Output-Feedback Input-Output Linearizing Controller for Nonlinear System Using Backward-Difference State Estimator

후방차분 상태 추정기를 이용한 비선형 계통의 입출력 궤환 선형화 제어기

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Abstract

This paper describes the design of a robust output-feedback controller for a single-input single-output nonlinear dynamical system with a full relative degree. While all the previous research works on the output-feedback control are based on dynamic observers, a new state estimator which uses the past values of the measurable system output is proposed. We name it backward-difference state estimator since the derivatives of the output are estimated simply by backward difference of the present and past values of the output. The disturbance generated due to the error between the estimated and real state variables is compensated using an additional robustifying control law whose gain is tuned adaptively. Overall control system guarantees that the tracking error is asymptotically convergent and that all signals involved are uniformly bounded. Theoretical results are illustrated through a simulation example of inverted pendulum.

요 약

본 논문은 단일입력 단일출력 비선형 계통에 대해서 강인한 출력궤환 제어기를 제시한다. 이전의 출력궤환 비선형 제어기가 모두 동적인 관측기 기반으로 설계된 반면 본 논문에서는 출력의 과거값들만을 이용하여 상태변수를 추정하는 새로운 방식을 제안하고 이를 후방차분 상태추정기라 명했다. 실제 상태변수값과 추정치와의 오차를 보상하기 위해서 제어입력에 강인제어항을 추가하였고 그것의 이득을 자동으로 조정하는 적응 알고리듬을 채택했다. 전체 폐루프 시스템은 출력 추종 오차가 점근적으로 안정하도록 그리고 모든 신호가 유계이도록 제어입력과 적응법칙이 설계된다. 제시된 제어기를 역진자 계통에 적용한 모의실험을 통해서 성능을 검증하였다.

keywords: input-output linearization, nonlinear system, state estimator

Ⅰ. Introduction

Nonlinear control has emerged as a research area of rapidly increasing activity. Especially, the theory of explicitly linearizing the input-output response of nonlinear systems to linear systems using the state feedback has received great attention [1]-[5]. Especially, the output-feedback control of nonlinear system has been an extensively researched issue

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since, in many practical systems, the full state variables are not available.

The output-feedback control schemes of nonlinear systems have been based on various dynamic state observers. In [4], [5], a dynamic observer for the systems which can be transformed into the output-feedback form is firstly built and then an input-output linearizing controller are designed based on the observer. The adaptive versions of these results are also proposed using filtered transformation or error augmentation method [6]-[8]. However, the method is restricted on the systems which can be transformed to output-feedback form where the system nonlinearities are the functions of system output only. Another form of observer which is known as high-gain observer [9]-[11] is widely adopted in the output-feedback scheme since its dynamics are linear and independent upon system dynamics. This high-gain observer is relatively simpler in its structure than proposed in [6]-[8]. However, due to a peaking phenomenon, additional saturating scheme is to be employed.

In this paper, a new state estimator is proposed for a single-input single-output (SISO) feedback linearizable nonlinear system with full relative degree. The proposed observer uses the present and past values of the measurable system output to estimate the time derivatives of the system output using backward difference approximation. We call this observer as backward difference state estimator (BDSE). The disturbances generated due to the error between the estimated and real state variables are compensated for by an additional sliding mode-like robustifying control law whose gain is tuned adaptively. Overall output-feedback control system guarantees that the tracking error is asymptotically convergent and that all signals involved are uniformly bounded. Theoretical results are illustrated through a simulation example of inverted pendulum system.

II. Controller Design and Stability Analysis

In this section, we first set up control objective, and then show how to design an input-output linearizing controller based on the BDSE to achieve the objectives.

2.1 Problem formulation

Consider the nth-order nonlinear system of the form

$$\begin{aligned} x^{(n)} &= f(x, \dot{x}, \cdots, x^{(n-1)}) + g(x, \dot{x}, \cdots, x^{(n-1)})u \\ y &= x \end{aligned} \tag{1}$$

where f and g are continuous functions, $u \in R$ and $y \in R$ are the input and output of the system, respectively, and $\mathrm{x} = [x_1\ x_2 \cdots x_n]^T = [x\ \dot{x} \cdots x^{(n-1)}]^T \in R^n$ is the state vector of the system. It is assumed that only the system output y is measurable. For (1) to be controllable, it is required that $g(\mathrm{x}) \neq 0$ for all $\mathrm{x} \in R^n$. Since $g(\mathrm{x})$ is continuous, without loss of generality, we assume that $g(\mathrm{x})$ is positive for all $\mathrm{x} \in R^n$. The control objective is to force the output $y(t)$ to track a given bounded reference signal $y_d(t)$, under the constraint that all the state variables must be stabilized.

2.2 State Estimator

All the previous research results on the output-feedback controller are based on the dynamic observer of the state-variables. In this paper, we propose a static state observer which estimates the state variables using the past values of the output.

The followings are from the finite difference theory [12]. From the definition of the derivative of the output

$$\dot{x}_1(t) = \lim_{d \to 0} \frac{x_1(t) - x_1(t-d)}{d} \tag{2}$$

where $d > 0$, we can imply, that for any $\varepsilon > 0$

there exists a $\delta(\varepsilon)$ such that $\forall d < \delta$,

$$\left| \dot{x}_1(t) - \frac{x_1(t) - x_1(t-d)}{d} \right| < \epsilon \tag{3}$$

which means that at arbitrary time t the $\dot{x}_1 = x_2$ can be approximated up to arbitrary accuracy by its values at two subsequent points. Thus, we define $\hat{x}_2(t) = (x_1(t) - x_1(t-d))/d$. Similarly, to compute the second derivative,

$$\begin{aligned} \ddot{x}_1(t) &= \lim_{d\to 0} \frac{\dot{x}_1(t) - \dot{x}_1(t-d)}{d} \\ &= \lim_{d\to 0} \frac{1}{d}\left[\lim_{d\to 0} \frac{x_1(t) - x_1(t-d)}{d} \right. \\ &\quad \left. - \lim_{d\to 0} \frac{x_1(t-d) - x_1(t-2d)}{d} \right] \end{aligned} \tag{4}$$

which implies that for any $\varepsilon > 0$ there exists a $\delta(\varepsilon)$ such that $\forall d < \delta$,

$$\left| \ddot{x}_1(t) - \frac{\hat{x}_2(t) - \hat{x}_2(t-d)}{d} \right| < \epsilon \tag{5}$$

Thus, we define $\hat{x}_3(t) = (x_2(t) - x_2(t-d))/d$. By induction, we propose the following BDSE:

$$\hat{\mathbf{x}} = [x_1(t)\ \hat{x}_2(t) \cdots \hat{x}_n(t)]^T \tag{6}$$

where

$$\begin{aligned} \hat{x}_2 &= \frac{x_1(t) - x_1(t-d)}{d} \\ \hat{x}_3 &= \frac{\hat{x}_2(t) - \hat{x}_2(t-d)}{d} \\ &\vdots \\ \hat{x}_n &= \frac{\hat{x}_{n-1}(t) - \hat{x}_{n-1}(t-d)}{d}. \end{aligned} \tag{7}$$

We can see that for any ε and time instant t there exists a δ such that the following inequality is satisfied

$$|\mathbf{x}(t) - \hat{\mathbf{x}}(t)| \le \epsilon(d),\ 0 < d < \delta. \tag{8}$$

Assumption 1. The time interval d is sufficiently small so that the following inequality holds

$$g(\mathbf{x}(t)) < 2g(\hat{\mathbf{x}}(t)) \tag{9}$$

for all $t \ge 0$

2.3 Controller Design

A control law is proposed as

$$u = \frac{1}{g(\hat{\mathbf{x}})}\left(-f(\hat{\mathbf{x}}) + y_d^{(n)} + \mathbf{k}^T\hat{\mathbf{e}} + \beta \right) \tag{10}$$

where $\mathrm{k} = [k_n \cdots k_1]^T$ is determined such that the polynomial $h(s) = s^n + k_n s^{n-1} + \cdots + k_1$ is Hurwitz, $\hat{\mathrm{e}} = \mathrm{x}_d - \hat{\mathrm{x}}$, $\mathrm{x}_d = [y_d\ \dot{y}_d \cdots y_d^{(n-1)}]^T$, and β is a robustness term which compensates for the disturbances due to the error between the state variables and their estimates.

Let $e = y_d - y$ and $\mathrm{e} = \mathrm{x}_d - \mathrm{x}$. Substituting (10) into (1) can yield the following error dynamics.

$$\begin{aligned} e^{(n)} &= y_d^{(n)} - y^{(n)} \\ &= -\mathbf{k}^T\hat{\mathbf{e}} - (f(\mathbf{x}) - f(\hat{\mathbf{x}})) \\ &\quad - (g(\mathbf{x}) - g(\hat{\mathbf{x}}))u - \beta. \end{aligned} \tag{11}$$

or

$$\begin{aligned} \dot{\mathbf{e}} &= A\mathbf{e} + \mathbf{b}\Big[\mathbf{k}^T(\mathbf{e} - \hat{\mathbf{e}}) - (f(\mathbf{x}) - f(\hat{\mathbf{x}})) \\ &\quad - (g(\mathbf{x}) - g(\hat{\mathbf{x}}))u - \beta\Big]. \end{aligned} \tag{12}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & & & \vdots & \\ -k_n & -k_{n-1} & -k_{n-2} & \cdots & -k_1 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}.$$

Since A is a stable matrix, there exist the positive-definite symmetric matrixes P and Q satisfying

$$A^T P + PA = -Q. \tag{13}$$

For the stability analysis, we need the following inequality

$$|h(\mathbf{x}) - h(\hat{\mathbf{x}})| \le l_h\epsilon,\ h = f, g \tag{14}$$

where $l_h, h = f, g$ are Lipschitz constants. The proof of (14) is straightforward from the smoothness of the functions f and g and (8).

Since, in general, the constants ε is unknown and it is hard to calculate the Lipschitz constants l_f and

l_g even if the functions f and g are known, we adopt an adaptive scheme for the lumped constant defined as

$$\psi^* = \frac{\max\{|\mathbf{k}|\epsilon + l_f\epsilon, l_g\epsilon\}}{1-\lambda} \tag{15}$$

where λ is the constant satisfying

$$\frac{|g(\mathbf{x}) - g(\hat{\mathbf{x}})|}{g(\hat{\mathbf{x}})} \le \lambda < 1,\ \forall t \ge 0. \tag{16}$$

It is obvious that there exists the constant λ in (16) from Assumption 1. We denote the estimate of ψ^* as $\hat{\psi}$ in what follows.

2.4 Stability Analysis

The following is the main theorem of this paper.

Theorem 1. Consider the affine nonlinear system (1) and the control input (11) with the robustifying control term as

$$\beta = \hat{\psi} s\,\mathrm{sgn}(\mathbf{e}^T P\mathbf{b}) \tag{17}$$

where $s = 1 + |u_a|$ and

$$u_a = \frac{1}{g(\hat{\mathbf{x}})}\left(-f(\hat{\mathbf{x}}) + y_d^{(n)} + \mathbf{k}^T\hat{\mathbf{e}}\right) \tag{18}$$

The update law for $\hat{\psi}$ is determined as

$$\dot{\hat{\psi}} = \gamma s\left|\mathbf{e}^T P\mathbf{b}\right| \tag{19}$$

where $\gamma > 0$ is the adaptation rate at designers' deposal. Then, the tracking error $\mathbf{e}$ is UAS and the $\hat{\psi}$ is bounded.

proof. Consider the Lyapunov function

$$V = \frac{1}{2}\mathbf{e}^T P\mathbf{e} + \frac{1-\lambda}{2\gamma}\tilde{\psi}^2 \tag{20}$$

where $\tilde{\psi} = \hat{\psi} - \psi^*$. Differentiating V along the solution of (13), we obtain

$$\begin{aligned}\dot{V} &= -\frac{1}{2}\mathbf{e}^T Q\mathbf{e} + \mathbf{e}^T P\mathbf{b}\Big[\mathbf{k}^T(\mathbf{e} - \hat{\mathbf{e}}) - (f(\mathbf{x}) - f(\hat{\mathbf{x}})) \\ &\quad - (g(\mathbf{x}) - g(\hat{\mathbf{x}}))u - \beta + \frac{1-\lambda}{\gamma}\tilde{\psi}\dot{\hat{\psi}} \\ &\le -\frac{1}{2}\mathbf{e}^T Q\mathbf{e} + |\mathbf{e}^T P\mathbf{b}|\left(|\mathbf{k}|\epsilon + l_f\epsilon + l_g\epsilon|u_a|\right) \\ &\quad -\mathbf{e}^T P\mathbf{b}\left(1 + \frac{g(\mathbf{x}) - g(\hat{\mathbf{x}})}{g(\hat{\mathbf{x}})}\right)\beta \\ &\quad + \frac{1-\lambda}{\gamma}\tilde{\psi}\dot{\hat{\psi}}\end{aligned} \tag{21}$$

Using (17), (19) and the following inequality

$$1 - \lambda \le 1 + \frac{g(\mathbf{x}) - g(\hat{\mathbf{x}})}{g(\hat{\mathbf{x}})} \tag{22}$$

we can further describe the inequality (24) as

$$\begin{aligned}\dot{V} &\le -\frac{1}{2}\mathbf{e}^T Q\mathbf{e} + |\mathbf{e}^T P\mathbf{b}|\psi^*(1-\lambda)s \\ &\quad -|\mathbf{e}^T P\mathbf{b}|(1-\lambda)s\hat{\psi} + \frac{1-\lambda}{\gamma}\tilde{\psi}\dot{\hat{\psi}} \\ &= -\frac{1}{2}\mathbf{e}^T Q\mathbf{e} - |\mathbf{e}^T P\mathbf{b}|\tilde{\psi}(1-\lambda)s + \frac{1-\lambda}{\gamma}\tilde{\psi}\dot{\hat{\psi}} \\ &= -\frac{1}{2}\mathbf{e}^T Q\mathbf{e}\end{aligned} \tag{23}$$

Eqs. (20) and (23) guarantee that $|\mathbf{e}|$, and $\tilde{\psi}$ are bounded since $V(t)$ is nonincreasing. Because all the variables in the right-hand side of (12) are bounded, $\dot{\mathbf{e}}(t)$ is also bounded. Integrating both sides of (23) and after some manipulations, we have

$$\int_0^\infty |\mathbf{e}(t)|^2\,dt \le \frac{2}{\lambda_{min}(Q)}\left(V(0) - V(\infty)\right). \tag{24}$$

Since the right side of (24) is bounded, $|\mathbf{e}(t)| \in L_2$. Using Barbalat's lemma [13], we have $\lim_{t\to 0}|\mathbf{e}| = 0$. This completes the proof.

Remark 1. In many applications, the $sgn(\cdot)$ in (17) is replaced by a saturation function of the form

$$\mathrm{sat}\left(\mathbf{e}^T P\mathbf{b}\right) = \begin{cases}\mathrm{sgn}\left(\mathbf{e}^T P\mathbf{b}\right) & \text{if } \left|\mathbf{e}^T P\mathbf{b}\right| \ge \epsilon_s \\ \mathbf{e}^T P\mathbf{b}/\epsilon_s & \text{if } \left|\mathbf{e}^T P\mathbf{b}\right| < \epsilon_s\end{cases} \tag{25}$$

or a smooth function $\tanh\left(\frac{\bar{\mathbf{e}}^T P\mathbf{b}}{\varepsilon_s}\right)$ where $\varepsilon_s > 0$ is a small design constant in order to remedy the control chattering.

III. Simulation Example

To illustrate the control procedure and

performance we apply the proposed robust adaptive controller to control the inverted pendulum to track a sinewave trajectory. The dynamic equations of the system are given by [2].

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{G\sin(x_1) - \frac{mlx_2^2\cos(x_1)\sin(x_1)}{m_c+m}}{l\left(\frac{4}{3} - \frac{m\cos^2(x_1)}{m_c+m}\right)} + \frac{\frac{\cos(x_1)}{m_c+m}}{l\left(\frac{4}{3} - \frac{m\cos^2(x_1)}{m_c+m}\right)}u \end{aligned} \tag{26}$$

where $x_1=\theta$ represents the angle of the pendulum, x_2 represents the angular velocity, $G=9.8\,m/s^2$ acceleration due to gravity, m_c is the mass of cart, m is the mass of pole, l is the half length of pole, and u is the applied force (control). We choose $m_c=1kg$, $m=0.1kg$ and $l=0.5m$ in the following simulations. Clearly, (26) is of the same form of (1), thus our control scheme can be implemented on this system. We also choose the reference signal $y_m(t)=\frac{\pi}{30}\sin(\ t)$ in the following simulations.

The design parameters are specified as follows. Let $k_1=2, k_2=1$ (so that $s^2+k_1s+k_2$ is stable), and $Q=I$, then we have the Lyapunov equation (14) and obtain

$$P = \begin{bmatrix} 1.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix} \tag{27}$$

which is positive-definite with $\lambda_{\min}=0.2929$. We also choose $\gamma=0.1$ and the initial values as $x(0)=[-0.05\ 0]^T$, $\phi(0)=0$.

For a comparison purpose, two simulations are performed with the values of $d=0.1$sec and $d=0.01$sec. The results of the first simulation with $d=0.1$sec are illustrated in Fig.1-2 and with $d=0.01$sec in Fig. 3-4. From the results, it can be inferred that the system output tracks the desired output well by the proposed controller. Comparing the two controllers also reveals that tracking performance is slightly degraded in the former case, i.e., with the bigger value of d The smaller the value of d is, the better the tracking performance is, which is expected.

IV. Conclusions

This paper describes the design of a robust output-feedback controller for a SISO nonlinear dynamical system with a full relative degree. The proposed state observer called BDSE estimates the unmeasurable states by backward difference of the present and past values of the output. No restrictive dynamic observer as in [6]-[11] is used. The disturbance generated due to the error between the estimated and real state variables is compensated using the additional sliding-mode control law whose gain is tuned adaptively. With the proposed BDSE and conventional input-output linearizing controller, stability analysis for the closed-loop system has been performed. Overall control system guarantees that the tracking error is asymptotically convergent and that all signals involved are uniformly bounded.

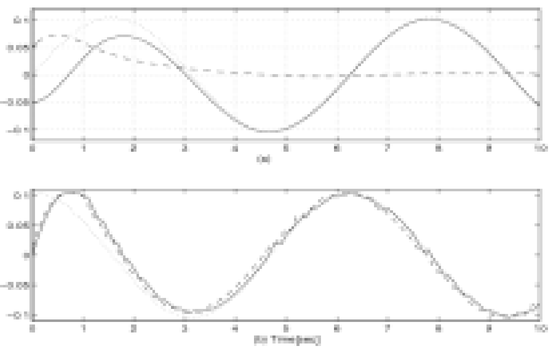

Fig. 1. Simulation results with $d=0.1$sec. (a) y (line), y_d (dotted line) and e (dashed line). (b) x_2 (line), $\dot{y}_d$ (dotted line), $\hat{x}_2$ (dashed line).

그림 1. $d=0.1$sec.인 경우의 모의실험 결과 (a) y (실선), y_d (점선) and e (쇄선). (b) x_2 (실선), $\dot{y}_d$ (점선), $\hat{x}_2$ (쇄선).

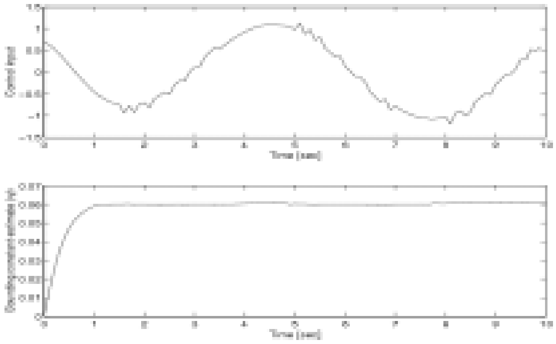

Fig. 2. Simulation results with $d=0.1$sec. (a) control input. (b) trajectory of $\hat{\psi}$

그림 2. $d=0.1$sec.인 경우의 모의실험 결과 (a) 제어입력 (b) $\hat{\psi}$의 궤적.

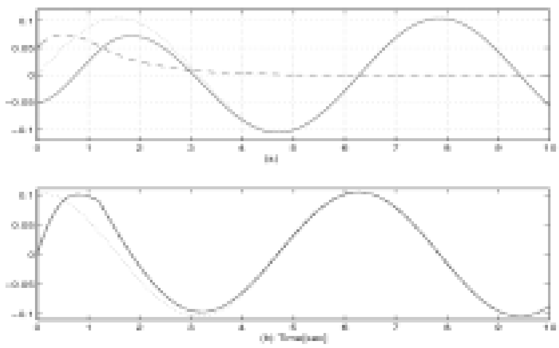

Fig. 3. Simulation results with $d=0.01$sec. (a) y (line), y_d (dotted line) and e (dashed line). (b) x_2 (line), $\bar{y}_d$ (dotted line), $\hat{x}_2$ (dashed line).

그림 3. $d=0.01$sec.인 경우의 모의실험 결과 (a) y (실선), y_d (점선) and e (쇄선). (b) x_2 (실선), $\bar{y}_d$ (점선), $\hat{x}_2$ (쇄선).

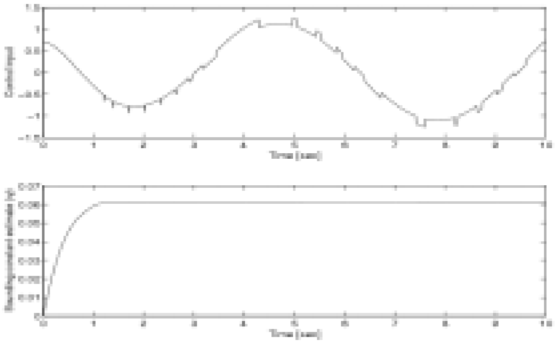

Fig. 4. Simulation results with $d=0.01$sec. (a) control input. (b) trajectory of $\hat{\psi}$

그림 4. $d=0.01$sec.인 경우의 모의실험 결과 (a) 제어입력 (b) $\hat{\psi}$의 궤적.

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