-a_i^0)\cdot x_i \\ &- b(t)\cdot\left(\sum_{i=1}^{n}\Psi_i\cdot x_i + \Psi_z\cdot z(t)\right) \\ &- b(t)\cdot\delta\cdot sgn(s(x,z)) + d(t) - b(t)\cdot\kappa\cdot s(x,z)\end{aligned} \tag{42}$$

and

$$\begin{aligned}\dot{s}(x,z) =& -\sum_{i=1}^{n}\Delta a_i(t)\cdot x_i \\ &-(\Delta b/b^0)\cdot \sum_{i=1}^{n}(c_{i-1}-a_i^0)\cdot x_i \\ &- b(t)\cdot\left(\sum_{i=1}^{n}\Psi_i\cdot x_i + \Psi_z\cdot z(t)\right) \\ &- b(t)\cdot\delta\cdot sgn(s(x,z)) + d(t) \\ &- b(t)\cdot\kappa\cdot s(x,z)\end{aligned} \tag{43}$$

and

$$\begin{aligned}\dot{s}(x,z) =& -b(t)\cdot\Psi_0\cdot z(t) \\ &- \sum_{i=1}^{n}\left\{\begin{array}{c}\Delta a_i(t)+\Delta b/b^0 \\ \cdot(c_{i-1}-a_i^0)+b(t)\cdot\Psi_i\end{array}\right\}\cdot x_i \\ &- b(t)\cdot\delta\cdot sgn(s(x,z)) + d(t) \\ &- b(t)\cdot\kappa\cdot s(x,z)\end{aligned} \tag{44}$$

At this point, it is noted that the original control problems is converted to the stabilizing problems against Δa, Δb, and $d(t)$, which means that the robustness problem is separated from the performance design. Finally, using the (44) and (36a)-(36d), the following equation can be derived

$$\dot{V}(x,z) = s(x,z)\cdot\dot{s}(x,z) < -b^{-}\cdot\kappa s^2(x,z) \tag{45}$$

and represents that $\dot{V}(x,z) < 0$ for all times and completes the proof.

Because of Theorem 2, for the control strategy (34) with (16), the controlled system can slide from a given initial condition to origin with the ideal sliding dynamics (22). Therefore, the optimal output previously designed to the ideal sliding dynamics (22) by Theorem 1 becomes the real output despite of the existence of the parameter variations and disturbances by means of the property of the sliding mode, which is also one important issue in the area of the optimal control. The previous approaches of the optimal design of the sliding surface mentioned in the introduction basically use the classical optimal theory. Moreover, their designed optimal performance is not globally guaranteed because of the reaching phase. In the IOVSS algorithm of this study, the VSS theory is well combined with the advanced optimal control theory in order to take the advantages of both approaches for uncertain plants. To show the explained effectiveness of the algorithm, an example will be presented.

III. Simulation Studies

Consider an uncertain following plant:

$$\begin{aligned}\dot{x}(t) =& \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1\pm0.3 & 2\pm0.6 & 3\pm0.9\end{bmatrix} \\ &\cdot x(t) + \begin{bmatrix}0\\0\\5\pm1.5\end{bmatrix}\cdot u(t) + \begin{bmatrix}0\\0\\\pm70.0\end{bmatrix} \\ \dot{z}(t) =& x_1(t) \\ y =& x_1(t)\end{aligned} \tag{46}$$

where '±' means the maximum and minimum variations of parameters. An initial condition $x^0=[2\ 1\ 0]^T$ for (46) is given. To show the robustness of the IOVSS

표 1 공칭값 과 $case\,i$와 $case\ ii$의 조건 값

Table 1 Nominal value and conditions for $case\,i$ and $case\ ii$

	a_1	a_2	a_3	b	$d(t)$
case i	−1.3	−2.6	−3.9	6.5	−70.0
case ii	−0.7	−1.4	−2.1	4.5	+70.0
nominal	−1.0	−2.0	−3.0	5.0	0.0

표 2 슬라이딩 면의 최적 계수와 폐루프의 극
Table 2 Optimal coefficients of integral sliding surface and its closed loop poles

Algorithm		c_0	c_1	c_2	c_3	closed loop eigenvalues	z^0
IOVSS	$N=0$	158.117	109.258	21.943	1	-15.583, -3.180 ± j1.842	-1.521
IOVSS	$N=1$	131.738	73.302	13.230	1	-3.193,-5.018 ±j 4.009	-1.213
IOVSS	$N=2$	108.363	57.7946	11.724	1	-4.064,-3.830 ±3 .464	-1.175
IOVSS	$N=3$	96.105	51.710	11.195	1	-4.412,-3.392 ±j 3.265	-1.193
IOVSS	$N=4$	90.156	49.385	11.026	1	-4.507-3 .260 ±3. 063	-1.142
IOVSS	$N=5$	87.951	48.898	11.041	1	-4.534,-3.253 ±j 2,969	-1.238
conventional		--	20	10	1	0,-10	--

algorithm, the conventional VSS and proposed IOVSS with (16) and (34) will be comparatively designed for the two cases of the different condition on $a_i(t)$, $b(t)$, and $d(t)$ as in Table 1.

The first design step, the weighting matrices, Q and γ in (27) are chosen as

$$Q=\begin{bmatrix} 100 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \gamma=0.1 \tag{47}$$

Then, by Theorem 1, the optimal coefficients of the integral sliding surface can be obtained for $N=0,1,\ldots,5$ and summarized in Table 2 with its closed loop eigenvalues

표 3 제어입력의 스위칭 이득
Table 3 Switching gains for control inputs

VSS		z	x_1	x_2	x_3	δ	χ
conventional	α_i	--	3.6	26.3	3.8	23.8	--
conventional	β_i	--	-23.6	-2.3	-3.8	-23.8	
IOVSS	α_i	0.05	6.5	9.2	3.0	23.8	0.2
IOVSS	β_i	-0.05	-6.5	-9.2	-3.0	-23.8	

of the ideal sliding dynamics (22) and the initial values for the integral of (17). In turn, the switching gain in (35) are chosen as in Table 3 for the conventional VSS and IOVSS from the feasible set satisfying the inequalities of (36a)-(36d).

The computer simulation studies have been carried out using the sampling interval of 2 $[msec]$. The simulation results of the conventional VSS and IOVSS are shown in Fig. 1 through Fig. 6. Fig. 1 shows the outputs of the three states by the conventional VSS, x_1 is depicted in (a), x_2 in (b), and x_3 in (c) under the two conditions in Table 1. As can be seen, all the outputs in each state are disturbed as change of the parameter conditions, which implies the robustness of the whole output is not guaranteed due to the reaching phase. Hence, the output can not be predictable. Fig. 2 shows the output states when $N=0$ for the same initial conditions and $z^0=-1.521$ under the two case conditions, x_1 is depicted in (a), x_2 in (b), x_3 in (c), and z in (d). In spite of the existence of the parameter variations and disturbances, the outputs in each state are exactly same, which implies that complete robustness can be obtained by the IOVSS algorithms. Therefore, the output can be predictable. The phase trajectories of x_3 vs. x_2 and the ideal sliding surfaces projected to that plane by the conventional VSS are shown in Fig. 3. The expected reaching phases exist and the trajectories are disturbed during the reaching phase. Only after touching the sliding surface, the outputs are robust. By the IOVSS, the phase trajectories of x_3 vs. x_2 and the ideal sliding surfaces projected to that plane are shown in Fig. 4. There exists no reaching phase, but the controlled system slide from the given initial condition to origin because the integral sliding surface is defined from the given initial state to origin.

In the IOVSS, the predictability of output and the effect of N in (27) to the output is investigated. Fig. 5 shows the three states predicted by the solutions of the ideal sliding dynamics (22) for the given initial conditions as one increases N in (27) from 0 to 5. Fig. 6 shows the real state outputs by the IOVSS for case i condition. The predicted output and real output are exactly equal, except little difference in x_3 because of the chattering of the input. Thus, not only the output itself is predetermined, but the output can be precisely predicted. Moreover, by increasing N in (27) if the performance when $N=0$ is not satisfied, the output performance can be improved by means of the advanced optimal algorithm with the time-multiplied performance index.

From the simulation results, the IOVSS algorithm perfectly solves the reaching phase problems, so the robustness for the whole trajectory is guaranteed, since the optimal performance designed by the advanced optimal theory in the sliding dynamics is preserved, the prediction of output is feasible.

VI. Conclusions

In this paper, the design of the IOVSS is presented for the prescribed output control of uncertain SISO systems under persistent disturbances. This algorithm basically concerns with removing the problems of the reaching phase and combines with the advanced optimal control theory. By means of the integral sliding surface and Theorem 2, the reaching phase is completely removed, and the integral sliding surface can be defined from a given initial state to origin. The ideal sliding dynamics of the integral sliding surface is obtained in the form of the state equation and designed in the sense of the optimal by using the advanced optimal control theory in Theorem 1 reflected in the design of the integral sliding surface and equivalent control input. The corresponding control input is selected in order to generate the sliding mode on the predetermined integral sliding surface. As a result, the whole sliding output from a given initial state to origin is completely safeguarded against persistent disturbances. Moreover the prediction/predetermination of output is enabled due to the property of the sliding mode, whereas it is difficult to solve the problems of the performance robustness in the previous optimal VSS's. Through the illustrative example, the usefulness of the algorithm is shown. In the IOVSS, the advanced optimal theory is effectively incorporated to take the advantages of both algorithms. Finally, the attractive performance of the IOVSS are pointed out in view of no reaching phase, complete robustness, output prediction/predetermination, separation of the performance design and robustness problem.

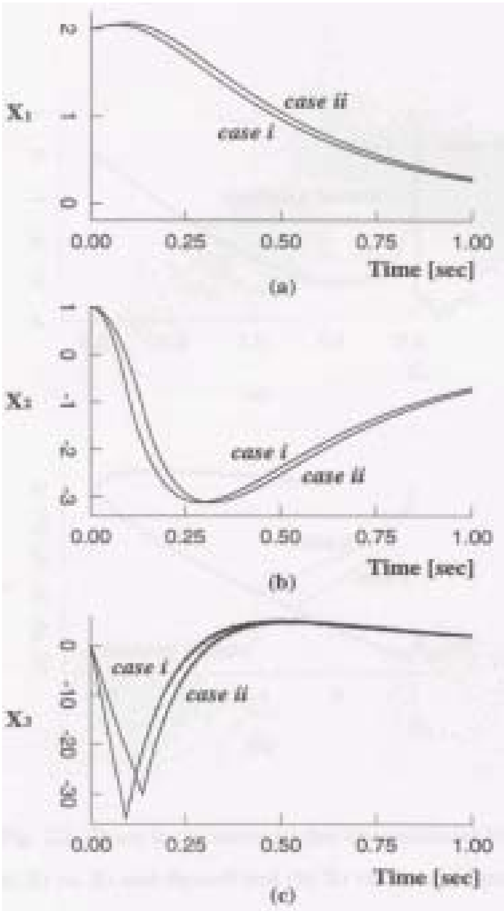

그림 1 기존 VSS에 의한 상태변수 응답

FIg. 1 State output responses by the conventional VSS (a) x_1 (b) x_2 (c) x_3 for case i and case ii

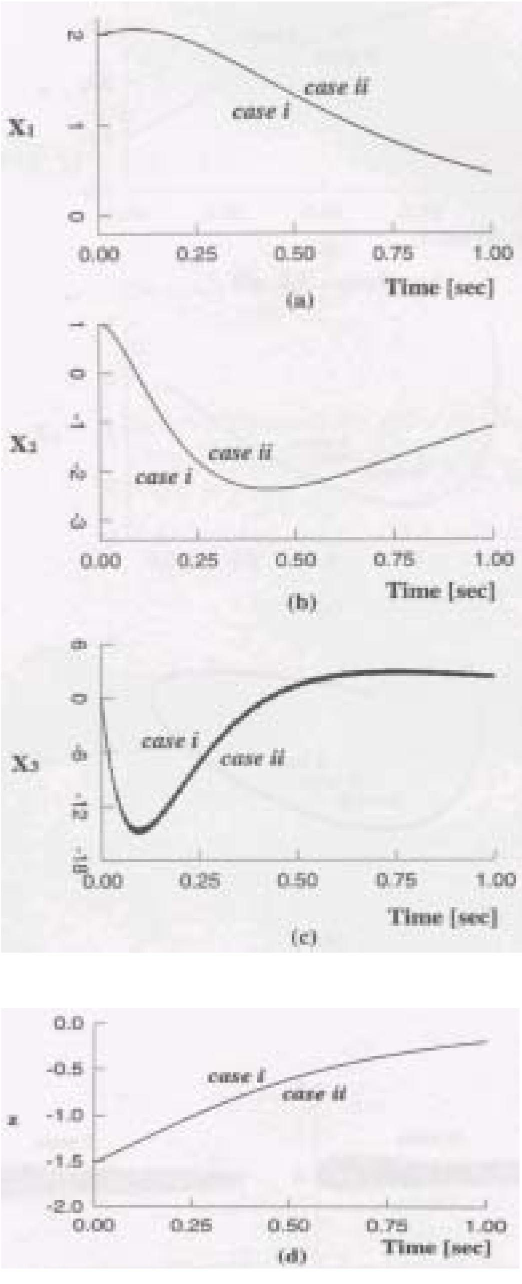

그림 2 제안된 IOVSS에 의한 상태변수 응답
FIg. 2 State output responses by the IOVSS
(a) x_1 (b) x_2 (c) x_3 (d) z for case i and case ii

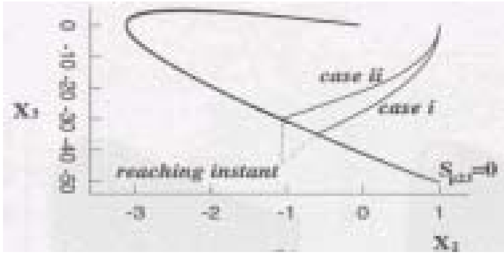

그림 3 기존 VSS에 의한 투영된 상궤적
Fig. 3 Projected phase trajectories by the conventional VSS

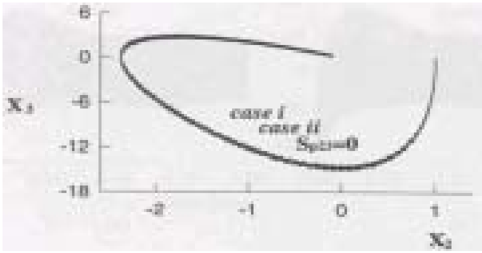

그림 4 제안된 IOVSS에의한 투영된 상궤적
Fig. 4 Projected phase trajectories by the IOVSS

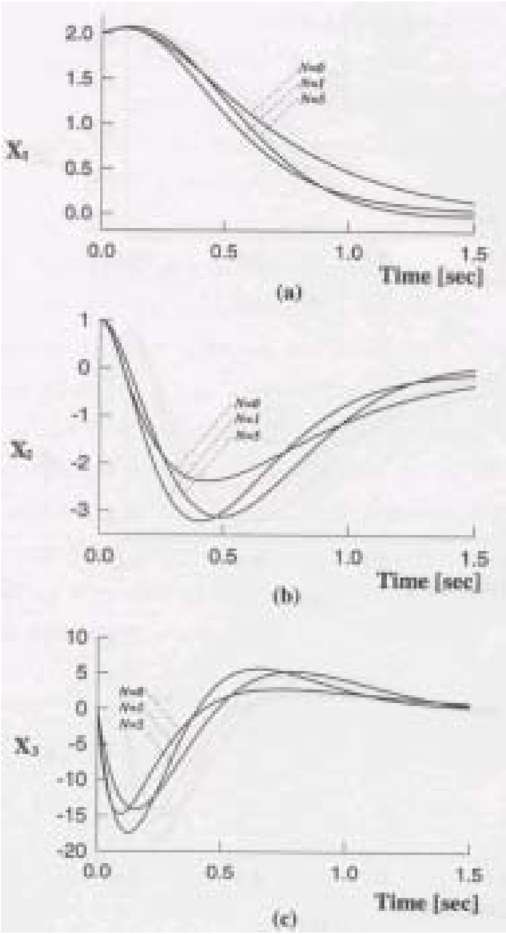

그림 5 제안된 IOVSS의 이상 슬라이딩 동특성으로 예측된 상태변수 출력
Fig. 5 Predicted state outputs by the ideal sliding dynamics of the IOVSS

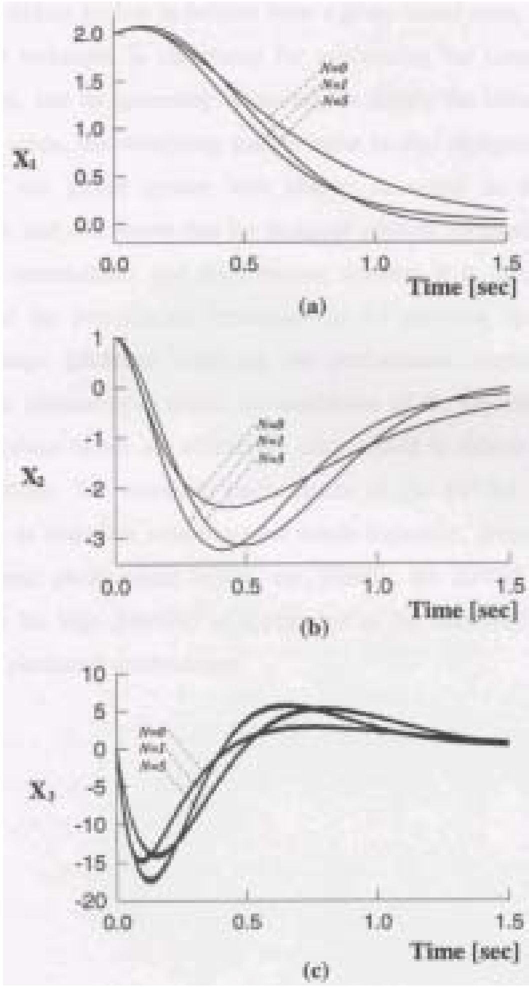

그림 6 제안된 IOVSS의 상태변수 실제 출력
Fig. 6 Real state outputs by the IOVSS

Appendix: Proof of Lemma 1

Let $x(t),\ t\geq 0$ satisfies (3a), then

$$\dot{V}(x,t)\equiv\frac{dV(x,t)}{dt}$$

$$=\frac{\partial V(x,t)}{\partial t}+\nabla V(x,t)\cdot(\Lambda x(t)+\Gamma V(t)) \qquad (48)$$

Hence, it follows from (11d) that

$$\dot{V}(x,t)\ <\ 0,\quad x(t)\neq 0,\quad t\geq 0 \qquad (49)$$

Thus, from (11a)-(11d) and (48), it follows that $V(\cdot)$ is a Lyapunov function, which proves the global asymptotic stability for the solution $x(t)=0,\ t\geq 0$ by the uniform continuity of $\dot{V}(x,t)$ based on Lyapunov-like Lemma in pp.124-127 of [29]. Consequently, $x(t)\to 0$ as $t\to\infty$ for all initial condition x^0. Equation (48) implies that

$$-\dot{V}(x,t)+\frac{\partial V(x,t)}{\partial t} + \nabla V(x,t)\cdot(Ax(t)+\Gamma\nu(t))=0 \tag{50}$$

and hence, by (11e)

$$L(x,\nu,t)=-\dot{V}(x,t)+L(x,\nu,t)+\frac{\partial V(x,t)}{\partial t} + \nabla V(x,t)\cdot(Ax(t)+\Gamma\nu(t)) \tag{51}$$

Integrating over $[0\quad t]$ leads to

$$\int_0^t L(x(\omega),\nu,\omega)d\omega = -V(x,t)+V(x^0,0) \tag{52}$$

Now, letting $t\to\infty$ and noting that $V(x,t)\to 0$ yields (13). To prove (14), let $\nu(\cdot)\in U(x)$, and let $x(\cdot)$ be the solution to (3a). Then, using (51) and the fact that $\nu(\cdot)\in U(x^0)$, along with (11f) and (13), one can obtain

$$\begin{aligned} J(x,\nu(\cdot)) &= \int_0^\infty \left\{ \begin{array}{l} -\dot{V}(x,t)+L(x,\nu,t) \\ +\frac{\partial V(x,t)}{\partial t}+\nabla V(x,t) \\ \cdot(Ax(t)+\Gamma\nu(t)) \end{array} \right\} dt \\ &= -\lim_{t\to\infty} V(x,t)+V(x^0,0) \\ &\quad + \int_0^\infty \left\{ H(x,\nabla V(x,t),\nu,t)+\frac{\partial V(x,t)}{\partial t} \right\} \\ &\geq V(x^0,0) \\ &= J(x^0,\phi(\cdot)) \end{aligned} \tag{53}$$

which yields (14).

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